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MEDITATIONS ON EQUI-STRENGTH, EQUI-STABILITY, EQUI-RELIABILITY AND OPTIMAL DESIGN

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We provide some thoughts on three interrelated concepts of equi-strength, equi-stability, and equi-reliability in conjunction with optimal design. Hopefully, these thoughts might spark some discussion.

Keywords: equi-strength, equi-stability, equi-reliability, reliability.

1. Introduction

The problem of rationally distributing available resources among the elements of a complex system is a fundamental task that arises in a wide variety of situations. In the strength calculation of engineering structures, the solution to this problem sometimes takes the form of the principles of equistrength, equistability, or equireliability of the system components.

Such an approach has deep historical roots and is associated with the idea of fully utilizing the material's strength properties, which, at first glance, seems not only reasonable but also the only correct one. This approach to engineering decision-making has firmly established itself as a tradition. This tradition is closely related to the ideas of determinism and complete predictability of the behavior of the created structural complex, and the possibility for the designer to "foresee everything." However, as experience shows, this is not always the case, and in this paper, we will attempt to analyze the validity and possible universality of this "equalizing" paradigm.

This question gains particular significance when the subject of analysis is not the structure's performance under "normal" operating conditions (including possible limit states), but rather its extreme states accompanied by local damage, and the assessment of their resistance to disproportionate (progressive) collapse.

2. Equi-strength

In his principal book [1], the "father" of modern science, Galileo Galilei, placed the science of material resistance on the notion of failure. Moreover, he found the ideal parabolic shape for a beam subjected to a transverse force, a beam we now call equistrength (or fully stressed). The problem of the compressed column of minimum volume, with equi-strength shape, was posed and solved by Joseph-Louis Lagrange in 1770–1773 [2]. The famous Maxwell-Michell structures [3, 4, 5] were also equistrength.

The optimization approach to structural mechanics problems began with the problem of selecting the cross-sections of members in a system of minimum volume. Fundamental results were obtained here by Maurice Levy, who proved that the solution can be expressed as a statically determinate system.

The first general works in the field of searching for equistrength truss structures belong to A. Pippard [6] and H. Heimann [7]. They contained a prescriptive description of a method for calculating statically indeterminate trusses, which consists of specifying the forces in the redundant (conditionally necessary) members, based on which the sections of all other members are selected from the condition that the stress in them equals the limiting value. Thus, the truss (with the exception of the redundant members) turns out to be "fully stressed".

A small brochure by Heinrich Heimann [7], who proposed using the areas of the cross-sections of the truss elements as unknowns when resolving static indeterminacy, led Rabinovich [8] to the idea of

solving the optimal truss design problem using the method of installed stresses, which is based on the idea of equistrength [8]. It turned out that achieving complete equistrength in a statically indeterminate system is impossible. This result was formulated as an addition to Levy's theorem in the form [8, p. 53]: "The highest achievement possible in the area of stress distribution is that the stresses in at least the members of the chosen primary system of the force method have a specified magnitude".

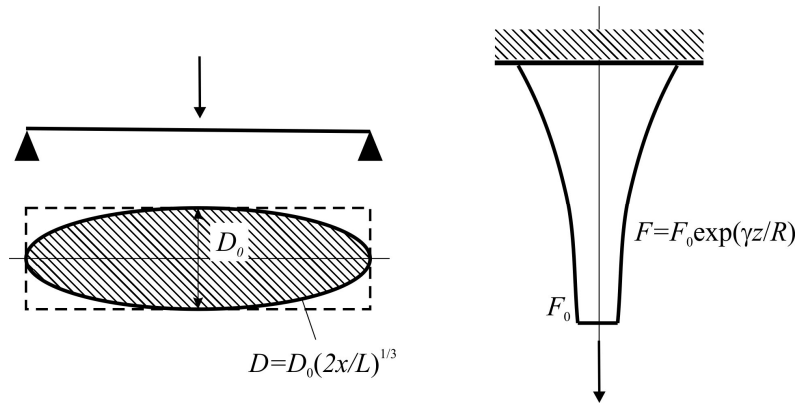


Fig. 1. Equally stressed structures under bending and tension

An important proposition was proven regarding the possibility of creating a truss of minimum weight on the condition of using prestressing (this result was rediscovered much later by Hofmeister and Felton [9]).

But an equistrength and fully stressed structure is intuitively appealing because every element "gives all its strength" to support the loads. Therefore, the search for such structures actively continued. Special interest in fully stressed design was sparked by the aerospace industry in the 1940s and 1950s, where it was known as the "use-one-boss" principle or simultaneous failure mode design [10].

