UDC 621.87

DYNAMIC ANALYSIS OF THE SLEWING MECHANISM OF A JIB CRANE WITH A PAYLOAD SUSPENSION IN THE FORM OF A DOUBLE PENDULUM

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DOI: 10.32347/2410-2547.2026.116.393-405

Abstract. Gripper devices with weights close to the payload weight are used when jib cranes perform installation work on complex structures. For such cases, the flexible payload suspension is represented by a double mathematical pendulum model. The paper deals with the dynamics of the slewing mechanism of a jib crane with a flexible payload suspension in the form of a double pendulum. The purpose of this study is to construct a mathematical model and perform a dynamic analysis of the jib crane slewing mechanism with a double mathematical pendulum payload suspension. The slewing mechanism is represented by a dynamic model with four degrees of freedom (4-DoF). Based on this model, a mathematical model of the jib crane slewing mechanism is constructed using Lagrange equations of the second kind, forming a system of second-order ordinary differential equations. In this model, the driving torque of the electric motor is described by its dynamic mechanical characteristic. As a result of numerical solving of the equations, the kinematic, dynamic, and energy characteristics of the jib crane slewing mechanism are determined. The study investigates the main movement of the drive mechanism, as well as high-frequency oscillations of drive elements and low-frequency oscillations of the payload and gripper on the flexible suspension. It is revealed that the dynamics of the slewing mechanism depend on the nature of the driving torque change, while low-frequency oscillations of the payload and gripper practically do not dampen and continue throughout the entire movement cycle.

To reduce dynamic loads and high-frequency vibrations in the drive transmission mechanism, as well as low-frequency vibrations of the gripping device and load on a flexible suspension, it is recommended to select modes of smooth change of the drive torque during start-up and braking, which ensure the desired movement of the load on a flexible suspension.

Keywords: jib crane, slewing mechanism, double pendulum, oscillatory processes, driving torque, dynamic loads.

Introduction. The efficiency of using jib cranes largely depends on increasing their productivity and reliability during loading, unloading, transport, and installation operations. One way to increase the reliability of jib cranes is to identify actual loads and establish directions for their reduction. Gripper devices, whose weight is close to the weight of the payload, are frequently used for installation work of complex structures. In such cases, the flexible suspension is represented as a double mathematical pendulum. There is a necessity for mathematical modeling of the movement of mechanisms, specifically the slewing mechanism, and conducting its dynamic analysis for cranes with such a suspension design

Analysis of publications. In [1] a dynamic analysis of a tower crane was performed using the multibody modeling method, taking into account the flexibility of the boom, traction ropes, and hoisting mechanism. The authors demonstrated that neglecting the elastic properties of crane structural elements leads to a significant decrease in peak dynamic loads, especially during transitional starting and braking modes. The authors of [2] investigated the dynamic behavior of a jib crane, considering the spatial geometry of the payload fixed on a flexible suspension and a multi-rope system. The research results showed that the shape and eccentricity of the payload have a significant impact on oscillatory processes and dynamic loads in hoisting mechanisms.

Work [3] developed a mathematical model of a fail-safe crane mechanism, which allows for the evaluation of the impact of operating modes and corresponding damping elements on the magnitude of dynamic loads. Researchers proved that the use of hydraulic or controlled dampers can significantly reduce peak dynamic loads. The authors of [4] analyzed the dynamics of industrial cranes under eccentric loading and proposed control methods to reduce oscillation amplitude. The results obtained are important for improving the safety and durability of crane structures.

In study [5], an analysis of the dynamic stability of tower cranes used for the installation of wind turbines was performed. It was shown that the combined action of wind and inertial loads can lead to critical operating modes not accounted for by standard static calculations.

Having developed a mathematical model that considers payload movement and the interaction of structural elements, the authors of [6] conducted a dynamic analysis of a tower crane. They performed modal analysis, evaluated natural frequencies, oscillation modes, and force reactions, which allows for the identification of critical points and the assessment of dynamic loads on safety and efficiency. This source is essential for understanding tower crane mechanics and serves as a basis for research on payload movement control to counteract oscillations.

Work [7] developed a mathematical model of a crane with a double pendulum using Lagrange methods to describe the motion of all system links. Vibration control methods were proposed to reduce the amplitude of pendulum movements and increase safety. In publication [8], dynamic modeling of a gantry crane with a double pendulum and two cables was presented, with simulations confirmed by experimental data. The study showed how system parameters, such as cable length and mass distribution, affect payload oscillations and loads. In article [9], the authors analyzed the dynamics of a long payload transported by two bridge cranes, taking into account the double mathematical pendulum effect. This study allowed for the evaluation of payload oscillations and rope forces, which is important for the safe transport of long and heavy loads.

In work [10], a 3D dynamic model of a gantry crane was constructed, describing the movement of the payload and structural elements in all three dimensions. The model accounts for the inertial and elastic characteristics of structural elements and allows for the analysis of forces and accelerations during operation. The main part of the work is devoted to payload and mechanism movement analysis, including mass distribution, geometry, and the influence of speed on dynamic loads.

In article [11], authors consider the dynamic behavior of tower cranes during slewing operations when the direction of boom and payload movement changes. They analyze how various movements affect dynamic forces and show that traditional static methods underestimate the loads arising during dynamic movements. The results show the need to use more realistic models of structural dynamics that can take into account the effect of rotational motion on the tension in crane components under various boom configurations. The authors of [12] proposed a multi-mass dynamic model of a tower crane where the slewing mechanism drive is considered as a separate mass body. His allows for evaluating dynamic forces, elastic and damping characteristics during payload slewing. Numerical calculations show the dependence of structural forces on inertial characteristics and drive parameter, which is important for assessing loads in difficult crane operating conditions..

The author of [13] conducted a dynamic analysis via simulation of a crane for wind turbine installation operating in complex conditions (low wind speed, limited space). A rigid-flexible coupling model for ropes and crane structure was used with ADAMS and ANSYS systems. The analysis determined maximum dynamic loads in various hoisting modes, allowing for safety and efficiency evaluation.

In work [14], the dynamic behavior of an offshore knuckle-boom crane is investigated under different load application laws. The authors demonstrate how load changes affect the accelerations and the dynamic response of the structure, specifically the velocity and acceleration of the payload and the mechanism reactions. The work shows that the dynamic characteristics of the system vary depending on the load type and direction of movement, which must be taken into account when designing structures for a wide range of cranes. In article [15], the authors investigated the dynamics of a knuckle-boom crane during payload movement under wind load conditions. The authors proposed a dynamic model that accounts for the flexibility of the boom elements and the compliance of the drives.

