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UNDERESTIMATION OF VERTICAL STRESSES IN SOIL FOUNDATIONS USING ANALYTICAL METHODS: NUMERICAL SUBSTANTIATION CONSIDERING NONLINEAR BASE BEHAVIOR

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The paper presents a comparative analysis of the stress state in soil base under a raft foundation using analytical methods and numerical simulation in Midas GTS NX. The study was conducted for a load range of 100–300 kPa, considering sandy ($E=30$ MPa) and clayey ($E=15$ MPa) soils with the application of Elastic, mM-C, and HSsmall constitutive models. It was established that classical analytical methods lead to a systematic underestimation of vertical stresses at depths exceeding 1.5B. The most significant discrepancy was recorded for weak subgrades when accounting for small-strain stiffness within the HSsmall model. The necessity of correcting the stress distribution coefficients α is substantiated to enhance the accuracy of foundation design.

Keywords: raft foundation, geotechnical medium, vertical stresses, numerical simulation, Midas GTS NX, Hardening Soil Small Strain (HSsmall), stiffness degradation, small-strain stiffness, stress distribution.

1. Introduction

Modern construction is characterized by a continuous increase in foundation loads and the growing complexity of geotechnical conditions. The design of raft foundations requires highly accurate prediction of the stress-strain state (SSS) of the subgrade, as the reliability of soil stress determination directly influences settlement calculations and the assessment of the structure's bearing capacity [6].

Traditionally, in engineering practice, soil stress determination relies on analytical methods, specifically the corner point method based on Boussinesq's solution [2] for a linearly elastic half-space. However, this approach treats the soil as an idealized elastic body with constant stiffness parameters, which contradicts the actual physical nature of the soil mass.

The advancement of finite element method (FEM) software packages, such as Midas GTS NX [3], has enabled the implementation of nonlinear constitutive models. Models such as Hardening Soil (HS) [4] and Hardening Soil Small Strain (HSsmall) [1] account for plastic soil hardening, the dependence of stiffness on stress levels, and increased stiffness at small strain levels. Despite the global widespread application of these models, the question of the quantitative discrepancy between results obtained through modern numerical techniques and classical analytical calculations [5] remains insufficiently addressed.

The aim of this work is to conduct a comparative analysis of the vertical stress distribution in sandy and clayey subgrades, obtained using the analytical corner point method and numerical simulation in Midas GTS NX software. The study employs a hierarchy of models: ranging from linear-elastic to the advanced nonlinear Hardening Soil Small Strain (HSsmall) model.

The object of the study is the stress-strain state (SSS) of sandy and clayey subgrades beneath a raft foundation.

2. Research Methodology

To achieve the research objectives, a comprehensive approach was adopted, combining classical computational algorithms of soil mechanics with modern numerical analysis of the "foundation–soil base" system's stress-strain state (SSS).

2.1. Analytical Method (Corner Point Method)

The analytical determination of vertical stresses was performed in accordance with the principles of the linearly deformable half-space theory. The method is based on Boussinesq's solution for a concentrated point load, integrated over the area of a rectangular foundation. The vertical stresses at depth

z beneath the corner of a rectangular loaded area (1) are determined based on the following assumptions: the soil is considered a homogeneous, isotropic, and linearly elastic medium; the deformation modulus E is assumed to be a constant value, independent of the stress level and the depth of the soil layer

$$\sigma_{zp} = \alpha \cdot p. \quad (1)$$

Here α is the influence factor that accounts for the relative depth (z/B) and the foundation aspect ratio (L/B); p - average contact pressure.

2.2. Numerical Modeling in Midas GTS NX

The numerical study was conducted using the finite element method (FEM) in a three-dimensional (3D) formulation (Fig. 1). The computational domain dimensions were established to eliminate boundary effects on the results, extending at least 3–5 times the foundation width B in each lateral direction and in depth.

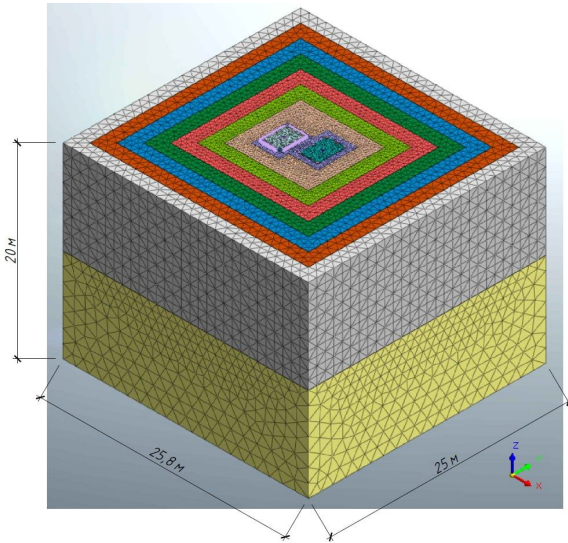


Fig.1. Finite element model for SSI numerical simulation in Midas GTS NX

For the comparative analysis, a hierarchical sequence of soil constitutive models was utilized:

- Elastic Model: Employed as a digital counterpart to the analytical method to verify the finite element mesh. This model operates solely with the Young's modulus E and Poisson's ratio ν ;

- Modified Mohr-Coulomb (mM-C): Accounts for limited soil strength through the cohesion c and internal friction angle φ parameters. Furthermore, it differentiates between stiffness during loading and unloading stages;

- Hardening Soil Small Strain (HSsmall): An advanced hardening model that extends the standard HS model features (stress-level dependency of stiffness via the m parameter; differentiation of three stiffness moduli: the secant E_{50} , oedometer E_{oed} , and unloading/reloading E_{ur} moduli; and plastic

hardening strains). Crucially, it incorporates the nonlinear stiffness increase in the small-strain region (up to $10^{-6} \dots 10^{-4}$), characterized by the initial shear modulus G_0 and the modulus degradation parameter γ_{07} .

3. Main Body of Research

The study is based on a comparative analysis of the results for three loading stages applied by the raft foundation to the soil, with intensities ranging from 100 to 300 kPa. To evaluate the influence of soil type, two subgrade models were examined: a sandy subgrade with a deformation modulus of $E=30$ MPa and a clayey subgrade with a modulus of $E=15$ MPa.