It was proven that statically determinate equistrength structures (truss systems, membranes, membrane shells) are optimal in terms of material consumption [11], but this fact is debatable for statically indeterminate systems. There are examples where equistrength structures were optimal, but this is not the case in general. One positive example is the solution to the problem of a rational elastic simply supported plate obtained by Mushtari [12]. The volume of an equi-strength plate of variable thickness amounted to 74% of the volume of a plate of constant thickness. For a plate clamped along the contour, this indicator is 79.4%.

For a single load case, a number of other problems were solved for creating equistrength structural elements (members, plates, cutout reinforcement, discs, shells of revolution) [13, 14]. In the spheroidal droplet-shaped reservoir shown in Fig. 2 as an example, all structural elements are stretched by the same force, making it an equi-stressed shell.

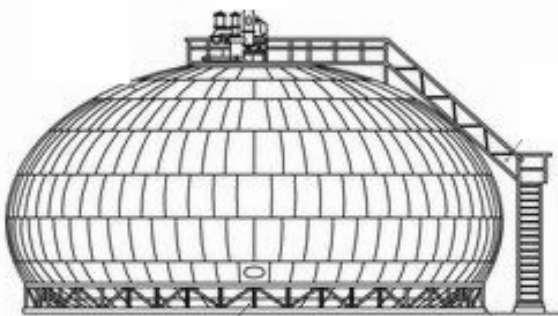


Fig. 2. Teardrop-shaped reservoir

This and other examples, as well as considerations of simplicity of implementation, led to the fact that the striving for a situation where the limit stress state is realized in every element in at least one load case (the principle of discrete equistrength [11]) became a widely used design technique.

The following considerations were adopted as the basis for the calculation algorithm:

- If the structure is statically determinate, the size of each element can be

chosen independently so that it can be fully stressed.

- For a statically indeterminate system, the iterative process is based on the idea that if an element is overstressed, its size should be increased, and if it is under stressed, its size can be reduced to save materials.

Fig. 3 shows the geometric representation of the calculation process in the space of stiffness parameters x_1 and x_2 . Curves 1 and 2 correspond to the boundary strength conditions (the inadmissible region is shaded), and the first approximation, represented by point A_0 , indicates the elements are overstressed. All points on the ray passing through A_0 correspond to the stiffness ratio at which the internal forces were found, and resizing means the sections of elements 1 and 2 move to points B_1 and B_2 , respectively. They are located on the boundary of the admissible region (the limit stress is maintained) and on the original ray (the proportionality to the forces is maintained for the previously adopted stiffness ratio).

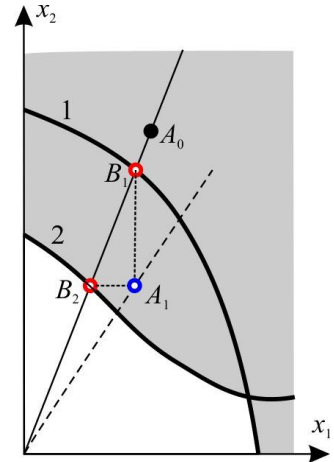


Fig. 3. The geometric interpretation of the iterations

The stiffnesses corresponding to the new sections define point A_1 (the ray changes), and it becomes necessary to account for this in the new iteration. The convergence of this iterative process is investigated in [15], and the result depends primarily on the initial structure used to initiate the iterative method. This work presents three illustrative examples of possible outcomes of iterations, in which fully stressed sections are selected at each step (Fig. 4).

- In the problem in Fig. 4 (a), the iterative process converges to the **equi-strength solution** A_1 , which is also the minimum weight solution.

- The result of the iterative process for the problem in Fig. 4 (b) depends on the state from which the iterations start. If we start with $I_2 > I_1$, we arrive at solution B_1 in the form of two cantilevers with a degenerate middle section; otherwise, the iterations converge to the degenerate solution B_2 , in which a balanced moment load acts on the middle beam section. One of these solutions corresponds to the minimum weight system.

- Finally, in problem 4 (c), depending on the starting distribution of stiffnesses, the iterations also converge to different equistrength solutions, but only solution B_2 is also the minimum weight system.