The equations of motion were derived based on Lagrange equations. Wind pressure is modeled as an external disturbance; both steady and time-varying wind gusts are considered. To quantitatively assess the wind's influence, root-mean-square (RMS) load indicators in the drives and payload positioning errors were applied. It was shown that analyzing only the motion of the center of mass is insufficient, and accounting for the displacements of characteristic points of the payload provides a more correct assessment of the accuracy and safety of crane operation under wind disturbances.

Researchers in [16] developed a method for calculating dynamic loads and energy losses in an overhead crane hoisting mechanism. They created a model considering drive parameters, payload mass, crane dimensions, and inertial characteristics. It was shown that energy losses depend heavily on hoisting height and payload mass. This allows for more accurate assessment of crane operating parameters and efficiency. In article [17], the hoisting mechanism of an overhead crane is viewed as a complex electromechanical system. The model allows for the analysis of dynamic forces and torques in the crane structure that arise during accelerations and changes in the movement speeds of the mechanisms. The results demonstrate the features of the interaction of moving parts, the transmission of torques, and inertial forces, which is critical for evaluating the reliability and durability of the equipment under intensive operating modes.

In work [18], a mathematical approach for modeling dynamic loads during transient processes (start/finish of hoisting) was proposed. Authors use equations of motion to describe force and inertia changes, emphasizing the complexity of calculating loads during sharp regime changes. They highlight the importance of accounting for oscillations and accelerations that arise during abrupt changes in operating modes, demonstrating that these factors constitute the primary complexity in calculating dynamic loads in crane machinery.

Authors of [19], analyze dynamic and energy parameters of a tower crane in various modes, including structural reaction measurements. The research encompasses measuring structural responses and the variation of forces and accelerations arising during mechanism operation. This work enables the determination of the energy characteristics transmitted through the system and their impact on dynamic loads, aiding in the optimization of structural solutions for crane design. In work [20], a model was developed to evaluate the dynamic loads on a tower crane jib under different load movement conditions and velocity changes. The model incorporates the primary mechanical characteristics of the jib, allowing for the analysis of the dynamic force distribution along the structure. This approach demonstrates how transient regimes and load movement alter stresses and reactions within jib elements, which is vital for safe design and calculation.

In study [21], the dynamics of the outreach and slewing mechanisms of a tower crane were modeled, and the start-up mode of these mechanisms was assessed. The optimization performed significantly reduced dynamic overloading during the movement of the outreach and slewing mechanisms. Work [22] investigated the movement dynamics of the derricking and hoisting mechanisms of a jib crane. He dynamics of mechanism motion were modeled, and a dynamic analysis was conducted, establishing that the jib crane operates under overloading during their operation. The authors of [23] performed modeling of the movement dynamics of the derricking and jib extension mechanisms during steady crane slewing, based on which a dynamic analysis was conducted. This analysis identified force overloads in structural elements and payload oscillations.

Based on the literature review, it can be concluded that to identify loads and oscillations in structural elements during the operation of a jib crane slewing mechanism with a flexible payload suspension in the form of a double pendulum, there is a necessity for mathematical modeling and a subsequent dynamic analysis of the acting loads.

Purpose of the paper. The purpose of this study is to construct a mathematical model of the movement dynamics of the jib crane slewing mechanism with a double pendulum payload suspension and to perform a dynamic analysis of its motion.

Research results. When constructing the dynamic model of the jib crane slewing mechanism, the main movement of the drive electric motor and the oscillatory movements of the drive transmission mechanism are taken into account, as well as low-frequency oscillations of the gripper device and the payload on the flexible suspension.

The dynamic model of the jib crane slewing mechanism is represented as a system of absolutely rigid and dissipative links (Fig. 1). It is assumed that the drive transmission mechanism, as well as the

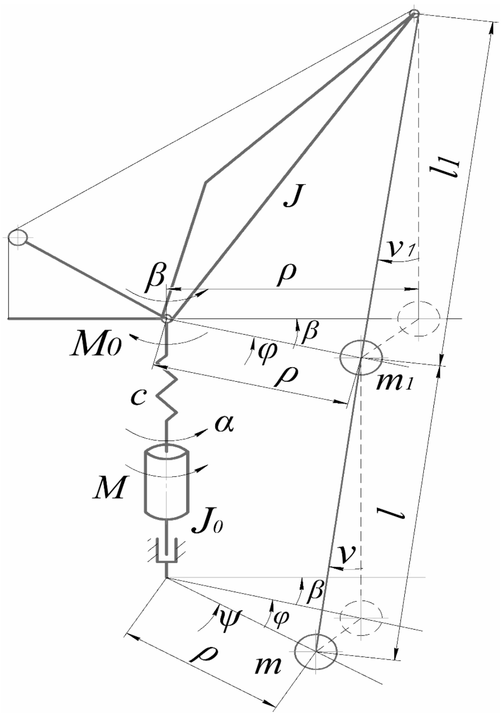


Fig. 1. Dynamic model of the crane slewing mechanism with a payload suspension in the form of a double mathematical pendulum

ropes of the flexible suspension for the gripper and the payload, possess dissipative properties, while the jib, payload, and gripper are considered absolutely rigid bodies. The gripper and the payload on the flexible suspension perform pendulum oscillations in the crane's slewing plane. The links of the slewing drive and the jib are reduced to the crane's slewing axis. The driving torque of the slewing mechanism drive is determined from the dynamic mechanical characteristic.

Thus, the dynamic model of the jib crane slewing mechanism is represented by a mechanical system with four degrees of freedom (4DOF) (Fig. 1). The generalized coordinates of the provided dynamic model are the angular coordinates of the electric motor rotor rotation – α, the crane – β, e gripper rotation – φ and the payload– ψ. The crane slewing mechanism is subject to the static resistance torque – M_0 and the driving torque of the electric motor – M, oth of which are reduced to the crane's slewing axis. Additionally, dissipative moments act in the drive transmission elements and the flexible suspension of the gripper and payload when they deviate from the vertical.