3.1. Simulation Scenarios

The computations were performed for two types of soil conditions (conditional sand and conditional clay) under varying external pressure levels:

- Stage I (100 kPa): Soil behavior is predominantly within the elastic range.
- Stage II (200 kPa): Initiation of plastic deformation zones beneath the edges of the raft.
- Stage III (300 kPa): Development of significant nonlinear deformations and a substantial deviation of stiffness from its initial values.

The results were monitored by constructing vertical stress profiles σ_z along the central, lateral, and corner vertical axes of the foundation to a depth of 10 m.

The parameter set, adapted to the input data (single-layer sandy and clayey subgrades) and the specifics of the selected models, includes basic physical and mechanical characteristics. These base parameters are utilized across all models, including the analytical method and the Elastic model (Table 1), alongside additional stiffness and nonlinearity parameters for deformation.

Table 1

Parameter	Sandy soil	Clayey soil
Young's modulus, E [MPa]	30	15
Unit weight, γ [kN/m ³]	18	16
Cohesion, c [kPa]	1	25
Internal friction angle, φ [deg]	35	25
Poisson's ratio, ν	0.30	0.37

In the Hardening Soil (HS) models, the constant Young's modulus is replaced by three primary stress-dependent stiffness moduli:

- E_{50} (Secant stiffness in standard triaxial test: for sandy soils $E_{50} \approx E$, for clayey soils may be lower than the compression modulus E_{oed}).

- E_{oed}^{ref} (Tangent stiffness for primary oedometer loading: $E_{oed}^{ref} = E_{50}^{ref}$).

- E_{ur}^{ref} (Unloading/reloading modulus: according to standard modeling practice, typically assumed to be $E_{ur}^{ref} = (3...5) \times E_{50}^{ref}$. This parameter defines the soil's elastic response during stress reduction and subsequent reloading cycles, preventing the overestimation of plastic deformations).

- m (Stiffness stress-dependency exponent: 0.5 for sandy soils, 0.7-1.0 for clayey soils).

Additional parameters for the HSsmall model (this model accounts for the increased soil stiffness at very small strain levels, which is critical for the accurate determination of stress distribution):

- G_0^{ref} (Initial shear modulus: $G_0 \approx (2...10) \times E_{50}$).

- $\gamma_{0.7}$ (Threshold shear strain: The level of shear strain at which the secant shear modulus G degrades to 70% of its initial value (G_0): $\gamma_{0.7} \approx 1 \times 10^{-4} ... 2 \times 10^{-4}$).

The object of the study is a monolithic reinforced concrete raft foundation with plan dimensions of 1.8×2.6 m and a thickness of 0.3 m. The numerical model assumes a uniformly distributed load applied over the entire base area of the foundation.

The distribution of vertical stresses was investigated along characteristic vertical axes of the raft, specifically: the geometric center; the corners; the midpoints of the external edges (lateral axes).

3.2. Analytical Calculations

The analytical calculation of stresses was performed for the corresponding characteristic points of the subgrade. Based on the obtained data, vertical stress profiles σ_{zp} were constructed to a depth of 10 m. This depth exceeds five times the foundation width ($>5B$), allowing for a comprehensive evaluation of stress attenuation within the compressible strata (Fig. 2-3).

Since most analytical methods (specifically the corner point method) are based on the Boussinesq solution for a half-space, vertical stress at any point in the soil mass depends solely on geometric parameters (depth, distance from the loading axis) and the magnitude of the applied load. Equation (1) contains no parameters for soil stiffness (deformation modulus) or shear strength. Consequently, for both the sandy subgrade with a modulus of $E=30$ MPa (Fig. 2) and the clayey subgrade with $E=15$ MPa (Fig. 3), the stresses are distributed according to identical geometric energy dissipation laws. The analytical stress profile remains identical across different soil types because such a model does not account for differences in their resistance to deformation.

3.3. SSI Numerical Modeling (Midas GTS NX)

For the numerical simulation, a finite element model was developed that incorporates all elements of the "foundation–subgrade" system (Fig. 1). The external load was applied as a static, uniformly distributed pressure on the foundation surface.

Due to the geometric symmetry and the concentric nature of the applied load, the simulation results (stress iso-surfaces) are presented for one-half of the structure (Fig. 4-5). This approach is justified by the fact that the symmetrical stress distribution in a homogeneous soil base under uniform loading allows for the visualization of numerical analysis results on a half-section without any loss of informative value. The

iso-surfaces are shown for the central cross-section of the foundation.

