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ANALYSIS OF THE STRESS-STRAIN STATE OF A CYLINDRICAL MULTILAYER SHELL OF A PROTECTIVE STRUCTURE UNDER EXPLOSIVE LOADS

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The work describes modeling and analysis of the reaction of multilayered shell bodies at quasi-static and dynamic loads. The quasi-static and non-stationary stress-strain state of heterogeneous building elements of structures made of significantly different structural materials is investigated. A layer-by-layer strength assessment of a heterogeneous structure is performed using the Tsai-Wu strength criterion.

Testing of the applied calculation methods is performed by comparing the results of solutions obtained for two-dimensional and three-dimensional models of a multilayer cantilever beam.

The response of a cylindrical multilayer shell of a protective structure to an explosive load in a quasi-static and non-stationary setting is analyzed. The stress-strain state of the structure is described, and the strength is assessed using the Tsai-Wu criterion at the most loaded points. When modeling an explosive load in a non-stationary setting, the phases of positive and negative pressure are taken into account. The analysis of vibrations occurring in the structure under such loads is carried out. The damping of vibrations is modeled using Rayleigh damping. The dynamics of changes in displacements and stresses at selected points of the structure is described and analysed.

Keywords: multilayer structures, explosive loading, quasi-static formulation, Tsai-Wu strength criterion, blast wave profile, unsteady vibrations.

Introduction. Ensuring the structural integrity and reliability of civil defense structures is a critically important task that requires detailed research. One of the most difficult aspects of designing such structures is assessing their ability to with stand extreme dynamic effects, in particular explosive loads [12, 15, and 19]. The use of multi-layer shell elements allows you to significantly increase the energy-absorbing properties and strength of structures, but requires a deep analysis of their deformation processes under dynamic loads.

Modern approaches to calculating protective structures are based on a detailed study of the unsteady dynamic deformation of spatial bodies and shells. For example, in [13], calculation formulas for three-layer inhomogeneous cylindrical shells on an elastic base are given. The response of multi-layer shell elements of structures to unsteady loading was studied in [5, 7, 8, 16, and 18]. The propagation of non-stationary disturbances in thick-walled multilayer and functionally inhomogeneous structural elements was analyzed in [4, 9, and 17]. The influence of boundary conditions on the propagation of dynamic disturbances in elastic layers was studied in [11].

For modeling of such processes, the finite element method (FEM) [6] is widely used, which allows taking into account thermal-force loading and complex geometric and material parameters of structures [1, 2, and 17].

An important stage of strength assessment is the analysis of interlayer stresses in composite structures [10] and the use of modern strength criteria for anisotropic materials [14]. The current state of research on the response of composite materials to impact loading is described in [3].

At the moment, the issue of fast and accurate prediction of the stress-strain state of multilayer shells under explosive loads is gaining particular relevance. The need to take into account specifics of pulsed explosive loading requires further improvement of existing calculation models.

This article is devoted to the numerical study of deformation processes and strength assessment of multilayer shell elements of protective structures under the action of intense dynamic loads. The main attention is paid to the analysis of the heterogeneity of the structure and geometry of elements influence on their bearing capacity and resistance to failure under conditions of a complex stress state. The determination of the magnitude of the load from the shock wave is performed in accordance with the regulatory requirements of the DBN.

1. Input relations

Basic formulas of the Mindlin-Reisner shell theory. The Mindlin-Reisner shell theory is an extension of the classical Timoshenko-type beam theory to plates and shells and takes into account transverse shear deformations and the possibility of non-perpendicularity of the normal to the median surface after deformation.

The displacement field in the Mindlin-Reisner shell is given by**:**

$$u_1(x_1,x_2,z)=u(x_1,x_2)+z\psi_1(x_1,x_2),\ u_2(x_1,x_2,z)=v(x_1,x_2)+z\psi_2(x_1,x_2),\ u_3(x_1,x_2,z)=w(x_1,x_2), \quad (1)$$

where (u,v,w) - displacements of points of the middle surface; ψ_1,ψ_2 - angles of rotation of the normal to the median surface; (x_1,x_2) - curvilinear coordinates on the median surface; z - coordinate along the thickness of the shell.