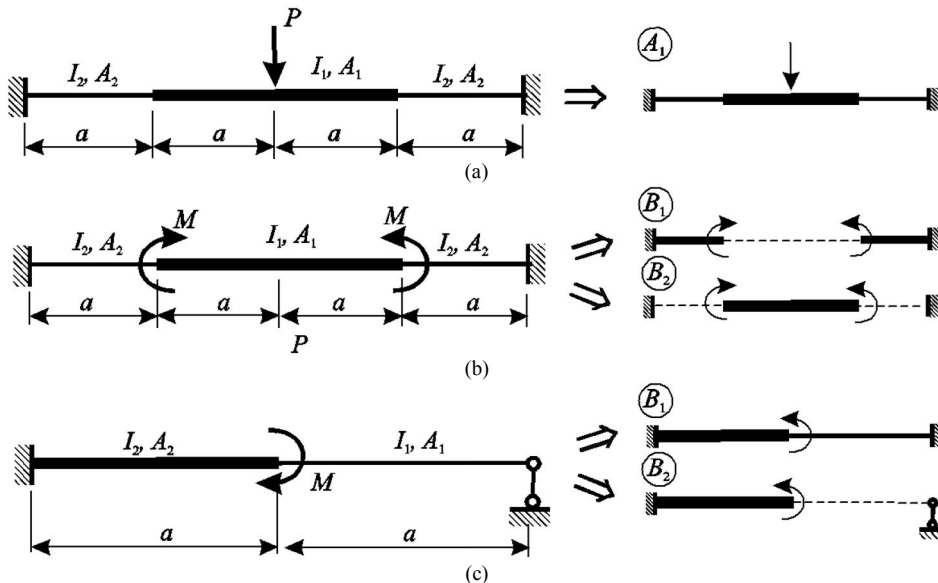


Fig. 4. Variants of possible optimal solutions

In the literature on optimal design, there are methods based on optimality criteria [16], in which, for example, instead of minimizing the objective function, a solution to the equations and inequalities of the Kuhn-Tucker optimality criterion is sought. Sometimes, as in the search for a fully stressed structure, other conditions (conditions of rationality) are used instead of the optimality criterion, but then it cannot be stated that the optimal solution is being sought [17]. However, structures designed in this way are often close to minimum volume structures (rational) in many practical problems.

It should be noted that the deviation from the optimum that may arise from using a fully stressed structure is quite significant for aerospace technology objects, where the reduction in weight launched into orbit more than covers all additional costs from the complication of the structural solution. For most other objects, the issue is less critical, and other problems may come to the fore.

A characteristic example is the work [18], which proposes a method for solving the problem of optimal grouping of structural elements into unified groups for a fully stressed structure (Fig. 5). The solution moves even further away from the material-consumption optimum, but the economic effect arises due to the simplification of the manufacturing technology.

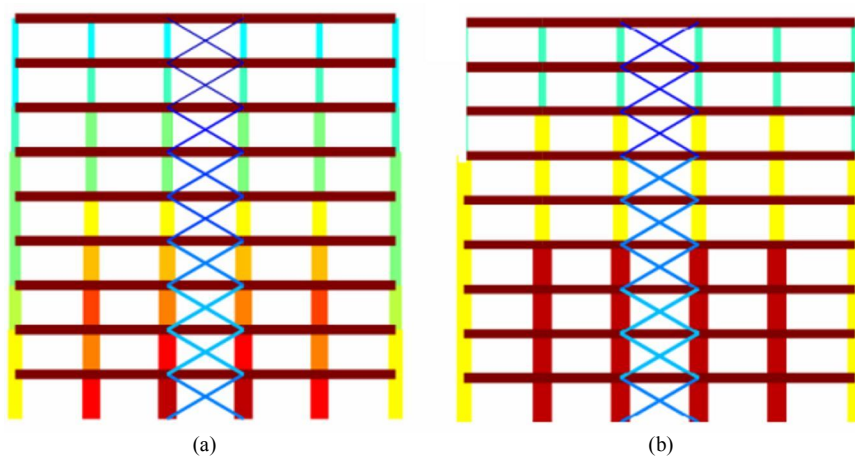


Fig. 5. Grouping result: (a) minimum weight system, (b) system with unified element groups

It is easy to see that the equi-strength of structural elements silently assumes their equal role in ensuring the reliability of the system as a whole, where the failure of at least one element is a critical event. Such systems are called series systems (in terms of reliability) of their constituent elements; these include, for example, all multi-element statically determinate structures.

It should be noted that the method for assessing the bearing capacity of a structure, which is enshrined in design codes, is based on the scheme of searching for the weakest link in the structure, the search for which is implemented by sequential checks of all design sections. From this point of view, the system is viewed as a series-connected chain of design sections, which is far from reality in all cases. And the designer's desire to ensure that the acting forces equal the bearing capacity for all design sections is, in essence, a striving to build an equistrength structure.

The desire for the practical implementation of equistrength (and not only for statically determinate systems) is associated with the possibility of using prestressing, which was discovered as early as [8]. In work [19], this issue is examined in detail, and methods for determining the necessary prestressing are indicated.