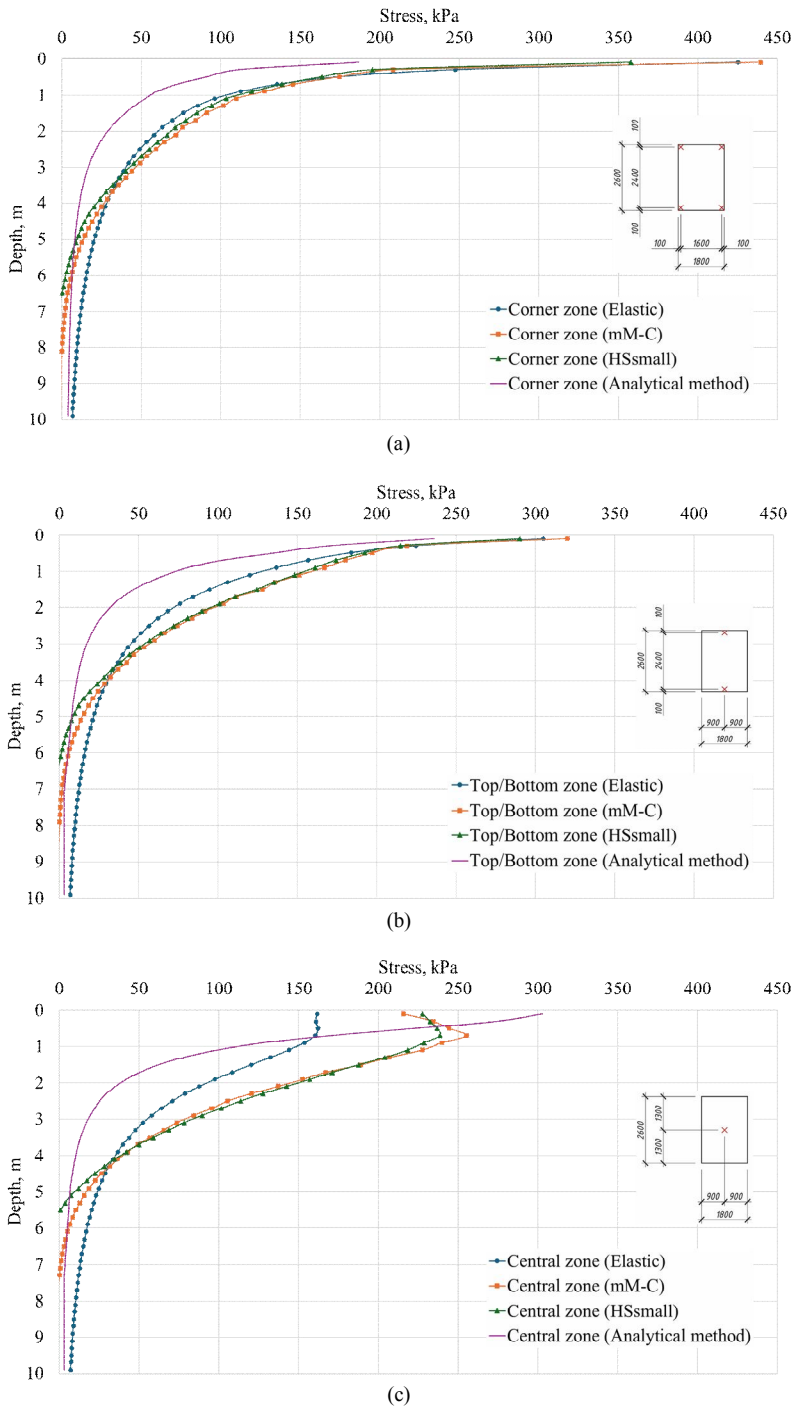


Fig. 2. Vertical stress (σ_{zp}) distribution in sandy soil ($E=30$ MPa) under 300 kPa loading (analytical method vs. numerical simulation): (a) - in the corner zone of the slab; (b) – top/bottom; (c) - center

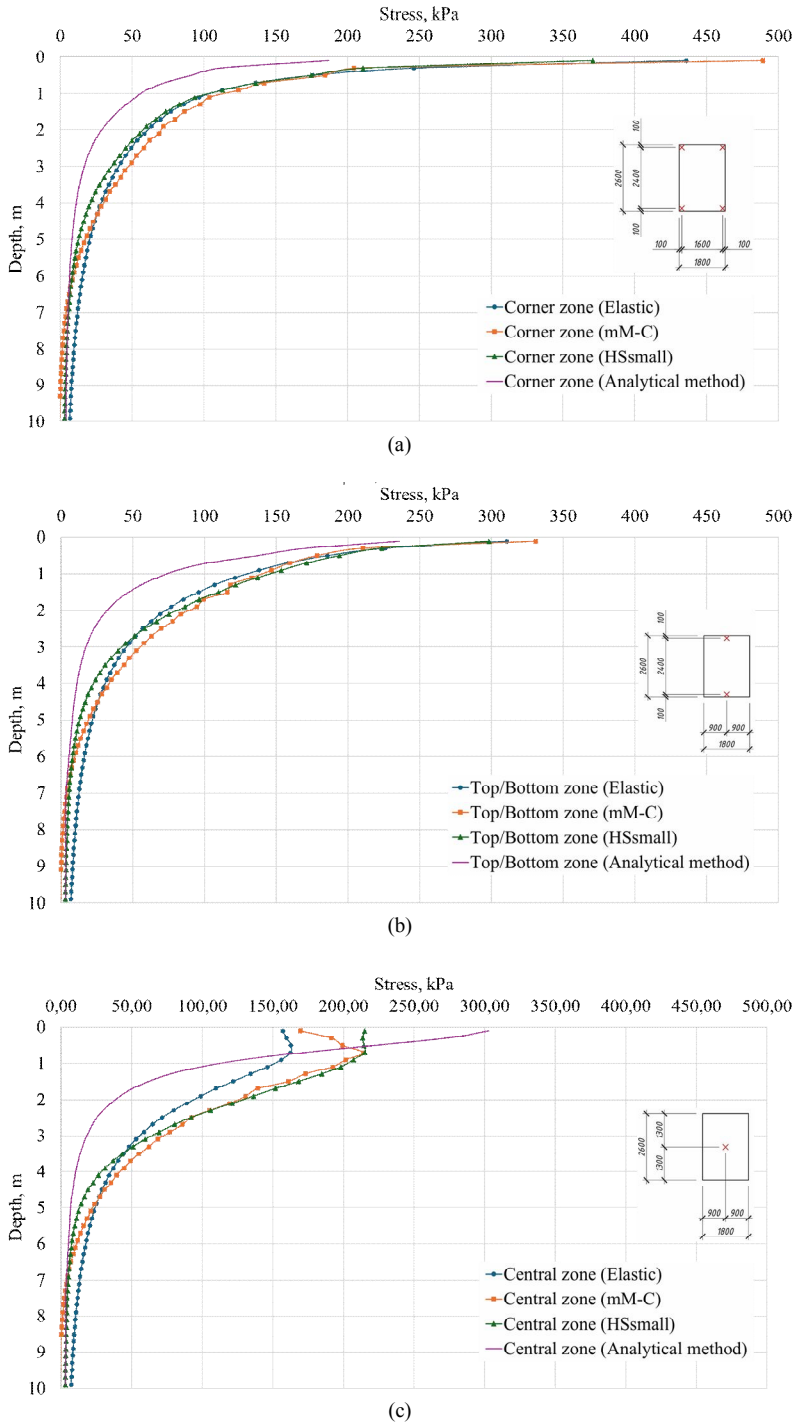


Fig. 3. Vertical stress (σ_{zp}) distribution in clayey soil ($E=15$ MPa) under 300 kPa loading (analytical method vs. numerical simulation): (a) - in the corner zone; (b) – top/bottom; (c) - center

The iso-surfaces obtained using different soil constitutive models provide an opportunity to evaluate the differences in the geometry of the "stress bulbs" formed within the soil base.

For the sandy subgrade scenario ($E=30$ MPa) the results obtained from different constitutive models revealed the following:

– Elastic Model (Fig. 4(a)): Exhibits a standard, geometrically consistent distribution where the shape of the iso-surfaces remains unchanged regardless of the pressure level, only scaling in magnitude.