To construct the mathematical model of the jib crane slewing mechanism with a double pendulum suspension (Fig. 1), Lagrange equations of the second kind are used:

$$\begin{aligned}
&\frac{d}{dt}\frac{\partial T}{\partial \dot\alpha}-\frac{\partial T}{\partial \alpha}=\mathrm{M}-\frac{\partial \Pi}{\partial \alpha}-\frac{\partial R}{\partial \dot\alpha};\\
&\frac{d}{dt}\frac{\partial T}{\partial \dot\beta}-\frac{\partial T}{\partial \beta}=-\mathrm{M}_0-\frac{\partial \Pi}{\partial \beta}-\frac{\partial R}{\partial \dot\beta};\\
&\frac{d}{dt}\frac{\partial T}{\partial \dot\varphi}-\frac{\partial T}{\partial \varphi}=-\frac{\partial \Pi}{\partial \varphi}-\frac{\partial R}{\partial \dot\varphi};\\
&\frac{d}{dt}\frac{\partial T}{\partial \dot\psi}-\frac{\partial T}{\partial \psi}=-\frac{\partial \Pi}{\partial \psi}-\frac{\partial R}{\partial \dot\psi},
\end{aligned} \tag{1}$$

where T, Π, R – are the kinetic energy, potential energy, and Rayleigh dissipation function of the crane slewing mechanism, respectively; M, M_0 – are the driving torque and the resistance torque, reduced to the crane's slewing axis.

The kinetic energy of the crane slewing mechanism is represented by the following relationship

$$T=\frac{1}{2}\cdot \mathrm{J}_0\cdot\dot\alpha^2+\frac{1}{2}\cdot \mathrm{J}\cdot\dot\beta^2+\frac{1}{2}\cdot \mathrm{m}_1\cdot\rho^2\cdot\dot\varphi^2+\frac{1}{2}\cdot \mathrm{m}\cdot\rho^2\cdot\dot\psi^2, \tag{2}$$

where J_0, J – the moments of inertia of the drive links and the rotating part of the crane, respectively, reduced to the slewing axis; m, m_1 – are the masses of the payload and the gripper; ρ – the payload outreach at which the crane slews.

The potential energy of the crane slewing mechanism is determined by the following relationship

$$\Pi=\frac{1}{2}\cdot \mathrm{c}\cdot(\alpha-\beta)^2+(\mathrm{m}_1+\mathrm{m})\cdot g\cdot l_1\cdot(1-\cos\nu_1)+\mathrm{m}\cdot \mathrm{g}\cdot l\cdot(1-\cos\nu), \tag{3}$$

where g – acceleration of gravity; c – stiffness coefficient of the drive transmission mechanism reduced to the crane slewing axis; l_1, l – lengths of the flexible suspension of the gripper and payload, respectively; ν_1, ν – angular coordinates of the deviation from the vertical for the gripper and payload flexible suspensions, respectively.

The Rayleigh dissipation function of the elastic elements of the crane slewing mechanism is as follows

$$R=\frac{1}{2}\cdot \mathrm{b}_2\cdot\left(\dot{\alpha}-\dot{\beta}\right)^2+\frac{1}{2}\cdot \mathrm{b}_1\cdot\dot{\nu}_1^2+\frac{1}{2}\cdot \mathrm{b}\cdot\dot{\nu}^2, \tag{4}$$

where b_2, b_1, b – damping coefficients of the drive transmission mechanism elements and the flexible suspension of the gripper and the payload, respectively, when they deviate from the vertical.

The angular coordinates of the deviation from the vertical for the flexible suspension of the gripper and the payload are determined by the following relationships:

$$\nu_1=\frac{(\beta-\varphi)\cdot\rho}{l_1};\ \nu=\frac{(\varphi-\psi)\rho}{l}, \tag{5}$$

We determine the partial derivatives of the kinetic energy (2) with respect to the generalized coordinates of the jib crane slewing mechanism:

$$\frac{\partial T}{\partial\alpha}=\frac{\partial T}{\partial\beta}=\frac{\partial T}{\partial\varphi}=\frac{\partial T}{\partial\psi}=0. \tag{6}$$

We also find the partial derivatives of the kinetic energy (2) with respect to the generalized velocities of the crane slewing mechanism:

$$\frac{\partial T}{\partial\dot{\alpha}}=\mathrm{J}_0\cdot\dot{\alpha};\ \frac{\partial T}{\partial\dot{\beta}}=\mathrm{J}\cdot\dot{\beta};\ \frac{\partial T}{\partial\dot{\varphi}}=\mathrm{m}_1\cdot\rho^2\cdot\dot{\varphi};\quad \frac{\partial T}{\partial\dot{\psi}}=\mathrm{m}\cdot\rho^2\cdot\dot{\psi}. \tag{7}$$

Subsequently, we determine the full time derivatives of the expressions (7), resulting in the following:

$$\begin{aligned}
&\frac{\mathrm{d}}{dt}\frac{\partial T}{\partial\dot{\alpha}}=\mathrm{J}_0\cdot\ddot{\alpha};\\
&\frac{\mathrm{d}}{dt}\frac{\partial T}{\partial\dot{\beta}}=\mathrm{J}\cdot\ddot{\beta};\\
&\frac{\mathrm{d}}{dt}\frac{\partial T}{\partial\dot{\varphi}}=\mathrm{m}_1\cdot\rho^2\cdot\ddot{\varphi};\\
&\frac{\mathrm{d}}{dt}\frac{\partial T}{\partial\dot{\psi}}=\mathrm{m}\cdot\rho^2\cdot\ddot{\psi}.
\end{aligned} \tag{8}$$

We determine the partial derivatives of the potential energy (3) with respect to the generalized coordinates:

$$\frac{\partial\Pi}{\partial\alpha}=\mathrm{c}\cdot(\alpha-\beta);\quad \frac{\partial\Pi}{\partial\beta}=-\mathrm{c}(\alpha-\beta)+(\mathrm{m}_1+\mathrm{m})\cdot g\cdot l_1\cdot\frac{\partial\nu_1}{\partial\beta}\cdot\sin\nu_1; \tag{9}$$

$$\frac{\partial\Pi}{\partial\varphi}=(\mathrm{m}_1+\mathrm{m})\cdot g\cdot l_1\frac{\partial\nu_1}{\partial\varphi}\cdot\sin\nu_1+\mathrm{m}\cdot g\cdot l\frac{\partial\nu}{\partial\varphi}\cdot\sin\nu;\ \frac{\partial\Pi}{\partial\psi}=\mathrm{m}\cdot g\cdot l\frac{\partial\nu}{\partial\psi}\sin\nu. \tag{10}$$

We determine the partial derivatives of the angular coordinates of the flexible suspension of the gripper and the payload with respect to the generalized coordinates of the system and substitute them into expressions (9) and (10), and also use the assumption that $\sin\nu_1\approx\nu_1, a\sin\nu\approx\nu$. As a result, we obtain:

$$\frac{\partial\Pi}{\partial\alpha}=\mathrm{c}\cdot(\alpha-\beta);\ \frac{\partial\Pi}{\partial\beta}=-\mathrm{c}\cdot(\alpha-\beta)+\frac{(\mathrm{m}_1+\mathrm{m})\cdot g\cdot\rho^2\cdot(\beta-\varphi)}{l_1}; \tag{11}$$

$$\frac{\partial\Pi}{\partial\varphi}=-\frac{(\mathrm{m}_1+\mathrm{m})\cdot g\cdot\rho^2\cdot(\beta-\varphi)}{l_1}+\frac{\mathrm{m}\cdot g\cdot\rho^2\cdot(\varphi-\psi)}{l};$$

$$\frac{\partial \Pi}{\partial \psi} = -\frac{\mathrm{m} \cdot g \cdot \rho^2 \cdot (\varphi - \psi)}{l}. \tag{12}$$