The equations of motion for shells of small curvature are written in the form

$$\frac{\partial N_{11}}{\partial x_1}+\frac{\partial N_{12}}{\partial x_2}+\frac{1}{R_1}Q_1+X_1=\rho h\frac{\partial^2 u}{\partial t^2}, \qquad \frac{\partial N_{12}}{\partial x_1}+\frac{\partial N_{22}}{\partial x_2}+\frac{1}{R_2}Q_2+X_2=\rho h\frac{\partial^2 v}{\partial t^2},$$

$$\frac{\partial Q_1}{\partial x_1}+\frac{\partial Q_2}{\partial x_2}-\frac{N_{11}}{R_1}-\frac{N_{22}}{R_2}+X_3=\rho h\frac{\partial^2 w}{\partial t^2}, \qquad \frac{\partial M_{11}}{\partial x_1}+\frac{\partial M_{12}}{\partial x_2}-Q_1+Y_1=\rho\frac{h^3}{12}\frac{\partial^2\psi_1}{\partial t^2},$$

$$\frac{\partial M_{12}}{\partial x_1}+\frac{\partial M_{22}}{\partial x_2}-Q_2+Y_2=\rho\frac{h^3}{12}\frac{\partial^2\psi_2}{\partial t^2},$$

where N_{ij} - tensile forces; M_{ij} - bending moments; Q_i - transverse forces; R_1,R_2 - main radii of curvature of the shell; X_i,Y_i - external loads; ρ - material density; h - shell thickness.

Physical relations for an isotropic material:

$$N_{11}=\frac{Eh}{1-\nu^2}(\varepsilon_{11}+\nu\varepsilon_{22}), \quad N_{22}=\frac{Eh}{1-\nu^2}(\varepsilon_{22}+\nu\varepsilon_{11}),; \quad N_{12}=\frac{Eh}{2(1+\nu)}\gamma_{12},$$

$$M_{11}=\frac{Eh^3}{12(1-\nu^2)}(\kappa_{11}+\nu\kappa_{22}), \quad M_{22}=\frac{Eh^3}{12(1-\nu^2)}(\kappa_{22}+\nu\kappa_{11}), \quad M_{12}=\frac{Eh^3}{24(1+\nu)}\kappa_{12},$$

$$Q_1=kGh\gamma_{13}, \quad Q_2=kGh\gamma_{23},$$

where E - Young's modulus; G - shear modulus; ν - Poisson's ratio; k - shear coefficient (usually taken to be 5/6).

Geometrical relations are written as follows:

$$\varepsilon_{11}=\frac{\partial u}{\partial x_1}+\frac{w}{R_1}; \quad \varepsilon_{22}=\frac{\partial v}{\partial x_2}+\frac{w}{R_2}; \quad \gamma_{12}=\frac{\partial u}{\partial x_2}+\frac{\partial v}{\partial x_1}; \quad \kappa_{11}=\frac{\partial\psi_1}{\partial x_1};$$

$$\kappa_{22}=\frac{\partial\psi_2}{\partial x_2}; \quad \kappa_{12}=\frac{\partial\psi_1}{\partial x_2}+\frac{\partial\psi_2}{\partial x_1}; \quad \gamma_{23}=\psi_2+\frac{\partial w}{\partial x_2}; \quad \gamma_{23}=\psi_2+\frac{\partial w}{\partial x_2}.$$

When modeling multilayer shells, it is assumed that all layers of the structure are perfectly interconnected, there is no sliding and delamination at the contact boundaries, and the deformations are small. Internal forces are defined as integral characteristics of stresses over the thickness of the shell:

$$N_i=\sum_{k=1}^{n}\int_{z_{k-1}}^{z_k}\sigma_i^{(k)}\,dz, \quad Q_i=k\sum_{k=1}^{n}\int_{z_{k-1}}^{z_k}\tau_{iz}^{(k)}\,dz, \quad M_i=\sum_{k=1}^{n}\int_{z_{k-1}}^{z_k}\sigma_i^{(k)}z\,dz.$$

The relationship between the generalized force vector and the deformations of the midsurface is written as:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix}=\begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix}\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}.$$