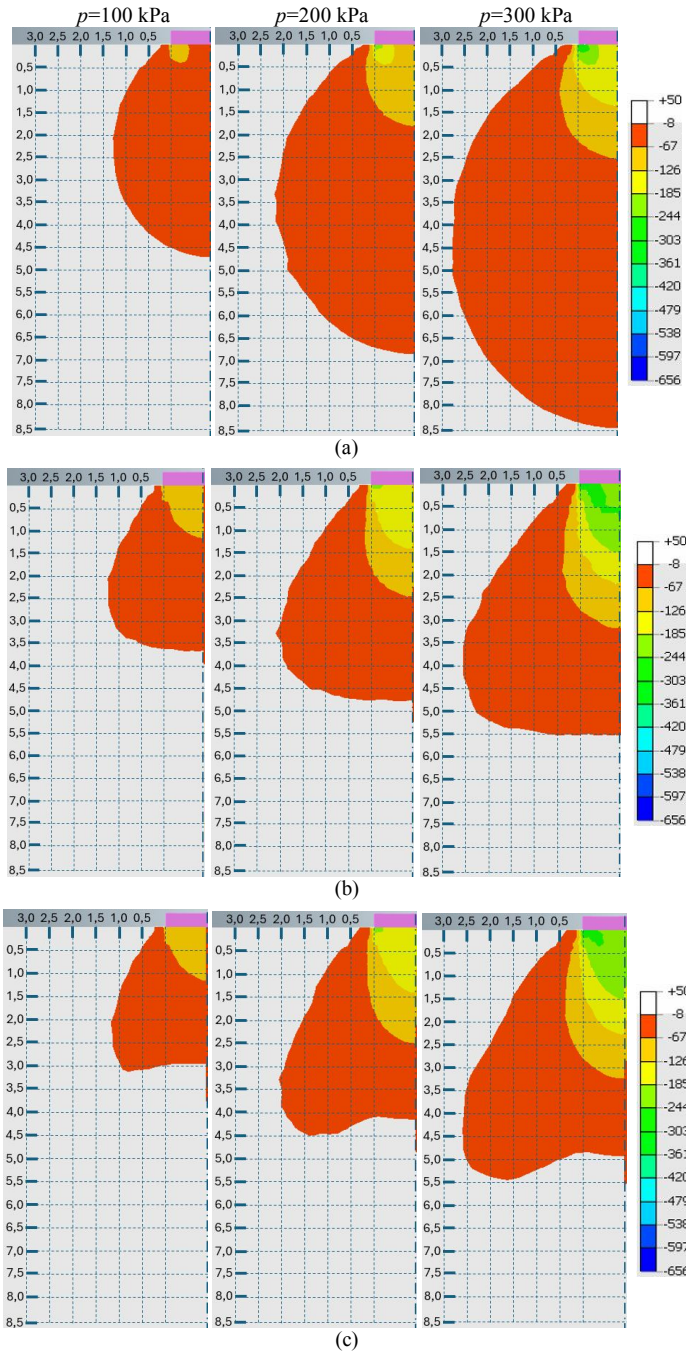


Fig. 4. Iso-surfaces of vertical stress distribution σ_{pp} in sandy soil ($E=30$ MPa) obtained by FEA with Elastic (a), mM-C (b) and HSsmall (c) models under varying pressure levels (100, 200, 300 kPa)

– Modified Mohr-Coulomb (mM-C) (Fig. 4(b)): A slight variation in the isofield geometry is observed, resulting from the shear strength limits defined by the Mohr-Coulomb criterion.

– HSsmall Model (Fig. 4(c)): A distinct "stress concentration" is evident here. The zones of high

stress in the HSsmall model are more localized and extend deeper. This is a direct consequence of incorporating small-strain stiffness, which accounts for the soil's increased resistance in zones with minimal deformation.

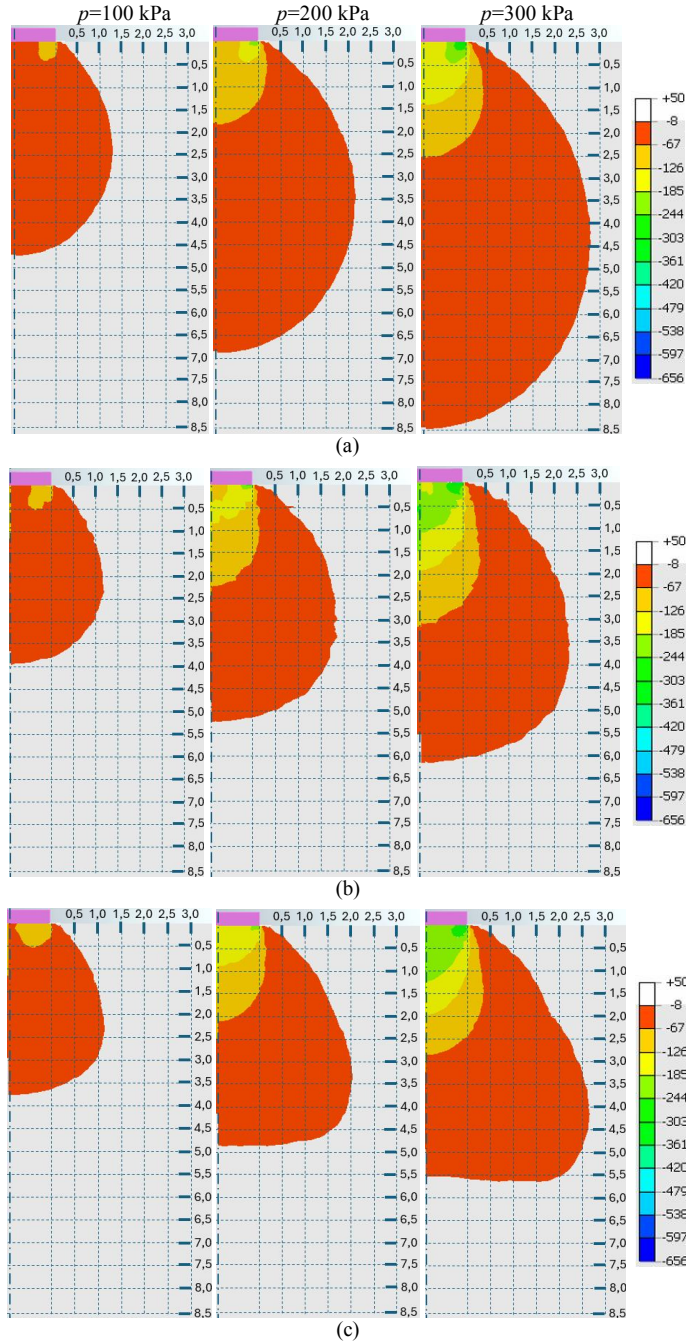


Fig. 5. Iso-surfaces of vertical stress distribution σ_{pp} in clayey soil ($E=15$ MPa) obtained by FEA with Elastic (a), mM-C (b) and HSsmall (c) models under varying pressure levels (100, 200, 300 kPa)