In essence, equistrength means that all elements of the system fail simultaneously under the design load. The drawback of such designs is that they formally do not account for the possibility of even a small change (increase) in external loads, as this change entails a complete loss of bearing capacity of the structure as a whole. The danger of such a situation lies in the fact that, in addition to the loads accounted for in the design, there may be non-design loads that are difficult, and often practically impossible, to foresee (personnel errors, gross defects, etc.).

The structural elements forming the load-bearing system interact in a complex manner during the object's operation. They perform different functions, vary in load, and differ in wear rate. While the

differences between these indicators across various elements of the system are relatively small in normal operation mode, this conclusion changes sharply when considering possible extreme situations.

The fact is that building elements with large safety margins or operating in a lighter mode (under stressed) can be considered, in a certain sense, redundant; they may be necessary to prevent general collapse in emergency situations not directly provided for by the design. This is the basis for the strategy of protecting a building from disproportionate collapse using the alternative load path method, which significantly improves the system's survivability [20, 21]. On the other hand, the opposite problem statement is also possible, where the conscious reduction in the strength of some system elements, which take on protective functions and prevent the overloading of other elements by their own failure, allows influencing the assumed collapse pattern and achieving the localization of possible troubles through this [22].

Thus, it is quite logical to conceive of a conscious violation of the equistrength principle with respect to elements (blocks, subsystems) that play different roles in ensuring the overall reliability of the structure. However, if the corresponding subsystem is structurally composed of several equally important parts, then the application of the equistrength principle to ensure the workability of their connection (a bolted or welded joint, for example) is undoubtedly logical. Creating joints based on their equistrength with the parts they connect is fully justified, and this approach has become a universally accepted design technique.

3. Equi-stability

The idea of equi-stability was put forward by Friedrich Bleich in 1924 [23]. According to this idea, the Euler critical stress of a centrally compressed member was equated to the critical stress of an ideal elastic plate within the member's cross-section. From this condition, a simple relationship was found between the slenderness of the member and that of the separate plate. For a long time, this relationship was considered acceptable and even included in some regulatory documents for the design of steel structures.

In contrast to strength checks, where the question of overload or underload is solved by a simple comparison of stresses, assessing the role of a particular element in ensuring overall stability is non-trivial. A method for solving this problem was found relatively recently [24]. But the main thing is that it is necessary to compare not the roles of individual elements, but different buckling modes, and only in a few cases are these modes clearly associated with the local instability of specific system elements.

In classical linear stability analysis, different buckling modes do not interact (do not influence each other) due to orthogonality. A deeper analysis, accounting for nonlinear effects, revealed the error of this assumption. This is clearly illustrated by the example of a compressed latticed column, whose buckling was studied in work [25]. In the design model (Fig. 6), it is assumed that failure can occur either due to the buckling of the chord, which has a cross-sectional area S_a and a radius of gyration R , in which case the critical load will be equal to

$$P_{cr,L} = 2\pi^2 Ear^2 / l^2,$$

or due to overall buckling at a critical load of

$$P_{cr,L} = 2\pi^2 EaR^2 / L^2.$$

The column will buckle at the load

$$P_{cr,s} = \min(P_{cr,L}, P_{cr,L}).$$

If we consider the system's behavior at different values of the parameter

$$x = P_{cr,L} / P_{cr,L} = (Rl)^2 / (rL)^2$$

and evaluate the result by the ratio $y = P_{cr,s} / P_{cr,L}$, we get the bilinear characteristic shown in Fig. 6 by the thin line. In reality, the wave formation of the chords, associated with local buckling, reduces their average axial stiffness, which affects the value of the critical force for overall buckling. This mechanism of interaction between buckling modes is nonlinear, giving the solution shown by the thick line in the figure. The maximum deviation from the classical linear solution is observed at $x=1$, i.e., in the equistable system.

The buckling of stiffened plates and shells has important implications for the optimal configurations obtained in calculations. They are characterized by the presence of multiple buckling

modes (overall, local). The dependencies of the critical loads for the overall and local modes on the structural parameters are, in general, of opposite sign. For example, increasing the thin-walledness parameter (the ratio of an overall cross-section dimension to the thickness) increases the critical load for overall modes and decreases it for local modes. Based on these considerations, it was proposed to design these structures such that the critical stresses for these modes were equal.