The components of the communication matrix are defined as:

$$A_{ij}=\sum_{k=1}^{n}\bar{Q}_{ij}^{(k)}(z_k-z_{k-1}),\quad B_{ij}=\frac{1}{2}\sum_{k=1}^{n}\bar{Q}_{ij}^{(k)}(z_k^2-z_{k-1}^2),\quad D_{ij}=\frac{1}{3}\sum_{k=1}^{n}\bar{Q}_{ij}^{(k)}(z_k^3-z_{k-1}^3).$$

Here $\bar{Q}_{ij}^{(k)}$ is the transformed stiffness matrix of the layer k. For a transversely isotropic layers, the stiffness matrix components are calculated through the elastic constants of the material:

$$Q_{11}=\frac{E_1}{1-\nu_{12}\nu_{21}},\quad Q_{22}=\frac{E_2}{1-\nu_{12}\nu_{21}},\quad Q_{12}=\frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}},\quad Q_{66}=G_{12}.$$

The coordinate of the neutral surface of the multilayer shells for the case of isotropic homogeneous layers is determined by the formula

$$z_0=\frac{\sum_{i=1}^{n}E_i(z_{i-1}+z_i)(z_i-z_{i-1})}{2\sum_{i=1}^{n}E_i(z_i-z_{i-1})}. \quad (2)$$

Application of FEM to shell deformation problems. Within each finite element, the displacement field $\{u\}=[N]\{q_e\}$ is approximated through the values $\{q_e\}$ at the nodes using a matrix of shape functions $[N]$. For shells, the displacement vector takes into account both translational displacements and normal rotation angles. The relationship between the deformation vector $\{\varepsilon\}$ and nodal displacements $\{q_e\}$ is determined through the differentiation matrix $[B]$ [6]:

$$\{\varepsilon\}=[L]\{u\}=[L][N]\{q_e\}=[B]\{q_e\}, \quad (3)$$

where $[L]$ is a differentiation operator specific to the shell geometry (e.g., cylindrical or conical). For multilayer anisotropic materials, stresses $\{\sigma\}$ are related to strains through the material stiffness matrix $[D]$:

$$\{\sigma\}=[D]\{\varepsilon\}=[D][B]\{q_e\}. \quad (4)$$

In the case of thermal force loading, the temperature deformation vector is added.

According to the principle of virtual displacements, the internal work of virtual deformations is equal to the external work of virtual displacements:

$$\int_{V_e}\{\delta\varepsilon\}^T\{\sigma\}dV=\{\delta q_e\}^T\{F_e\}. \quad (5)$$

Substituting expressions (2) and (3) into (5), we obtain:

$$\{\delta q_e\}^T\left(\int_{V_e}[B]^T[D][B]dV\right)\{\delta q_e\}=\{\delta q_e\}^T\{F_e\}.$$

Since this must hold for any $\{\delta q_e\}$, we obtain the basic FEM equation for the element

$$[K_e]\{q_e\}=\{F_e\}, \quad (6)$$

where $[K_e]=\int_{V_e}[B]^T[D][B]dV$ is finite element stiffness matrix.

After ensemble of equations (6) taking into account inertia forces and damping according to d'Alembert's principle, we obtain a FEM system of solving equations:

$$[M]\{\ddot{q}\}+[C]\{\dot{q}\}+[K]\{q\}=\{F(t)\}. \quad (7)$$

Here $[M]$ is the mass matrix; $[C]$ is the damping matrix; $\{F(t)\}$ is the vector of the unsteady load (shock wave).

The damping matrix in (7) is most often formed according to Rayleigh damping:

$$[C]=\alpha[M]+\beta[K],$$

$$\alpha=2\pi\frac{f_b f_e}{Q_d(f_b+f_e)},\quad \beta=\frac{1}{2\pi(f_b+f_e)Q_d}, \quad (8)$$

where $[f_b, f_e]$ is frequency range; Q_d is quality factor for the material.

Tsai–Wu failure criterion for orthotropic materials. To assess the strength of a multilayer structure, in the work it was used the Tsai-Wu strength criterion, which belongs to the energy tensor criteria for the failure of orthotropic materials. This criterion allows us to take into account the mutual influence of normal and tangential stresses and is a generalization of quadratic strength criteria for anisotropic media. It has become widely used to assess the limit state of multilayer composite materials.