Now we determine the partial derivatives of the Rayleigh function (4) with respect to the generalized velocities:

$$\frac{\partial R}{\partial \dot{\alpha}} = \mathrm{b}_2 \cdot \left(\dot{\alpha}\text{-}\dot{\beta}\right); \quad \frac{\partial R}{\partial \dot{\beta}} = -\mathrm{b}_2 \cdot \left(\dot{\alpha}\text{-}\dot{\beta}\right) + \mathrm{b}_1 \cdot \dot{\nu}_1 \cdot \frac{\partial \nu_1}{\partial \beta};$$
$$\frac{\partial R}{\partial \dot{\varphi}} = \mathrm{b}_1 \cdot \dot{\nu}_1 \cdot \frac{\partial \nu_1}{\partial \varphi} + \mathrm{b} \cdot \dot{\nu} \cdot \frac{\partial \nu}{\partial \varphi}; \quad \frac{\partial R}{\partial \dot{\psi}} = \mathrm{b} \cdot \dot{\nu} \cdot \frac{\partial \nu}{\partial \psi}. \tag{13}$$

The dependencies (13) include full time derivatives and partial derivatives with respect to the generalized coordinates of the angular coordinates of the deviation from the vertical for the flexible suspensions of the gripper and the payload; therefore, we determine them. To do this, we will use expressions (5) and determine the necessary derivatives:

$$\dot{\nu}_1 = \frac{\left(\dot{\beta} - \dot{\varphi}\right) \cdot \rho}{l_1}; \dot{\nu} = \frac{(\dot{\varphi} - \dot{\psi}) \cdot \rho}{l}; \tag{14}$$

$$\frac{\partial \nu_1}{\partial \beta} = \frac{\rho}{\mathrm{l}_1}; \frac{\partial \nu_1}{\partial \varphi} = -\frac{\rho}{l_1}; \frac{\partial \nu}{\partial \varphi} = \frac{\rho}{1}; \frac{\partial \nu}{\partial \psi} = -\frac{\rho}{l}. \tag{15}$$

As a result of substituting expressions (6), (8), (11)-(13), taking into account the relations (14) and (15), into system (1), we obtain the mathematical model of the motion dynamics of the crane slewing mechanism when modeling the payload suspension as a double mathematical pendulum:

$$\mathrm{J}_0 \cdot \ddot{\alpha} = M - \mathrm{c} \cdot (\alpha - \beta) - \mathrm{b}_2 \cdot (\dot{\alpha} - \dot{\beta});$$
$$\mathrm{J}\ddot{\beta} = -M_0 + \mathrm{c}\left(\alpha\text{-}\beta\right) - \frac{(\mathrm{m}_1 + \mathrm{m}) g \rho^2 (\beta - \varphi)}{l_1} + \mathrm{b}_2\left(\dot{\alpha}\text{-}\dot{\beta}\right) - \mathrm{b}_1 \frac{(\dot{\beta} - \dot{\varphi})\rho^2}{l_1^2};$$
$$\mathrm{m}_1 \rho^2 \ddot{\varphi} = \frac{(\mathrm{m}_1 + \mathrm{m}) g \rho^2 (\beta - \varphi)}{l_1} - \frac{\mathrm{m} g \rho^2 (\varphi - \psi)}{l} + \mathrm{b}_1 \frac{(\dot{\beta} - \dot{\varphi})\rho^2}{l_1^2} - \mathrm{b} \frac{(\dot{\varphi} - \dot{\psi})\rho^2}{l^2};$$
$$\mathrm{m}\rho^2 \ddot{\psi} = \frac{m g \rho^2 (\varphi - \psi)}{l} + b \frac{(\dot{\varphi} - \dot{\psi})\rho^2}{l^2}. \tag{16}$$

Research results

The system of differential equations (16) includes the driving torque of the drive electric motor, which is determined from the dynamic mechanical characteristic shown in Fig. 2. This mechanical characteristic is constructed according to the parameters of the drive motor of the jib crane slewing mechanism. In the dynamic characteristic, significant fluctuations of the driving torque occur, which are caused by electromagnetic transient processes in the electric drive during the start-up process.

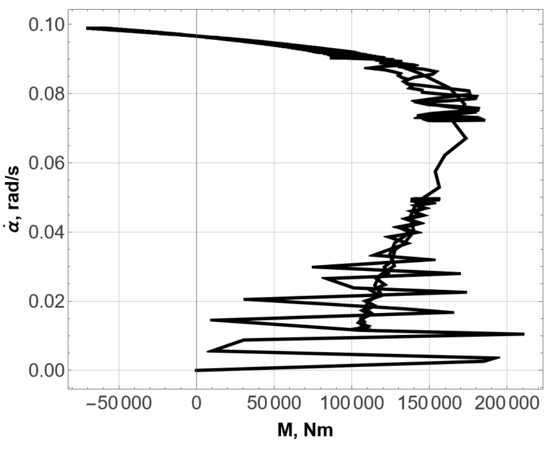


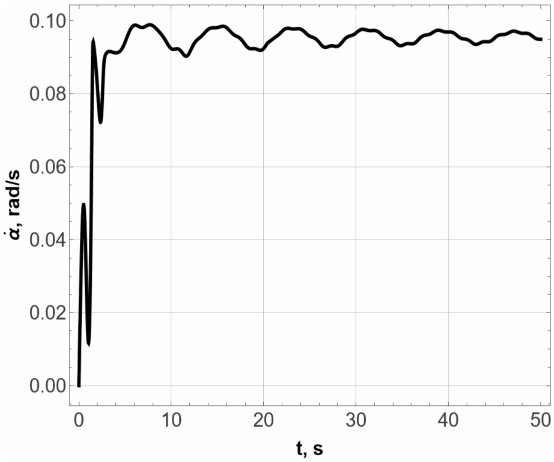
Fig. 2. Dynamic characteristic of the drive electric motor

The system of equations (16) for the motion dynamics of the crane slewing mechanism is a system of nonlinear second-order differential equations; therefore, a numerical method implemented in a developed computer program is used for its solution. The system of equations (16) is solved under the following initial conditions of the crane slewing mechanism motion:

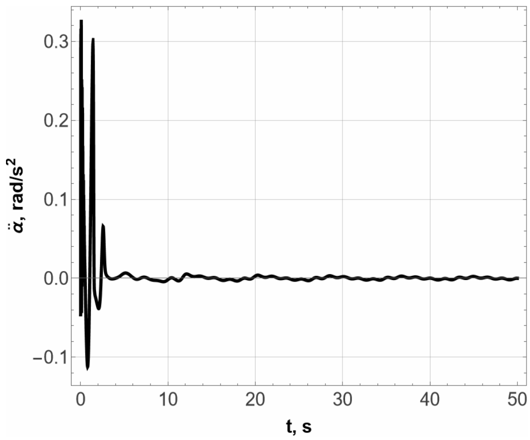
$$\mathrm{t} = 0: \ \alpha = \frac{\mathrm{M}_0}{\mathrm{c}}; \ \dot{\alpha} = 0; \ \beta = 0; \ \dot{\beta} = 0; \ \varphi = 0; \ \dot{\varphi} = 0; \ \psi = 0; \ \dot{\psi} = 0. \tag{17}$$

The dynamic analysis of the crane slewing was performed for the following parameter values of the crane jib system: $m=2000\,kg$, $m_1=500\,kg$, $J_0=557740\,kg{\cdot}m^2$, $J=1551700\,kg{\cdot}m^2$, $l=5.0\,m$, $l_1=10.0\,m$, $\rho=25.0\,m$, $u=1083.6$, $\omega_0=0.0966\,rad/s$, $\omega_n=0.0892\,rad/s$, $g=9.81\,m/s^2$, $M_0=25750\,N{\cdot}m$, $M_n=560290\,N{\cdot}m$, $b=0$, $b_1=0$, $b_2=0$.

After the numerical solution of the nonlinear system of differential equations (16) under the initial conditions (17) of the crane slewing mechanism motion, graphical dependencies of the kinematic (Figs. 3-8), dynamic (Fig. 9), and energy (Fig. 10) characteristics were constructed.



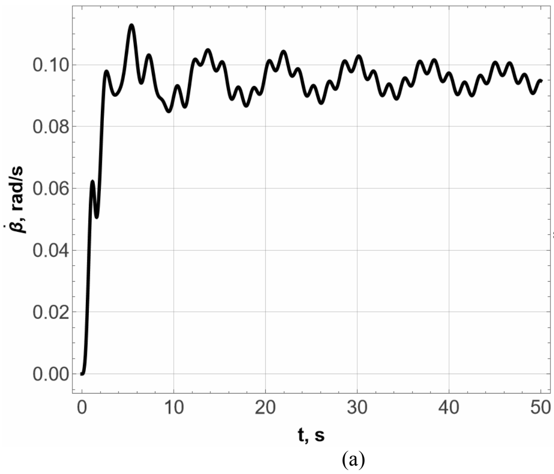
(a)



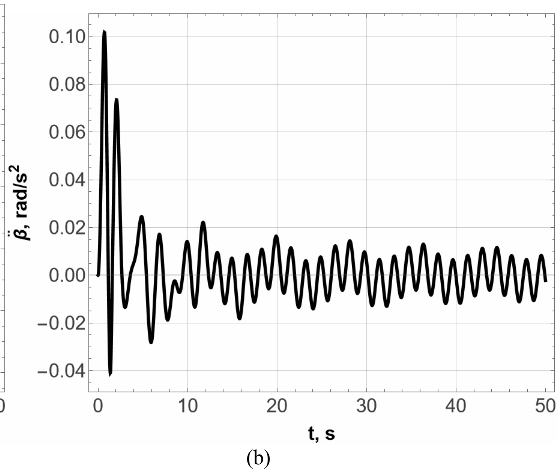
(b)

Fig. 3. Graphs of the angular velocity (a) and acceleration (b) of the electric motor rotor, reduced to the crane slewing axis

From Fig. 3,a, it can be seen that the electric motor rotor speed increases sharply to a steady-state value during the start-up phase. This speed increase occurs in an oscillatory mode. In the steady-state motion phase, the speed of the electric motor rotor changes periodically in an oscillatory mode with slight damping. The angular acceleration of the electric motor rotor (Fig. 3,b) changes rapidly in an oscillatory mode with significant damping during start-up. The maximum acceleration value is reached at the first oscillation peak and amounts to $0.33\,rad/s^2$. During the steady-state motion of the electric motor rotor, high-frequency oscillations of the rotor acceleration with a small amplitude are observed.



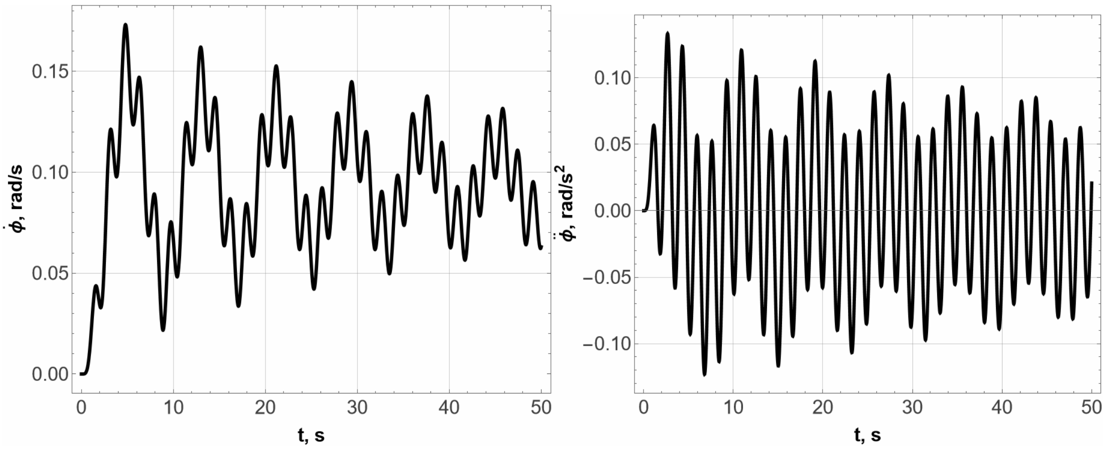
(a)



(b)

Fig. 4. Graphs of the angular velocity (a) and acceleration (b) of the crane's rotating part