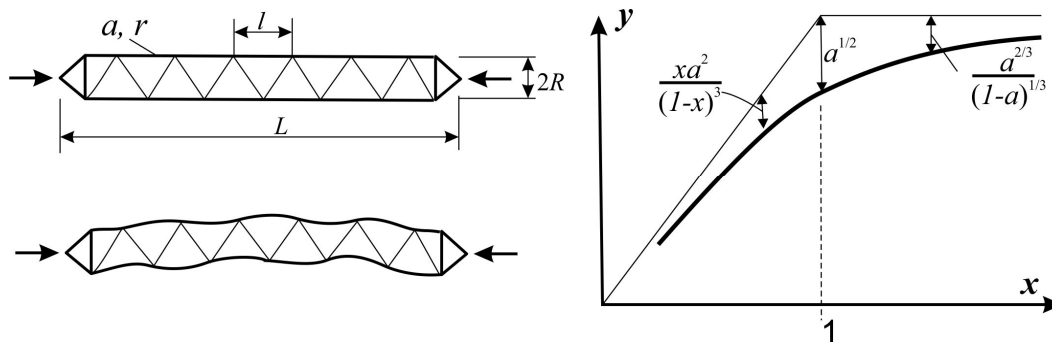


Fig. 6. A compressed latticed column

The equistability condition (like equistrength) was widely used in the construction of various optimization algorithms. Subsequently, it turned out that structures designed according to the equistability principle showed a particular tendency to be destabilized by initial imperfections. The danger of such a "naive" approach to the optimization problem, based on the equi-stability condition, was emphasized in the review [26], which indicated that, for a spatial system in which the equi-stability principle is satisfied, structural failure of avalanche-type is possible [27].

It was this feature of behavior that prompted J.M.T. Thompson [28] to call equi-stability a naive principle of optimization. He notes that symmetry and the equistability corresponding to optimization are, in some sense, pathological phenomena and are structurally unstable in the topological sense. Thompson notes that they are not observed in nature, and human attempts to violate this rule are met with resistance from nature in the form of symmetry-breaking imperfections.

A detailed study of a system with two degrees of freedom showed that at equal or sufficiently close critical loads corresponding to different buckling modes, it is supersensitive to initial imperfections, and the maximum bearing capacity tends to deviate from equi-stability upwards or downwards, depending on the level of initial imperfections [27].

The creation of the theory of post-buckling behavior of structures (W.T. Koiter [25]) made it possible to find out that the sensitivity of buckling to geometric imperfections significantly depends on the nonlinear interaction of buckling modes. It is clear that this conclusion should have changed the usual approach to structural optimization problems, requiring them to be solved in a nonlinear formulation that considers the interaction of buckling modes (coupled buckling).

A typical example here can be the work of Manevich [29], which considered the problem of weight optimization of compressed panels stiffened with longitudinal rectangular and tee cross-section ribs. The analysis was based on solutions to the nonlinear problem in the first asymptotic approximation, which is applicable to structures sufficiently close to equi-stable ones (the difference in critical stress is no more than about 2 times).

The critical loads of thin-walled structures, such as plates and cylindrical shells, sharply decrease in the presence of initial geometric imperfection, and the magnitude of the decrease in the bending load strongly depends on the shape of the imperfection. Therefore, extensive research has been conducted to identify the worst imperfections that reduce the critical load, within the limits set by some norm. The worst imperfections have been found for various types of critical points, including, very importantly, coincidence critical points.

4. Equi-reliability

For systems with a series connection of elements in terms of reliability (recall that design codes assume all structures are such), the overall reliability is determined by the least reliable element. If it is

impossible to increase its reliability for some reason, then requiring increased reliability for all other elements makes no sense. This property of series systems underlies the principle of equireliability.

However, when there is a need to increase the system's reliability, and the means for implementing this measure are limited, the problem of the optimal allocation of these means arises. Work [30] provides a solution to the problem of minimizing the overall reliability of a system with elements having a probability of failure $P_{f1}=P_{f2}=\dots=P_{fn}$ and whose conditional cost is $C_1=C_2=\dots=C_n$.

Ensuring the reliability of the load-bearing structure is associated with certain costs

$$C_f = f(P_f).$$

In the vicinity of the initial value

$$P_{fi} = (P_{0fi})^{-C_i/C_{0i}},$$

This dependence can be approximated by the expression (P_{0fi}, C_0) , which shows how the probability of failure changes with a change in the value of C_f .

If, without changing the total cost $C_0=C_1+C_2+\dots+C_n$, the values C_{0i} are redistributed to minimize the system's probability of failure, then the values

$$C_j = C_{0j} \left[1 + \frac{1}{\ln(1/P_{0j})} \left(\frac{\sum_{i=1}^m C_{0i} \cdot \ln C_{0i}}{C_0} - \ln C_0 \right) \right] \quad (j = 1, \dots, m).$$

should be used. Thus, the reliability of the system is increased by rejecting equireliability.