For a three-dimensional model, the Tsai-Wu criterion is written as a quadratic function of the components of the stress tensor:

$$F_i\sigma_i + F_{ij}\sigma_{ij} = 1,$$

where σ_i are components of the stress vector in the main mutually perpendicular directions; F_i, F_{ij} – strength coefficients determined based on the material characteristics.

For orthotropic materials with three planes of symmetry oriented along the coordinate directions, we assume that $F_{ij} = F_{ji}$, and there is no connection between the normal and shear stresses. Then the general form of the Tsai-Wu failure criterion reduces to the strength condition [14]:

$$F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 + F_4\sigma_4 + F_5\sigma_5 + F_6\sigma_6 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + F_{44}\sigma_4^2 + F_{55}\sigma_5^2 + F_{66}\sigma_6^2 + \\ +2F_{12}\sigma_1\sigma_2 + 2F_{13}\sigma_1\sigma_3 + 2F_{23}\sigma_2\sigma_3 \le 1. \quad (9)$$

Let us denote the components of the limit state under uniaxial tension or compression in the three directions of anisotropy as $\sigma_{1t}, \sigma_{1c}, \sigma_{2t}, \sigma_{2c}, \sigma_{3t}, \sigma_{3c}$. The shear strength in the three planes of symmetry is denoted by $\tau_{12}^s, \tau_{13}^s, \tau_{23}^s$. For isotropic materials $\sigma_{1t} = \sigma_{2t} = \sigma_{3t}$, $\sigma_{1c} = \sigma_{2c} = \sigma_{3c}$, $\tau_{12}^s = \tau_{13}^s = \tau_{23}^s$. Then the coefficients of the orthotropic Tsai-Wu failure criterion are equal to:

$$F_1 = F_2 = F_3 = \frac{1}{\sigma_t} - \frac{1}{\sigma_c}, \quad F_4 = F_5 = F_6 = 0, \quad F_{11} = F_{22} = F_{33} = \frac{1}{\sigma_c\sigma_t}, \quad F_{44} = \frac{1}{\tau_{23}^2}, \quad F_{55} = \frac{1}{\tau_{31}^2}, \quad F_{66} = \frac{1}{\tau_{12}^2}. \quad (10)$$

The coefficients F_{12}, F_{13}, F_{23} can be determined approximately. It is often used originally proposed by Tsai approximation:

$$F_{xy} = -\sqrt{F_{xx}F_{yy}}/2 .$$

This ensures the interaction term lies within the bounds for which the yield envelope is an ellipse. It can be shown that the Tsai-Wu criterion is a particular case of the generalized Hill yield criterion.

2. Testing of the methodic

The methodic was tested on the example of a multilayer cantilever beam loaded with a vertical concentrated force. There were compared results obtained by modeling the beam with three-dimensional and multilayer two-dimensional finite elements (Fig. 1).

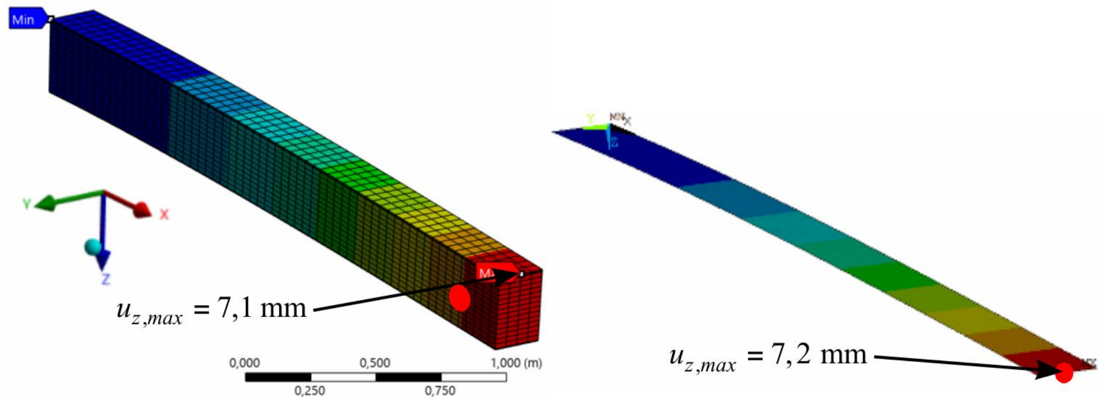
A cantilever beam of rectangular cross-section with dimensions $h = 0{,}294$ m, $b = 0{,}2$ m, length $L = 3$ m, fixed at one end and loaded with a concentrated vertical force $P = 20$ kN at the free end is investigated. The beam structure consists of five layers (from bottom to top): layer 1 - carbon fiber composite (thickness4 mm); layer 2 - fiber concrete (thickness 100 mm); layer 3 - foam concrete (thickness 80 mm); layer 4 - fiber concrete (thickness 100 mm); layer 5 − steel (thickness 10 mm).