1) *Central Foundation Zone.* In the immediate vicinity of the foundation base ($z < 1,0$ m), both the analytical method and the Elastic model demonstrate maximum stress values that approach the intensity of the applied load (300 kPa). In contrast, the mM-C and HSsmall models record slightly lower stress

values directly beneath the base, which is attributed to the redistribution of contact pressure resulting from the development of plastic deformations and non-linear stiffness.

However, at depths exceeding 2 m ($>B$), an opposite trend is observed: the HSsmall model exhibits a slower rate of stress attenuation. This indicates that incorporating small-strain stiffness leads to the formation of a deeper and more concentrated "stress bulb" compared to the classical analytical solution.

2) *Corner Zone of the Foundation*. The corner sections exhibit the most significant relative discrepancy in the results. Compared to non-linear models, the analytical method substantially underestimates the stresses in the upper part of the soil mass. The plots indicate that the HSsmall model yields higher stress values σ_{zp} at depths of 1–4 m. This suggests that the actual stress distribution in corner zones is more intensive than predicted by the theory of a linear-elastic half-space, particularly under high loading levels (300 kPa in this case).

3) *Stress Equalization Effect with Depth*. The plots clearly demonstrate a convergence point of the results. At a depth of 3 m (approximately 1.5B), the discrepancy between the stresses in the central and corner zones significantly decreases across all methods. However, the stress attenuation rate in the HSsmall model is the least intensive: the stress profile remains more "filled" (sustained) at greater depths (exceeding 6 m). This confirms the issue of stress underestimation when using simplified analytical dependencies.

It is noteworthy that at depths of 3–4 m, the stresses calculated via the HSsmall model still maintain a significant magnitude (approximately 40–50 kPa), whereas the analytical solutions show a much faster decay.

To investigate how stiffness and the soil constitutive model affect the spatial stress distribution, a simulation of a clayey subgrade scenario ($E=15$ MPa) was conducted (Fig. 5). In clayey soils, non-linear effects are significantly more pronounced than in sandy soils:

– Elastic Model (Fig. 5(a)): A classical stress distribution is observed, where the shape of the iso-surfaces remains identical for all loading stages (100, 200, and 300 kPa), with only their amplitude changing. The depth of stress penetration is directly proportional to the applied pressure. This represents an idealized scenario that fully aligns with analytical solutions but fails to account for changes in the stiffness of the soil mass.

– Modified Mohr-Coulomb (mM-C) (Fig. 5(b)): At a load of 300 kPa, a slight widening of the iso-surfaces is noticeable in the upper part directly beneath the foundation base. This is attributed to the stress state reaching the shear strength limit (the Mohr-Coulomb failure envelope) and the onset of plastic zone development. However, the overall configuration of the stressed zone remains close to the elastic solution.

– HSsmall Model (Fig. 5(c)): The most significant differences are demonstrated by the HSsmall model. As the load increases to 300 kPa, a "vertical stress concentration" effect is observed:

a) Elongation of iso-surfaces: The "stress bulb" becomes narrower and more elongated in depth compared to the elastic model. This is a direct consequence of incorporating increased soil stiffness at small-strain levels (at the periphery and at depth), which allows the soil mass to "transmit" stress deeper into the profile.

b) Depth of the active zone: It is clearly visible that the iso-surfaces of low stress values (e.g., 20–40 kPa) in the HSsmall model penetrate the soil mass 15–25% deeper than in the elastic formulation.

Thus, the visual analysis confirms that nonlinear constitutive models (particularly HSsmall) describe a more intensive stress state at significant depths. This explains why traditional methods based on the linear-elastic model tend to underestimate the stresses transmitted to the underlying layers of the soil foundation.

4. Results and Discussion

4.1. Vertical Stress Profiles

The analysis of the comparative vertical stress distribution plots σ_{zp} (Fig. 2) reveals significant discrepancies between the classical analytical method and numerical models utilizing different soil constitutive laws.

According to the results obtained via the corner point method, the distribution of vertical stresses indicates a concentration in the central zone directly beneath the foundation base, with a gradual decrease toward the periphery. Starting from a depth of 3 m (corresponding to the 2B range), a leveling

of stress values across the foundation area is observed. This indicates a transition toward a uniform pressure distribution in the deeper layers of the soil mass.

Analysis of the obtained graphical dependencies (Fig. 2) shows that at a load of 300 kPa, there is a significant deviation of the numerical model results from the classical analytical solution.

– *In the upper zone (up to a depth of $0.5B$):* Stress values according to the HSsmall model are slightly lower than the analytical ones. This is explained by the development of plastic deformation zones beneath the foundation edges under high pressure, leading to a redistribution of contact stresses and a reduction in peak values directly under the center of the foundation.

– *In the middle zone and at depth ($>B$):* The HSsmall model demonstrates higher stress values compared to the corner point method. This is due to the incorporation of high initial soil stiffness at small strains (G_0). Consequently, the "stress bulb" in numerical models appears more vertically elongated, indicating deeper stress penetration into the sandy subgrade mass.

– *Influence of nonlinearity:* The use of a constant modulus $E=30$ MPa in analytical calculations leads to an underestimation of actual stresses at depths exceeding 3–5 m. Specifically, at a pressure of 300 kPa, the discrepancy between the analytical solution and the HSsmall model at the depth of the active zone can reach 2 to 4 times, which is critical for the accurate prediction of settlement.