The formulation of identical reliability requirements (target reliability levels) for objects of approximately the same class of responsibility is perfectly logical, regardless of the material (steel, reinforced concrete, or others) from which they will be designed and the type of structural scheme [31]. However, the equality of target reliability levels is not an automatic requirement for creating an equi-reliable structure. The same applies to the requirement of equal probability of occurrence of design loads, regardless of their number in the design combination [26]. The entire standardization system stands on these positions.

Relatively recently, target reliability levels began to be differentiated for objects of different responsibility classes, taking into account the consequences of failure, including human casualties, economic damage, environmental losses, and loss of cultural and historical values (Eurocode provides for three classes: CC1, CC2, and CC3).

Today, setting target values for reliability measures at the structural element level when checking the corresponding limit states is practically universal. This is explained by the fact that, in traditional calculations, the designer applies the safety format at the element level, or more precisely, at the section level. And despite the fact that structural elements are subdivided into reliability classes (RC) based on the consequences of failure, these classes practically correspond to the CC classes based on the consequences of system failure.

As emphasized in [32], in the design process, depending on the adopted structural solution of the building and the expected failure mode, higher or lower consequence classes should be assigned to individual structural elements than to the structural system as a whole. And although [33] permits the use of different reliability classes for different structural elements, this is practically never done in conventional design.

The situation is different when the problem of ensuring system survivability is considered, and the ability of a damaged structural system to retain operability (possibly limited) after the occurrence of an abnormal action is evaluated. The probability of collapse is then calculated using the multi-component Bayes formula:

$$P(\text{Collapse}) = P_1(\text{Collapse} | \text{LD}) P_2(\text{LD} | \text{H}) P_3(\text{H}),$$

where $P(\text{Collapse})$ is the total probability of disproportionate collapse; $P_3(\text{H})$ is the probability of the H -th emergency action being realized; $P_2(\text{LD} | \text{H})$ is the probability of local destruction of a single structural element upon the occurrence of an emergency event H ; and $P_1(\text{Collapse} | \text{LD})$ is the

probability of disproportionate collapse occurring, given that local destruction of a single element occurs upon the realization of emergency event H.

The designer's possibilities are limited to influencing P_2 (strengthening elements) or P_1 (increasing survivability by improving the structure and system). It is evident that effective protection can be achieved with different values and ratios between P_1 and P_2 , and the solution depends on the adopted protection strategy.

- When using a strategy based on the application of key elements, their reliability is consciously increased against the background of the rest of the structure's elements.
- Another strategy related to system segmentation can be based either on weak segment boundaries or on strong segment boundaries. In both variants, the reliability of the boundary elements differs from that of all other elements. These two strategy variants are well-known in fire protection, where they are implemented in the form of a fire break or a fire wall.

5. Conclusions

The principles of equi-strength, equi-stability and equi-reliability of the constituent elements of a complex system, traditionally used in design practice, have a very specific area of rational use, where they provide acceptable engineering solutions [34-39], but cannot be considered universal.

A conscious deviation from these principles allows influence on certain performance indicators of load-bearing structures and can be an important tool in the decision-making process. A detailed analysis of its capabilities is required to improve this tool.