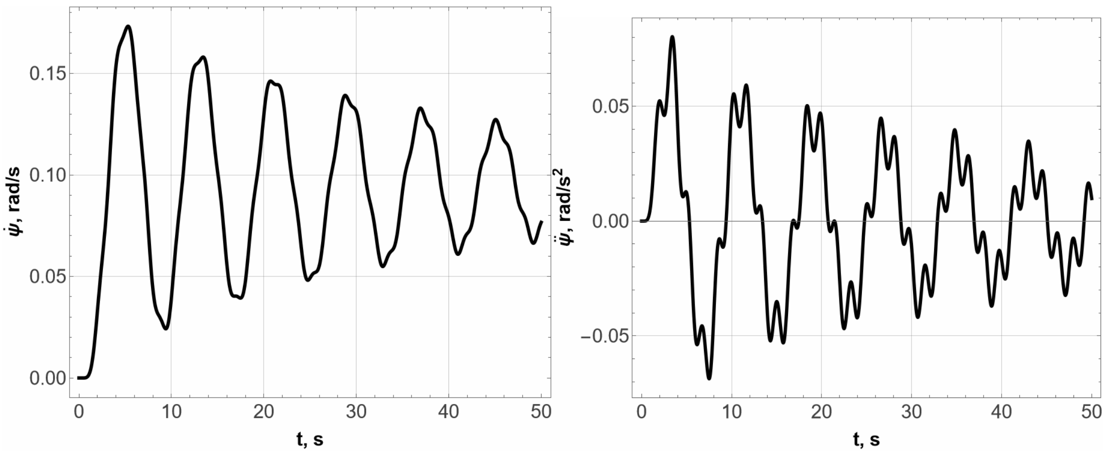
The angular velocity of the crane's rotating part (Fig. 4,a) increases in an oscillatory mode during start-up and reaches its maximum value of $0.115\,rad/s$ at the fifth second of movement. Subsequently, the velocity of the crane's rotating part reaches a steady-state value and varies in a low-frequency mode with slight damping. At the same time, high-frequency, virtually undamped oscillations are superimposed on the low-frequency damped oscillations. The acceleration of the crane's rotating part (Fig. 4,b) varies in an oscillatory mode with a significant amplitude during the start-up phase, reaching a maximum value of $0.105\,\mathrm{rad/s^2}$ at the first second of movement. Subsequently, the acceleration varies relative to the zero line with low-frequency damped oscillations, which also have high-frequency undamped vibrations superimposed.



(a) (b)

Fig. 5. Graphs of the angular velocity (a) and acceleration (b) of the gripper

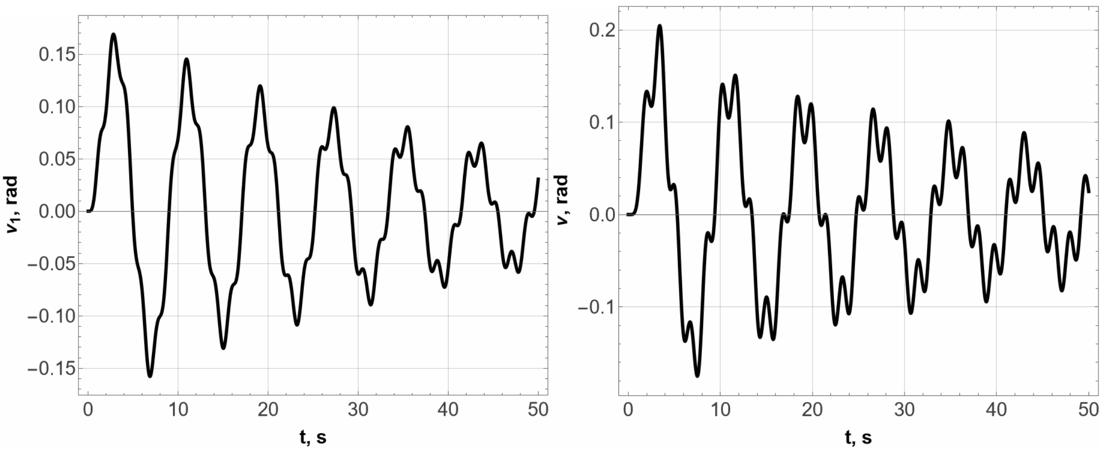
The angular velocity of the gripper rotation (Fig. 5,a) increases in an oscillatory mode during the start-up phase and reaches its maximum value of $0.18\,\mathrm{rad/s}$ at the fifth second of movement Subsequently, it varies relative to the steady-state value in a mode of low-frequency damped oscillations, with high-frequency oscillations further superimposed. He gripper acceleration (Fig. 5,b) reaches its maximum value of $0.135\,\mathrm{rad/s^2}$ at the time moment of $2.4\,\mathrm{s}$ during the start-up process. In the following stages, the acceleration varies relative to the zero line with low-frequency damped oscillations, which are superimposed by high-frequency oscillations of significant amplitude.



(a) (b)

Fig. 6. Graphs of the angular velocity (a) and acceleration (b) of the payload

The angular velocity of the payload rotation (Fig. 6,a) increases smoothly during the start-up phase to a maximum value of $0.175\,\mathrm{rad/s}$ over $5.5\,\mathrm{s}$. Subsequently, the payload velocity varies relative to the steady-state value in a low-frequency damped oscillation mode. The payload rotation acceleration (Fig. 6,b) reaches its maximum value of $0.82\ \text{rad/s}^2$ at the time moment of $3.4\ \text{s}$. Subsequently, the acceleration varies relative to the zero line in a mode of low-frequency oscillations, which are superimposed by high-frequency oscillations.

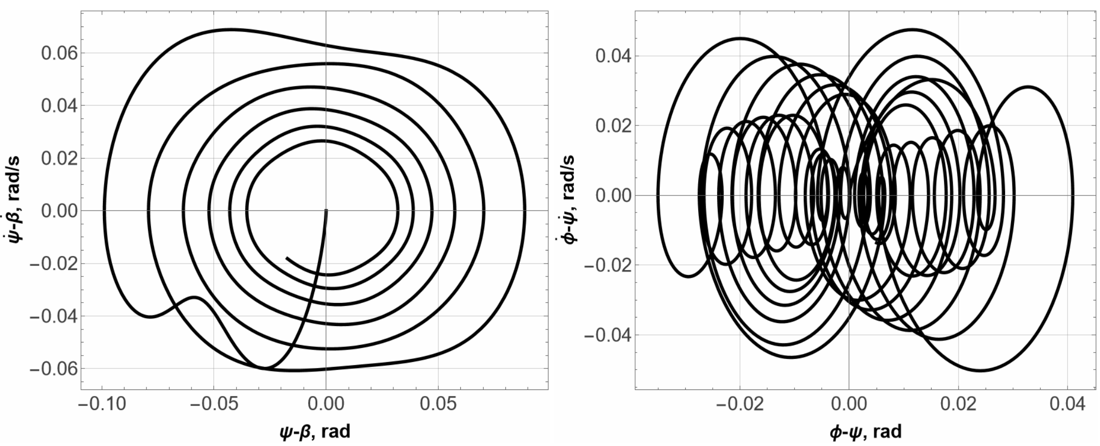


(a) (b)