The invariance of vertical stress profiles when varying the deformation modulus ($E=15...30$ MPa) in analytical calculations is explained by the nature of the mathematical framework employed. The traditional corner point method is based on the solution from the theory of elasticity for an isotropic half-space, where the stress distribution function is purely geometric and does not account for the physical and mechanical properties of the medium. This creates an illusion of an identical stress state across different soil types—a notion refuted by numerical modeling that incorporates the non-linear dependence of stiffness on strain levels. The HSsmall model accounts for stiffness degradation as the load increases (from 100 to 300 kPa), which leads to the observed discrepancies.

Based on the comparative analysis, a discrepancy coefficient was established (Fig. 6–7) between the vertical stress values obtained via the analytical method and numerical simulation. It was found that starting from a depth of $z=B$, this coefficient demonstrates high sensitivity to the applied pressure intensity (100–300 kPa). In the depth range from $z=B$ to $2B$, a peak increase in discrepancy (up to 2–4 times) was recorded, indicating the maximum inconsistency between the analytical model and the actual stress state. With further depth (beyond $2B$), a gradual convergence of results is observed. The equalization of the additional pressure influence was recorded at a depth of $z \approx 3B$ for the sandy subgrade and $2B$ for the clayey subgrade.

4.2. Analysis of Stressed Zone Transformation Depending on Subgrade Stiffness Characteristics

To describe the effect of soil type, it is essential to focus on the physics of the process: how the stiffness (E) and the constitutive model influence the spatial stress distribution. A comparative analysis of the vertical stress iso-surfaces for the sandy (Fig. 4) and clayey (Fig. 5) subgrades reveals a qualitative difference in the patterns of "stress bulb" formation as the external pressure increases from 100 to 300 kPa.

1) *Influence of the Deformation Modulus on the Geometry of the Stressed Zone.* In the sandy subgrade ($E=30$ MPa), the stress distribution is more localized. Due to higher stiffness, the soil mass offers greater resistance to deformation, which promotes stress concentration directly in the zone beneath the foundation base.

Conversely, in the clayey subgrade ($E=15$ MPa), an effect of "deepening" and widening of the stressed zone is observed. The lower deformation modulus means that a significantly larger volume of the soil mass must be engaged to balance the identical external load. This is visually reflected in the increased depth and width of the additional stress iso-surfaces.

2) Differentiation by Constitutive Model Type:

– *Elastic Model:* Demonstrates geometric similarity of the iso-surfaces for both soil types, ignoring actual changes in stiffness. This fails to reflect the physical essence of the process under high pressure levels.

– *HSsmall Model:* Most clearly illustrates the difference between soil types. In the clayey subgrade ($E=15$ MPa) at a pressure of 300 kPa, the intensive stress zone penetrates deeper than in the sand. This is

attributed to the more intensive stiffness degradation of clayey soil as strains accumulate. By accounting for small-strain levels at the periphery, the HSsmall model captures the "stiff skeleton" of the soil mass beyond the active zone, resulting in a more vertically elongated isofield shape.

3) *Nonlinear Stress Redistribution Effect.* It has been established that as the subgrade deformation modulus decreases by half (from 30 MPa to 15 MPa), the discrepancy between the analytical solution and the HSsmall model increases. In weaker soils (Fig. 5), a significant deviation in the shape of the stressed zone from the classical elastic half-space is observed. This confirms the hypothesis that traditional analytical methods most significantly underestimate the stress penetration depth precisely in subgrades with low strength and stiffness parameters, where the nonlinear behavior of the soil becomes the governing factor.

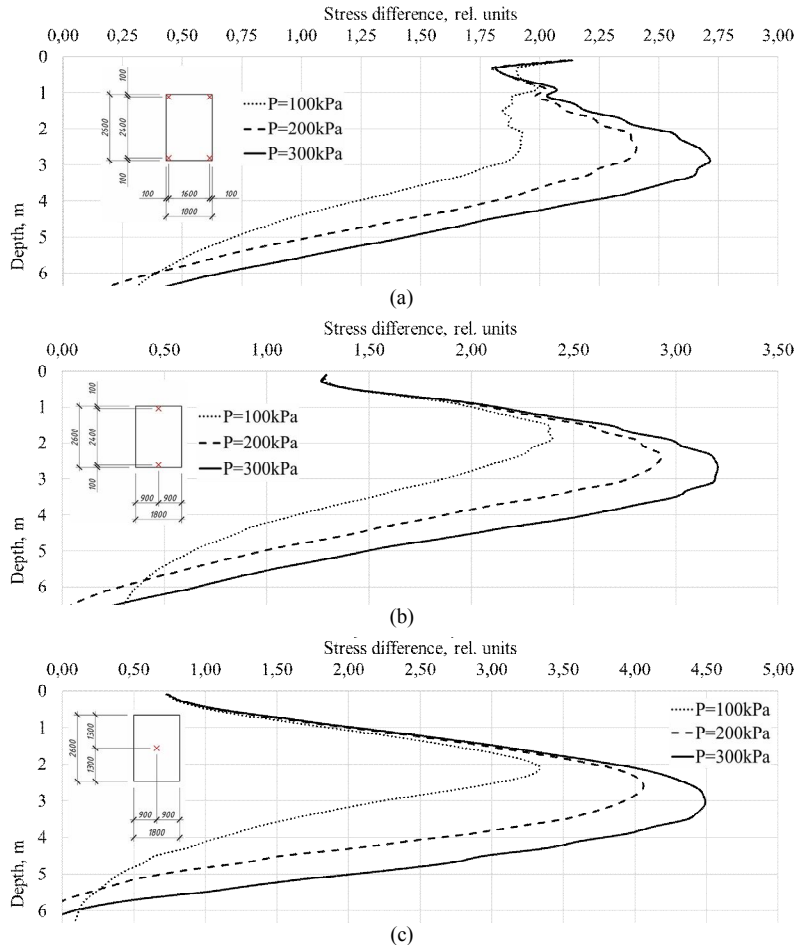


Fig. 6. A quantitative comparison of vertical stress (analytical method vs. numerical simulation) in sandy soil ($E=30$ MPa): (a) - in the corner zone of the slab; (b) - top/bottom; (c) - center

4.3. Key Research Findings

Quantitative discrepancies between methods have been identified. It was established that traditional analytical methods (based on the Boussinesq's solution) systematically underestimate vertical stress levels at depths exceeding $(1.0...1.5)B$. At a maximum load of 300 kPa, it was recorded that the discrepancy between analytical data and simulation results using the non-linear HSsmall model can reach 2 to 4 times. Such a difference is critical for the accurate determination of the compressible strata boundary (depth of the active zone).