Table 1

Material characteristics of the constituent layers

Name of the material		Carbon fiber composite	Fiber concrete	Foam concrete	Steel C255
Young's Modulus (E), GPa		600	30	1	206
Shear Modulus (G), GPa		230,8	12,5	0,42	80,8
Poisson's ratio (v)		0,3	0,2	0,2	0,28
Yield Stress, MPa	σ_c	1500	45	4	250
	σ_t	1500	10	0,6	250
Layer thickness, mm		4	100	80	10
Index Tsai-Wu		0,05	-0,65	-2,6	0,283

Fig. 1 presents the results obtained by modeling with three-dimensional finite elements SOLID185 and two-dimensional multilayer elements SHELL281, in what the Mindlin-Reisner shell theory (3)-(7) was implemented. The deviation between the results is within 1.5%.



(a) (b)

Fig. 1. Deflection of the beam

For both models, the stresses in the layers were determined. The obtained stress values σ_x at $x = 0,2$ m and a comparison of the results are given in Table 2. The maximum deviation is within 6%.

Table 2

Normal stresses of the beam (MPa)

Layer		2D	3D	Deviation, %
1. Carbon fiber composite	bottom	-164,85	-170,73	3,44
	top	-160,10	-165,76	3,41
2. Fiber concrete	bottom	-7,79	-8,59	0,47
	top	-2,02	-0,75	0,74
3. Foam concrete	bottom	-0,07	-0,03	0,02
	top	0,09	0,05	0,02
4. Fiber concrete	bottom	2,59	1,34	0,73
	top	8,35	8,60	0,15
5. Steel C255	bottom	57,28	59,12	3,11
	top	61,23	64,79	5,49

3. Analysis of the response of a protective structure to an explosive load in a quasi-static setting

The calculation of the multilayer cylindrical shell of the modular shelter was carried out in the FEA software package. The calculation scheme was given using 8-node multilayer two-dimensional finite elements SHELL281. SHELL281 can be used for modeling of layered structures, composite shells or sandwich panels. The element formulation is based on logarithmic measures of deformation and true stress.

There is considered the cylindrical multilayer shell, consisting of five layers of material composition similar to the considered beam: 1– carbon fiber; 2 – fiber concrete; 3 – foam concrete; 4 – fiber concrete; 5 – steel. The radius of curvature of the middle surface of the shell $R = 3$ m, the length of the structure $L = 6$ m. The blast wave is modeled by a uniformly distributed quasi-static pressure

$p = 400\,\text{kPa}$ applied to the right half of the surface of the structure. The boundary conditions of the shell fastening correspond to the hinged support of the shell structure.

Fig. 2 illustrates the distribution of displacements in the reduced surface of the shell. The maximum deviations in the direction of the X axis are $u_x = 3{,}61\,\text{mm}$, in the vertical direction (along the Y axis) $u_y = 3{,}05\,\text{mm}$. At a distance of more than 0,5 m from the ends, a zone of uniform displacements independent of the Z coordinate is observed. In the area of load application, maximum displacements $u_r = 4\,\text{mm}$ occur in the radial direction.

Fig. 3 illustrates the distribution of von-Mises equivalent stresses on the outer surface (layer 5, steel). The maximum stresses occur in the middle of the load application area and reach 90 MPa in the steel layer.