REFERENCES

1. Galileo Galilei. Discorsi e dimostrazioni matematiche intorno a due nuove scienze, In Leida: MDCXXXVIII
2. Lagrange J.-L. Sur la figure des colonnes // Ouvres de Lagrange, Paris: Gauthier-Villars, 1868 — P. 125-170.
3. Maxwell C. On reciprocal figures, frames, and diagrams of forces, Philosophical Magazine, 1864, Vol. 26 — pp. 250-261.
4. Michell A.G.M. The limits of economy of material in frame-structures, Philosophical Magazine, 1904, Vol. 8(47), pp. 589-597.
5. Mueller K.M., Burns S.A. Fully stressed frame structures unobtainable by conventional design methodology, International Journal for Numerical Methods in Engineering, 2001, Vol. 52, Issue 12, pp.1397-1409
6. Pippard A.J.S. On a method for the direct design of framed structures, having redundant bracing, Aeronautical Research Committee, Reports and memoranda No. 793, 1922.
7. Heimann H. Beitrag zur Berechnung statisch unbestimmter Fachwerke — Berlin: Springer-Verlag, 1928.
8. Rabinovich I.M. To the theory of statically indeterminate trusses, Moscow: Transzheldorizdat NKPS, 1933 (in Russian)
9. Hofmeister L., Felton L. Prestressing in Structural Synthesis, AIAA Journal, 1970, Vol. 8(2). 363-364.
10. Shanley F.R. Weight-strength analysis of aircraft structures, New York: McGraw-Hill, 1952 — 394 p.
11. Malkov V.P., Ugodchikov A.G. Optimization of elastic systems, Moscow: Nauka, 1981 — 288 p. (in Russian)
12. Mushtari H.A. On an inverse problem of the theory of bending of elastic plates of variable thickness // Engineering Journal, 1964, Vol. 4, No. 3, pp. 510-515 (in Russian).
13. Cherepanov G.P. Inverse problems of plane elasticity theory, Applied Mathematics and Mechanics, 1974, No. 6, pp. 963-979 (in Russian)
14. Cherepanov G.P. Equal-strength heavy beam: solution of the Galileo problem, Phys. Mesomech. - 2016, Vol. 19, No. 1, pp. 84-88. (in Russian)
15. Vinogradov A.I. The problem of optimal design in structural mechanics, Kharkov: Publishing house of Kharkov University, 1973 — 168 p. (in Russian)
16. Schmidt I.A. Structural optimization – some key ideas and insights // New directions in optimum structural design, Chichester: Wiley, 1984 — P. 1-45
17. Gallagher R.H. Fully stressed design, Optimum structural design, theory and application — Chichester: Wiley, 1973, pp. 19-32
18. Van Woudenbergab T. R. Van der Meera F.P. A grouping method for optimization of steel skeletal structures by applying a combinatorial search algorithm based on a fully stressed design // Engineering Structures, 2021, Vol. 249, No. 15, 113299
19. Perelmuter A.V. The physical achievement of an optimal prestressed system of beams // International Applied Mechanics, 1970, Vol. 6, No 7, pp. 795-797.
20. Starossek U. Typology of progressive collapse, Engineering Structures. 2007. Vol. 29, pp. 2302-2307.
21. D3-1: Design recommendations against progressive collapse in steel and steel-concrete buildings — Brussels: European Convention for Construction (ECCS), 2021 — 242 p
22. Perelmuter A.V., Veriuzhska T.Y. Optimization of the overload-protection degree // Engineering Optimization IV — London: Taylor & Francis Group, 2014. — P. 529-532
23. Bleich F. Theorie und Berechnung der eisernen Brücken, Berlin: Julius Springer, 1924 (in German).
24. Perelmuter A.V., Slivker V.I. The Problem of Interpretations of the Stability Analysis Results // ECCM-2001. 2nd European Conference on Computational Mechanics. Solid, Structures and Coupler Problems in Engineering. Cracow, Poland, June 26-29, 2001.— Abstracts, Vol. 2. Kraków: Vesalius, 2001, pp. 998-999.

25. Koiter W.T., Kalker G.D.C. The interaction between local buckling and overall buckling on the behavior of built-up columns // Rpt. No 447. Laboratory of Engineering Mechanics, Delft , 1971
26. Koiter W.T., Kalker G.D.C. The interaction between local buckling and overall buckling on the behavior of built-up columns, Report No. 447. Laboratory of Engineering Mechanics, Delft, 1971
27. Korchak M.D. and Galkin S.V. and Kartopolzev V.M., Basics of Instability and Theory of Catastrophes of Engineering Structures, Tomsk university Press, 1987 (in Russian).
28. Thompson J. M. T. Optimization as a generator of structural instability, Int. J. Mech. Sci., 1972, v.14, No. 9, pp. 627-629.
29. Manevich A.I. Weight optimization of a longitudinally compressed panel with T-shape stiffeners, Soviet Applied Mechanics, vol. 26, 1990, № 2, pp. 82-86.
30. Raizer V., Elishakoff I. Philosophies of Structural Safety and Reliability — Boca Raton, FL: CRC Press, 2022.
31. Tur V., Martynov Yu., Nadolski V., Veryovka F. Problems in Providing Equal Reliability of Reinforced Concrete and Steel Constructions Within the Existing Reliability Concept According to EN 1990. // Contemporary Issues of Concrete and Reinforced Concrete: Collected Research Papers — Minsk. Institute BelNIIS. Vol. 10. 2018, pp. 103–120.
32. Diamantidis D., Sykora M. Reliability Differentiation and Uniform Risk in Standards: A Critical Review and Practical Appraisal // Future Trends in Civil Engineering — Zagreb, Croatia: 17 Oct. 2019, pp. 59–78
33. EN 1990. Basis of Structural Design — Brussels: Eurocode European Committee for Standardization, 2002.
34. Prokudin A.N. and Burenin A.A., Design of Equi-Strength Nonhomogeneous Rotating Shafts // Mechanics of Solids, 2024. 59(7), pp. 3704-3711.
35. Cherepanov G.P. Optimum shapes of elastic bodies: equistrong wings of aircrafts and equistrong underground tunnels // Physical Mesomechanics, 2015. 18(4), pp.391-401.
36. Hein K. and Heinloo M. The design of nonhomogeneous equi-strength annular discs of variable thickness under internal and external pressures // International journal of solids and structures, 1990. 26(5-6), pp.617-630.
37. Alexandrov S., Rynkovskaya M. and Jeng Y.R. Design of equi-strength annular disks made of functionally graded materials // Mechanics-Based Design of Structures and Machines, 2024. 52(9), pp.7045-7062.
38. Gori I. and Hartung F. On equi-stability with respect to parameters in functional differential equations // Nonlinear Functional Analysis and Applications, 2016. pp.329-351.
39. Pinus B.L., April. Concept of design assurance for operational reliability of reinforced concrete structures in severe conditions. // IOP Conference Series: Earth and Environmental Science (Vol. 751, No. 1, p. 012066). IOP Publishing. 2021.