The transformation of the stressed zone geometry has been demonstrated. The visualization of stress iso-surfaces confirmed the hypothesis of "vertical stress concentration" in advanced constitutive models. Unlike the linear-elastic half-space, where the shape of the "stress bulb" remains constant, the HSsmall

model exhibits a vertical elongation of the iso-surfaces. This effect is most pronounced in clayey soils due to the more intensive stiffness degradation associated with the accumulation of plastic strains.

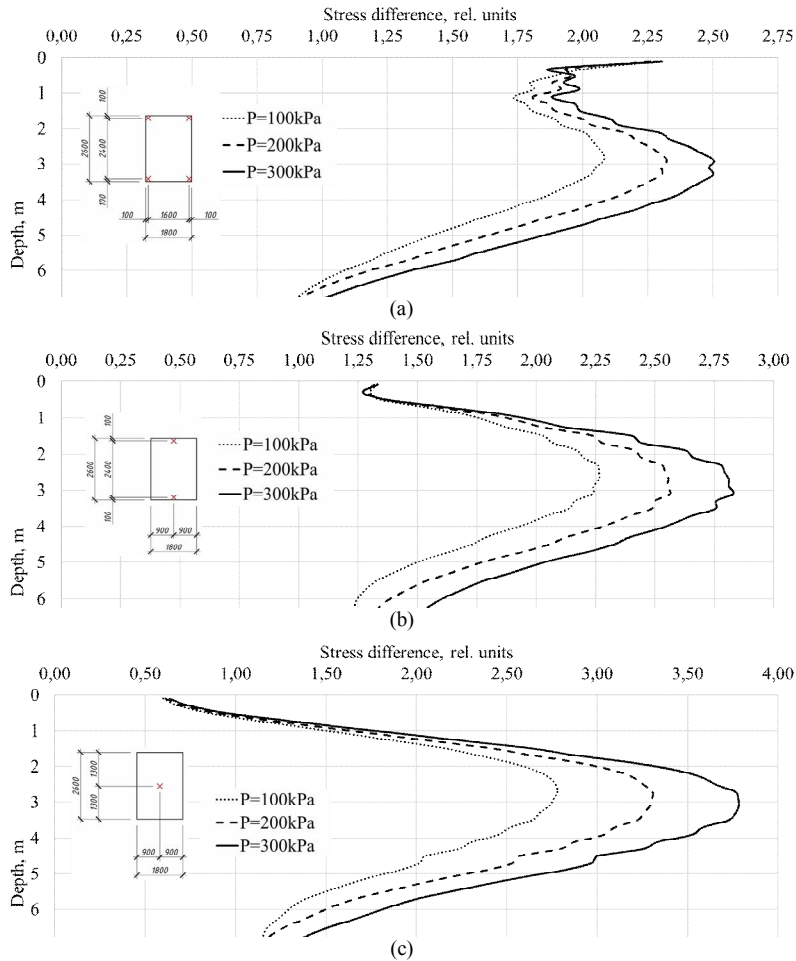


Fig. 7. A quantitative comparison of vertical stress (analytical method vs. numerical simulation) in clayey soil ($E=15\text{ MPa}$): (a) - in the corner zone of the slab; (b) - top/bottom; (c) - center

The zonal specificity of stress distribution has been confirmed. A non-uniform distribution of error within the analytical method across the foundation base was recorded. In the central zone directly beneath the base, non-linear models indicate a stress deconcentration effect due to the development of local plasticity. Conversely, in the corner zones and at the periphery, an opposite effect is observed: the analytical method yields significantly lower values than the numerical simulation. This indicates the actual engagement of a larger volume of soil in the subgrade's performance.

The necessity of correcting the stress dissipation coefficient has been justified. It has been proven that existing tabulated values for the stress dissipation coefficient (vertical stress factor) require refinement through the introduction of correlation functions. These functions should account for soil nonlinearity and the relative loading level. It was established that the error in determining the coefficient increases nonlinearly as the pressure on the subgrade increases and its deformation modulus decreases.

Verification of models for different soil types has been performed. Through numerical experimentation, it has been confirmed that the HSsmall model is the most adequate for predicting the stress-strain state (SSS). Among the models considered, it is the only one that accounts for increased soil stiffness beyond the zone of active deformation. This allows for a more accurate localization of the "stress core" and helps avoid errors when designing foundations on weak subgrades.

The nonlinear nature of the relative error in analytical calculations, depending on depth and loading level, has been established. A characteristic feature is that the influence of the pressure magnitude (100, 200, 300 kPa) on the stress distribution becomes a governing factor only at depths exceeding B (where B is the foundation width). The maximum amplitude of deviation between numerical and theoretical values peaks within the compressible strata at a depth of $z = (1...2)B$, where analytical methods demonstrate the greatest underestimation of stresses. A gradual decay of the discrepancy gradient occurs at a depth of $(3...4)B$, which corresponds to the lower boundary of the active zone of foundation-subgrade interaction.

5. Conclusions

Based on the conducted research and the comparison between analytical methods and numerical modeling (Midas GTS NX) for sandy and clayey subgrades, the following conclusions can be drawn:

1. The traditional corner point method, based on the linear-elastic half-space model, adequately describes stress distribution only at low loading levels (up to 100 kPa). As the pressure increases to 200–300 kPa, the analytical method exhibits significant deviations from the actual physics of soil behavior, as it fails to account for the nonlinear stiffness changes and the plastic properties of the soil mass.

2. The application of the HSsmall model established that incorporating increased soil stiffness at small strains leads to the formation of a more vertically elongated "stress bulb". Compared to analytical calculations, numerical modeling records vertical stress values that are 2 to 4 times higher at depths exceeding $1,5B$. This indicates a deeper penetration of the active compression zone.

3. The influence of soil type on calculation reliability has been demonstrated. In sandy subgrades ($E=30$ MPa), the discrepancy between the analytical method and the Finite Element Method (FEM) is noticeable but more localized. In clayey subgrades ($E=15$ MPa), due to lower stiffness and intensive stiffness degradation, the analytical method shows a critical underestimation of stresses. This proves that for weak soils, the use of simplified models leads to significant errors in settlement prediction.

4. It has been established that the greatest underestimation of stresses by the analytical method occurs in the corner zones of the foundation at intermediate depths. Conversely, in the central zone beneath the base, non-linear models record a pressure redistribution (deconcentration) resulting from the development of local plastic zones, a phenomenon ignored by the corner point method.