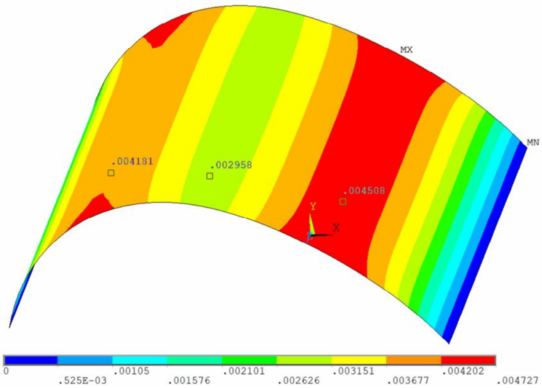


Fig. 2. Total deflection of the reduced surface of the shell

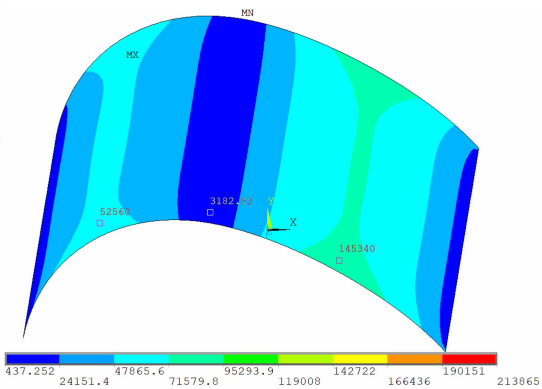


Fig. 3. Equivalent stresses on the external surface of the shell

For this shell model, the stresses in the layers were determined. For points 1 and 2 the values of the stress components were obtained in each layer and the structural strength was assessed using the Tsai-Wu strength criterion. The results are given in Table 3.

Table 3

Strength assessment of a cylindrical shell

Layer		p.1		p.2	
		σ_{eq}, MPa	Tsai-Wu	σ_{eq}, MPa	Tsai-Wu
1	bottom	31,82	0,00045	213,86	0,023
	top	31,89	0,00045	207,76	0,021
2	bottom	2,012	-0,13	7,35	0,997
	top	2,926	-0,092	6,07	0,420
3	bottom	2,575	16,09	0,423	0,458
	top	2,575	16,09	0,363	0,730
4	bottom	2,942	-0,095	1,406	0,068
	top	2,026	-0,14	7,295	0,850
5	bottom	11,75	0,002	75,67	0,087
	top	11,69	0,002	79,92	0,097

Thus, the strength of foam concrete is not ensured at point 1. At point 2, for fiber concrete, we have a state close to the limit, which requires additional structural solutions. To increase the load-bearing capacity of the shell, it is planned to install a discrete reinforcing frame in the areas of maximum stress concentration. In addition, local replacement of lightweight foam concrete aggregate with fiber concrete in critical sections allows to significantly increase the damping capacity and load-bearing capacity of the structure to dynamic impact.

4. Analysis of vibrations of a protective structure under impulse explosive loads

The dynamic behavior of the cylindrical shell described in previous section is investigated. The profile of the considered explosive load is shown in Fig. 4. The shock wave [12] consists of a positive phase with a sharp front of duration 10 ms with an intensity of 400 kPa and a negative phase of duration 8 ms with an intensity of 80 kPa. To take into account the damping of oscillations, Rayleigh damping is introduced with damping parameters (8), where $f_b = 40\,\text{Hz}$, $f_e = 100\,\text{Hz}$, $Q_d = 50$.

Fig. 5 shows the dependence of the horizontal and vertical displacements of points 1 and 2 of the cylindrical shell on time under impulse loading. The graph demonstrates a classic example of free damped oscillations after an impulse load. A gradual decrease in the amplitude of the displacements describes damping, which is due to internal friction in the shell material. The maximum amplitudes are recorded in the interval $20 < t < 70$ ms, where the values of u_x for point 2 reach a peak value near 5,4 mm. After load disappearing at $t = 50$ ms structure periodically vibrates with a frequency 26,5 Hz. Decrement of oscillations after load removal $\delta = 0{,}062$ is constant and corresponds to used quality factor for the package $Q_d = \pi / \delta \approx 50$.