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РОЗДУМИ ПРО РІВНОМІЦНІСТЬ, РІВНОСТІЙКІСТЬ, РІВНОНАДІЙНІСТЬ ТА ОПТИМАЛЬНЕ ПРОЕКТУВАННЯ

Розглядається традиційні підходи до проектування будівельних конструкцій, що ґрунтуються на повному використанні їхнього опору і спираються на пошук умов рівноміцності, рівностійкості або рівнонадійності компонентів системи.

Звертається увага на те, що рівноміцність елементів конструкції мовчазно припускає їхню однакову роль у забезпеченні безвідмовності системи в цілому, коли відмова хоча б одного елемента є критичною подією. Тобто конструкція вважається системою з послідовно (у сенсі безвідмовності) з'єднаними елементами, і це часто не відповідає дійсності. Саме тому умови оптимізації не співпадають з принципом рівноміцності. Наводяться приклади свідомого відхилення від рівноміцності з метою підвищення живучості конструкцій.

Проектування рівностійких конструкцій спирається на незалежність різних форм втрати стійкості, але така незалежність порушується при переході до нелінійного аналізу. Тоді виявляється, що конструкції, запроєктовані за принципом рівностійкості, мають особливу схильність до дестабілізуючого ефекту початкових недосконалостей, і тут виникає можливість руйнування конструкцій, що носить лавиноподібний характер.

Наголошується, що цілком логічним є формулювання однакових вимог щодо надійності функціонування (цільових рівнів надійності) до об'єктів приблизно однакового ступеня відповідальності, незалежно від матеріалу, з якого вони будуть запроєктовані, та від виду конструктивної схеми. Справедлива також умова щодо рівної ймовірності появи розрахункових навантажень, незалежно від кількості в розрахунковому поєднанні. Але рівнонадійність елементів конструкції може не відповідати їхньому функціональному призначення, що приводить до зниження загальної надійності. Простішим прикладом є забезпечення живучості шляхом використання ключових елементів з підвищеною надійністю.

Ключові слова: рівноміцність, рівностійкість, рівнонадійність, живучість.

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MEDITATIONS ON EQUI-STRENGTH, EQUI-STABILITY, EQUI-RELIABILITY AND OPTIMAL DESIGN

Traditional approaches to the design of building structures are considered, which are based on the full use of their resistance and rely on the search for conditions of equi-strength, equi-stability or equi-reliability of system components.

Attention is drawn to the fact that the equi-strength of structural elements tacitly assumes their equal role in ensuring the reliability of the system as a whole, when the failure of at least one element is a critical event. That is, the structure is considered a system with sequentially (in the sense of reliability) connected elements, and this often does not correspond to reality. That is why the optimization conditions do not coincide with the principle of equi-strength. Examples of deliberate deviation from equal strength in order to increase the survivability of structures are given.

The design of equi-stable structures is based on the independence of various forms of loss of stability, but such independence is violated when switching to nonlinear analysis. Then it turns out that structures designed according to the principle of equi-

stability have a special tendency to the destabilizing effect of initial imperfections, and here there is a possibility of destruction of structures, which has an avalanche-like character.

It is emphasized that it is quite logical to formulate the same requirements for the reliability of functioning (target reliability levels) for objects of approximately the same degree of responsibility, regardless of the material from which they will be designed and the type of structural scheme. The condition for the equi-probability of the occurrence of design loads is also valid, regardless of the number in the design combination. But the equal reliability of structural elements may not correspond to their functional purpose, which leads to a decrease in overall reliability. A simpler example is ensuring survivability by using key elements with increased reliability.

Keywords: equi-strength, equi-stability, equi-reliability, reliability.

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Fig. 6. Ref. 39.

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