5. The nonlinear nature of the deviation between numerical results and the analytical solution, depending on depth and loading level, has been established. It was found that the stress discrepancy coefficient becomes sensitive to the magnitude of external pressure at depths $z > B$. In the range of $z = (1...2)B$, a peak discrepancy was recorded, where values from the HSsmall model exceed analytical results by 3 to 4 times, indicating a maximum stress concentration within the compressible strata. At depths exceeding $3B$, a leveling of the additional pressure influence is observed, along with a gradual convergence of the results from both methods.

6. The research results confirm the necessity of transitioning from traditional analytical calculations to numerical modeling using Hardening Soil-type constitutive models. This is particularly critical for high-consequence raft-type structures with a contact pressure exceeding 200 kPa. Ignoring soil nonlinearity can lead to an incorrect determination of the compressible strata thickness and, consequently, to an underestimation of the calculated foundation settlement.

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НЕДООЦІНЕННЯ ВЕРТИКАЛЬНИХ НАПРУЖЕНЬ У ҐРУНТОВІЙ ОСНОВІ ПРИ ВИКОРИСТАННІ АНАЛІТИЧНИХ МЕТОДІВ: ЧИСЛОВЕ ОБҐРУНТУВАННЯ З УРАХУВАННЯМ НЕЛІНІЙНОЇ РОБОТИ ҐРУНТУ

Традиційні методи розрахунку основ часто базуються на лінійно-пружних моделях, що може призводити до некоректної оцінки стисливої товщі, особливо при високих інтенсивностях навантаження. Мета дослідження - кількісна оцінка похибки аналітичного методу кутових точок порівняно з сучасними нелінійними моделями ґрунту у різних інженерно-геологічних умовах. Для реалізації дослідження було виконано серію числових експериментів методом скінченних елементів для залізобетонної плити розміром 1,8×2,6 м. Розглянуто два типи основ: піщану ($E=30$ МПа) та глинисту ($E=15$ МПа). Розрахунки проводилися для різних моделей середовища, з акцентом на Hardening Soil Small Strain (HSsmall), що враховує деградацію жорсткості.

За результатами дослідження верифіковано гіпотезу про «вертикальну концентрацію» напружень у нелінійних моделях. Встановлено, що форма «цибулини напружень» суттєво трансформується залежно від типу ґрунту: у глинистих основах спостерігається інтенсивніше проникнення напружень у глибину, ніж передбачає пружний розв'язок. Зафіксовано перерозподіл контактних напружень у центральній та кутових зонах плити. Доведено, що похибка аналітичного методу зростає пропорційно навантаженню та зменшенню модуля деформації ґрунту. Результати моделювання доводять необхідність впровадження поправкових коефіцієнтів до табличних значень коефіцієнтів розсіювання напружень. Застосування моделі HSsmall дозволяє отримати найбільш достовірну картину НДС, що є критично важливим для уникнення недооцінки напружень у глибоких шарах основи.

Ключові слова: плитний фундамент, ґрунтова основа, вертикальні додаткові напруження, числове моделювання, Midas GTS NX, Hardening Soil Small Strain (HSsmall), деградація жорсткості, жорсткість при малих деформаціях, розподіл напружень.

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UNDERESTIMATION OF VERTICAL STRESSES IN SOIL FOUNDATIONS USING ANALYTICAL METHODS: NUMERICAL SUBSTANTIATION CONSIDERING NONLINEAR BASE BEHAVIOR

Traditional methods for foundation subgrade analysis are frequently based on linear-elastic models, which may lead to inaccurate estimations of the compressible thickness, particularly under high load intensities. This study aims to quantify the error of the analytical corner point method in comparison with advanced nonlinear constitutive soil models across diverse geotechnical conditions. To achieve this, a series of numerical experiments using the finite element method (FEM) was conducted for a reinforced concrete raft foundation with dimensions of 1.8×2.6 m. Two types of subgrades were analyzed: sandy soil ($E=30$ MPa) and clayey soil ($E=15$ MPa). Calculations were performed using various constitutive models, with a primary focus on the Hardening Soil Small Strain (HSsmall) model, which accounts for strain-dependent stiffness degradation.

The research results verify the hypothesis of "vertical stress concentration" within nonlinear models. It was established that the geometry of the "stress bulb" undergoes significant transformation depending on the soil type: in clayey subgrades, a more intensive stress penetration depth is observed compared to the predictions of the elastic solution. Furthermore, a redistribution of contact pressures in the central and corner zones of the raft was recorded. It is proven that the error of the analytical method increases proportionally to the load magnitude and the decrease in the soil's deformation modulus. The simulation results demonstrate the necessity of implementing correction factors for the standard tabular stress distribution coefficients α . The application of the HSsmall model provides the most reliable representation of the stress-strain state (SSS), which is critical for avoiding the underestimation of stresses in the deeper layers of the foundation base.

Keywords: raft foundation, geotechnical medium, vertical stresses, numerical simulation, Midas GTS NX, Hardening Soil Small Strain (HSsmall), stiffness degradation, small-strain stiffness, stress distribution

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У статті обґрунтовано систематичне недооцінення вертикальних напружень у ґрунті при використанні традиційних аналітичних методів. Шляхом числового моделювання у Midas GTS NX для піщаних та глинистих основ встановлено, що нелінійна модель HSsmall фіксує у 2-4 рази вищі значення напружень на глибинах понад 1,5Б. Доведено, що похибка аналітичного розв'язку зростає пропорційно навантаженню та зменшенню жорсткості ґрунту. Результати вказують на необхідність корекції коефіцієнтів розсіювання напружень для підвищення точності розрахунку стисливої товщі. Табл. 1. Іл. 7. Бібліогр. 6 назв.

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The paper substantiates a systematic underestimation of vertical soil stresses when employing traditional analytical methods. Through numerical simulation in Midas GTS NX for both sandy and clayey subgrades, it was established that the nonlinear HSsmall model records stress values 2-4 times higher at depths exceeding 1.5B. It is demonstrated that the analytical solution error increases proportionally to the applied load and the reduction in soil stiffness. The results indicate a necessity for correcting the stress distribution coefficients to improve the accuracy of compressible thickness calculations.

Tabl. 1. Fig. 7. Ref. 6.

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