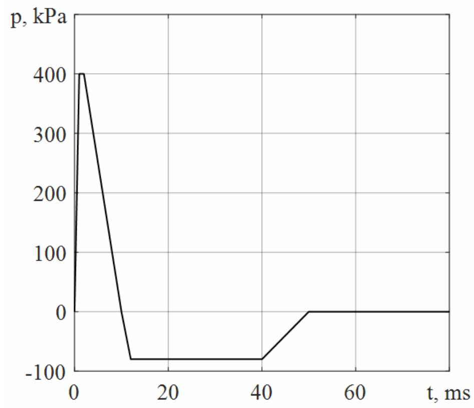


Fig. 4. Time dependence of blast wave intensity

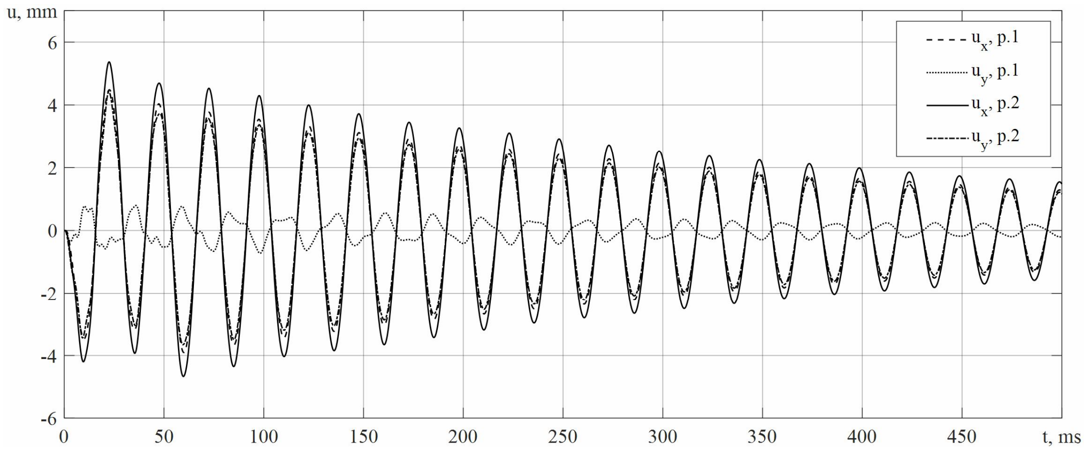


Fig. 5. Displacements of the shell points

Fig. 6 illustrates the dynamic dependence for the maximum equivalent stresses on the carbon fiber layer at points 1 and 2 of the structure. Analysis of the graphs shows that the peak values of the stresses are reached at the moment of passage of the primary shock wave, after which a series of damped oscillations are observed, caused by multiple reflections of deformation waves from the interface surfaces of the layers. The dynamic dependence of the equivalent stresses illustrate the complex wave nature of the structural response.

Maximum equivalent stresses reach 215 MPa, what is close to the carbon fiber stresses in quasi-static statement. The difference in amplitudes between points 1 and 2 indicates a high load capacity of the multilayer package. In particular, at point 2 a sharp jump in stresses is observed, corresponding to the wave travel time through the thickness of the package, while at point 2 the graph has a more blurred maximum, which is explained by the dispersion of waves in a nonhomogeneous medium.

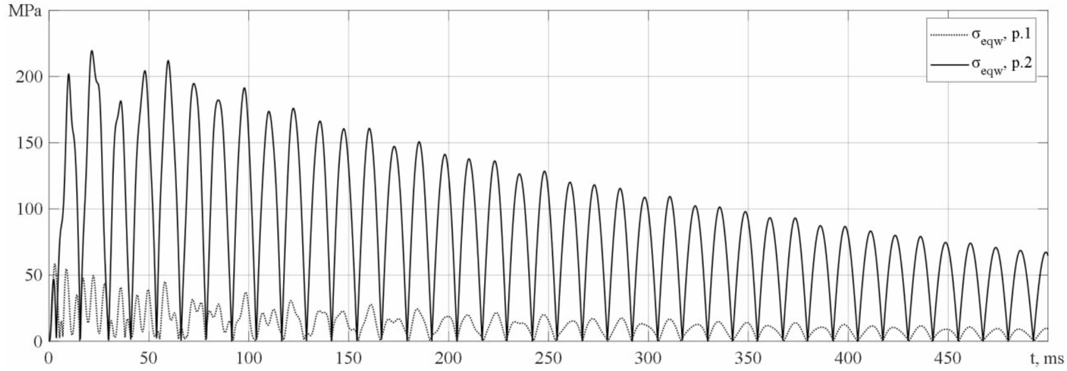


Fig. 6. Equivalent stress of the shell points

Conclusions. The results of the calculations showed that the use of shell elements provides high accuracy in predicting deformations with a significant reduction in computational costs compared to solid models. At comparing of quasistatic and nonstationary statement results can be considered as equivalent as soon as maximum displacements and equivalent stresses are little different. Nonstationary statement should be used for describing of the propagation of deformation waves, vibrations after load removal and damping processes in the structure.