Fig. 7.Graphs of the deviation from the vertical for the flexible suspension of the gripper (a) and the payload (b)

The deviation of the flexible gripper suspension from the vertical (Fig. 7,a) reaches its maximum value of $0.17\ \text{rad}$ at the time moment of $2.55\ \text{s}$. Subsequently, the deviation of the flexible gripper suspension varies in a low-frequency damped oscillation mode with the superposition of additional oscillations of small amplitude. The deviation of the flexible payload suspension from the vertical (Fig. 7,b) reaches its maximum value of $0.082\ \text{rad}$ at the time moment of $3.5\ \text{s}$. In the following stages, the deviation of the flexible payload suspension varies relative to the zero line in a low-frequency damped oscillation mode with the superposition of high-frequency oscillations.

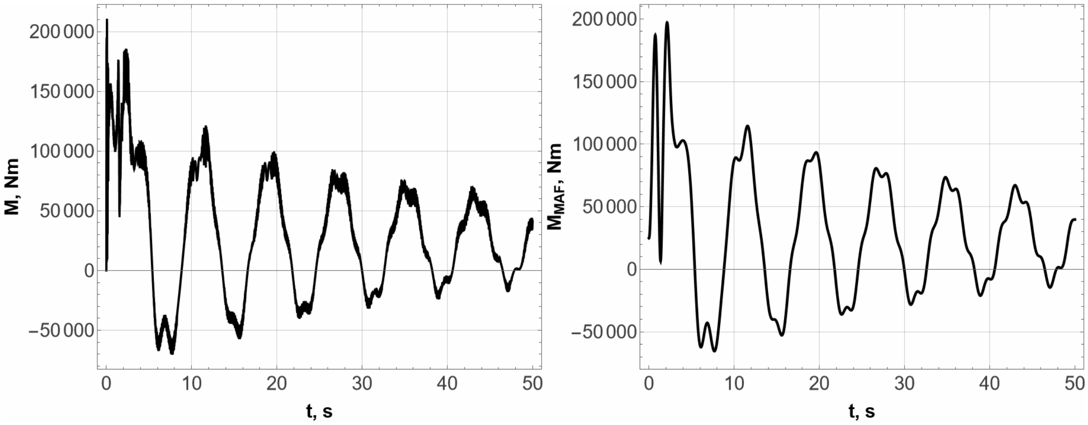
The phase portrait of the payload oscillations relative to the crane's rotating part (Fig. 8,a) indicates that these oscillations exhibit a stable damping tendency. At the same time, the maximum value of the oscillation amplitude deviation is $0.1\ \text{rad}$. From the phase portrait of the payload oscillations relative to the gripper (Fig. 8,b), it can be observed that a complex character of oscillations occurs. At the same time, these oscillations are also damped.



(a) (b)

Fig. 8. Phase portraits of the payload oscillations relative to the crane's rotating part (a) and the gripper (b)

The driving torque of the drive mechanism (Fig. 9,a) reaches its maximum value of $210\,\text{kN·m}$ at the start of start-up. Subsequently, the drive torque varies in a mode of low-frequency damped oscillations with superimposed high-frequency vibrations. The torque in the elastic coupling (Fig. 9,b) reaches its maximum value of $200\,\text{kN·m}$ at the time moment of $2.1\,\text{s}$, followed by damping in a low-frequency oscillatory mode. Unlike the driving torque, the torque in the elastic coupling is virtually free of high-frequency superimposed oscillations.



(a) (b)

Fig. 9. Graphs of the driving torque (a) and the elastic torque in the coupling (b), reduced to the crane's rotating part

In the start-up phase, the drive power (Fig. 10) varies in an oscillatory high-frequency mode and reaches a maximum value of $14.2\,\text{kW}$ at the time moment of $2.0\,\text{s}$. Subsequently, the power varies in a low-frequency damped oscillation mode toward a steady-state value, with high-frequency oscillations superimposed on it.

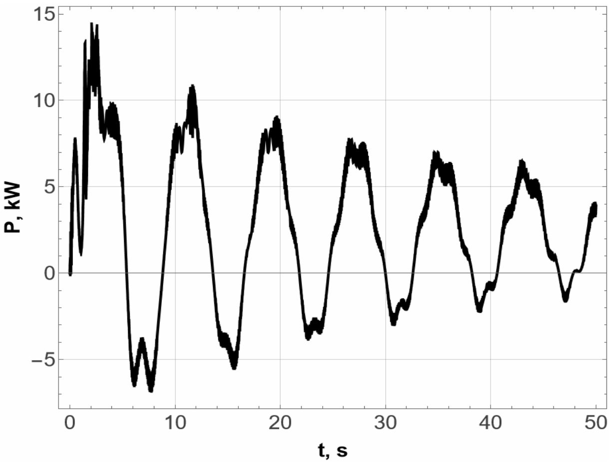


Fig. 10. Graph of the drive mechanism power variation

Conclusions. Based on the results of the dynamic analysis of the movement of the jib crane slewing mechanism with a flexible payload suspension in the form of a double pendulum, the following conclusions were made:

1. A dynamic model of the jib crane slewing mechanism with a flexible payload suspension in the form of a double pendulum has been developed. The developed dynamic model accounts for the main movement of the crane's slewing drive mechanism, high-frequency oscillations in the transmission mechanism of the drive, as well as low-frequency pendulum oscillations of the gripper and payload on the flexible suspension. The dynamic model incorporates the dynamic mechanical characteristic of the crane slewing drive's electric motor. Based on the dynamic model of the crane slewing mechanism, its mathematical model is constructed, which is described by a system of four nonlinear second-order differential equations. The resulting system of equations was solved numerically using a developed computer program.

2. Based on the solved system of mathematical model equations, a dynamic analysis of the movement of the jib crane slewing mechanism with a flexible payload suspension in the form of a double mathematical pendulum was performed. The dynamic analysis results revealed significant oscillations in the kinematic, dynamic, and energy characteristics of the slewing mechanism's drive

elements, gripper, and payload on the flexible suspension. The presence of oscillatory processes led to an increase in dynamic loads and energy consumption in the crane slewing drive and the overall jib crane structure. The instantaneous change in the driving torque of the slewing drive mechanism resulted in virtually undamped oscillations of the characteristics of the jib system, gripper, and payload. To minimize oscillations in the elements of the crane's jib system and reduce dynamic loads, it is necessary to control the start-up mode of the crane's slewing drive.

3. As a result of the dynamic analysis of the jib crane slewing mechanism with a flexible payload suspension represented by a double mathematical pendulum, the cause of the virtually undamped oscillations of the payload and its gripper has been identified, which is determined by the nature of the drive torque variation. In this case, the emergence of oscillatory processes is the result of the almost instantaneous change in the driving torque of the crane slewing drive mechanism. To reduce dynamic and energy loads and eliminate oscillations in the structural elements of the crane's jib system, it is proposed to optimize the start-up and braking modes of the slewing drive.

Acknowledgement

This paper was supported by the HEI-COPILOT project within the EIT

HEI Initiative "Innovation Capacity Building for Higher Education", funded by the European Union.

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Стаття надійшла 07.03.2026

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ДИНАМІЧНИЙ АНАЛІЗ МЕХАНІЗМУ ПОВОРОТУ СТРІЛОВОГО КРАНА З ПІДВІСОМ ВАНТАЖУ У ВИГЛЯДІ ПОДВІЙНОГО МАЯТНИКА

При виконанні стріловими кранами монтажних робіт складних конструкцій використовуються захватні пристрої, вага яких близька до ваги вантажу. Для такого випадку гнучкий підвіс вантажу представлено моделлю подвійного математичного маятника. В роботі розглядається динаміка механізму повороту стрілового крана з гнучким підвісом вантажу у вигляді подвійного маятника. Метою наведеного дослідження є побудова математичної моделі та здійснення динамічного аналізу механізму повороту стрілового крана з гнучким підвісом вантажу у вигляді подвійного математичного маятника. Механізм повороту стрілового крана представлений динамічною моделлю з чотирма ступенями свободи. На базі цієї моделі з використанням рівнянь Лагранжа другого роду побудовано математичну модель механізму повороту стрілового крану, яка являє собою систему звичайних диференціальних рівнянь другого порядку. В цій моделі рушійний момент приводного електродвигуна описується динамічною механічною характеристикою.. В результаті чисельного розв’язування системи рівнянь визначені кінематичні, динамічні та енергетичні характеристики механізму повороту стрілового крану з підвісом вантажу, представленим моделлю подвійного математичного маятника. . В роботі досліджено основний рух приводного механізму повороту крана, а також високочастотні коливання елементів приводу та низькочастотні коливання вантажу і захватного пристрою на гнучкому підвісі. Виявлено, що динаміка руху механізму повороту крана залежить від характеру зміни рушійного моменту приводного електродвигуна, а низькочастотні коливання вантажу та захватного пристрою на гнучкому підвісі практично не затухають і тривають протягом всього проміжку руху.

Для зниження динамічних навантажень та високочастотних коливань в передавальному механізмі приводу , а також низькочастотних коливань захватного пристрою і вантажу на гнучкому підвісі рекомендовано обирати режими плавної зміни рушійного моменту приводу під час пуску та гальмування, які забезпечують бажаний рух вантажу на гнучкому підвісі.

Ключові слова: стріловий кран**,** механізм повороту, подвійний маятник, коливальні процеси, рушійний момент, динамічні навантаження.

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DYNAMIC ANALYSIS OF THE SLEWING MECHANISM OF A JIB CRANE WITH A PAYLOAD SUSPENSION IN THE FORM OF A DOUBLE PENDULUM

*G*ripper devices with weights close to the payload weight are used when jib cranes perform installation work on complex structures. For such cases, the flexible payload suspension is represented by a double mathematical pendulum model. The paper deals with the dynamics of the slewing mechanism of a jib crane with a flexible payload suspension in the form of a double pendulum. The purpose of this study is to construct a mathematical model and perform a dynamic analysis of the jib crane slewing mechanism with a double mathematical pendulum payload suspension. The slewing mechanism is represented by a dynamic model with four degrees of freedom (4-DoF). Based on this model, a mathematical model of the jib crane slewing mechanism is constructed using Lagrange equations of the second kind, forming a system of second-order ordinary differential equations. In this model, the driving torque of the electric motor is described by its dynamic mechanical characteristic. As a result of numerical solving of the equations, the kinematic, dynamic, and energy characteristics of the jib crane slewing mechanism are determined. The study investigates the main movement of the drive mechanism, as well as high-frequency oscillations of drive elements and low-frequency oscillations of the payload and gripper on the flexible suspensionIt is revealed that the dynamics of the slewing mechanism depend on the nature of the driving torque change, while low-frequency oscillations of the payload and gripper practically do not dampen and continue throughout the entire movement cycle.

To reduce dynamic loads and high-frequency vibrations in the drive transmission mechanism, as well as low-frequency vibrations of the gripping device and load on a flexible suspension, it is recommended to select modes of smooth change of the drive torque during start-up and braking, which ensure the desired movement of the load on a flexible suspension.

Keywords: jib crane, slewing mechanism, double pendulum, oscillatory processes, driving torque, dynamic loads.

УДК 621.87
Ловейкін В.С., Ромасевич Ю.О., Почка К.І., Кадикало І.О., Стехно О.В. **Динамічний аналіз механізму повороту стрілового крана з підвісом вантажу у вигляді подвійного маятника** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2026. – Вип. 116. - С. 393-405.
У роботі розглянуто динаміку механізму повороту стрілового крана з гнучким підвісом вантажу, представленим моделлю подвійного математичного маятника. На основі рівнянь Лагранжа другого роду побудовано математичну модель з чотирма ступенями свободи та виконано чисельний аналіз кінематичних, динамічних і енергетичних характеристик. Встановлено вплив характеру зміни рушійного моменту на коливальні процеси та рекомендовано режими плавного пуску й гальмування.
Іл. 10. Бібліогр. 23 назв.

UDC621.87
Loveikin V.S., Romasevych Yu.O., Pochka K.I., Kadykalo I.O., Stekhno O.V. **Dynamic analysis of the slewing mechanism of a jib crane with a payload suspension in the form of a double pendulum** // Strength of Materials and Theory of Structures: Scientific-&-Technical collected articles – Kyiv: KNUBA, 2026. – Issue 116. - P. 393-405.
The paper investigates the dynamics of a jib crane slewing mechanism with a flexible payload suspension modeled as a double mathematical pendulum. A four-degree-of-freedom mathematical model based on Lagrange equations of the second kind was developed and numerically analyzed to obtain kinematic, dynamic, and energy characteristics. The influence of driving torque variation on oscillatory processes was established, and smooth start-up and braking modes were recommended to reduce dynamic loads.
Fig. 10. Ref. 23.

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