The obtained data confirm the effectiveness of the proposed approach for assessing the durability of protective structures and elements of special equipment exposed to extreme dynamic effects. The proposed method can be used to model vibrations of bodies of different geometries and materials of a layered structure. Further research can be aimed at taking into account the physical nonlinearity of layer materials and damping processes under the action of shock waves.

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АНАЛІЗ НАПРУЖЕНО-ДЕФОРМОВАНОГО СТАНУ ЦИЛІНДРИЧНОЇ БАГАТОШАРОВОЇ ОБОЛОНКИ ЗАХИСНОЇ СПОРУДИ ПРИ ВИБУХОВИХ НАВАНТАЖЕННЯХ

В роботі проведено моделювання та аналіз напруженого стану оболонкових тіл багатошарової структури при квазістатичних та динамічних навантаженнях. Досліджується квазістатичний та нестаціонарний напружено-деформований стан неоднорідних будівельних елементів конструкцій із суттєво різних конструкційних матеріалів. Проводиться пошарова оцінка міцності неоднорідної конструкції за критерієм міцності Цай-Ву.

Тестування застосованих методів розрахунку проводиться шляхом порівняння результатів розв'язків, отриманих для двовимірної та тривимірної моделі багатошарової консольної балки.

Проводиться аналіз реакції циліндричного багатошарового покриття захисної споруди на вибухове навантаження в квазістатичній та нестаціонарній постановці. У квазістатичній постановці досліджено переміщення та складний напружений стан, а також визначено найбільш навантажені точки. Виконано пошарову оцінку міцності неоднорідної конструкції за критерієм міцності Цай-Ву.

При моделюванні вибухового навантаження в нестаціонарній постановці враховано фази позитивного та негативного тиску. Проведено аналіз коливань, що виникають в конструкції при таких навантаженнях. Затухання коливань моделюється за допомогою демпфування за Релеєм. Описано динаміку зміни переміщень та напружень в вибраних точках конструкції.Визначено частоту та декремент вільних коливань після зняття навантаження.

Ключові слова: багатошарові конструкції, вибухове навантаження, квазістатична постановка, критерій міцності Цай-Ву, профіль вибухової хвилі, нестаціонарні коливання.

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ANALYSIS OF THE STRESS-STRAIN STATE OF A CYLINDRICAL MULTILAYER SHELL OF A PROTECTIVE STRUCTURE UNDER EXPLOSIVE LOADS

The work involves modeling and analyzing of the stress state of shell bodies of a multilayer structure under quasi-static and dynamic loads. The quasi-static and non-stationary stress-strain state of heterogeneous building elements of structures made of significantly different structural materials is investigated.

Testing of the applied calculation methods is performed by comparing the results of solutions obtained for a two-dimensional and three-dimensional model of a multilayer cantilever beam.

The response of a cylindrical multilayer coating of a protective structure to an explosive load in a quasi-static and non-stationary setting is analyzed. In quasistatic statement displacement and complex stress state was explored and most loaded points were defined. A layer-by-layer strength assessment of a heterogeneous structure is performed using the Tsai-Wu strength criterion.

When modeling an explosive load in a non-stationary setting, the phases of positive and negative pressure are taken into account. An analysis of the oscillations that occur in the structure under such loads is performed. The damping of vibrations is modeled using Rayleigh damping. The dynamics of changes in displacements and stresses at selected points of the structure are described. There were defined frequency and decrement of free vibrations after load removal.

Keywords: multilayer structures, explosive loading, quasi-static formulation, Tsai-Wu strength criterion, blast wave profile, non-stationary vibrations.

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Досліджується напружено-деформований стан та виконується оцінка міцності циліндричних багатошарових оболонок при вибухових навантаженнях в квазістатичній та нестаціонарній постановці.
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The stress-strain state and strength of cylindrical multilayer shells under explosive loads in quasi-static and non-stationary settings is investigated.
Fig. 6. Ref. 20.

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