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CRITICAL ANALYSIS OF ANALYTICAL AND NUMERICAL MODELS OF BOND BETWEEN REINFORCEMENT AND CONCRETE

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This paper presents a comprehensive critical analysis of existing approaches to modeling the bond between reinforcement and concrete, which is a fundamental factor in ensuring the reliability and durability of reinforced concrete structures. The relevance of this research is amplified in the context of the current challenges posed by the full-scale war in Ukraine, which demands accurate prediction of the behavior of protective structures under dynamic, impact, and blast loadings.

The key research areas have been systematized and analyzed: experimental methods, analytical models, and numerical simulations. The limitations of classical experimental methods, such as pull-out tests and beam-end tests, are reviewed, and the advantages of modern monitoring technologies are highlighted. These include distributed fiber optic sensing (DFOS) for quasi-continuous measurement of reinforcement strains and digital image correlation (DIC) for analyzing crack kinematics, both of which provide detailed data on local bond behavior.

A critical review of analytical bond-slip models is conducted, ranging from semi-empirical relationships like the BPE model to more theoretically grounded approaches based on the thick-walled cylinder theory and the fictitious crack model. It is demonstrated that due to their dependence on specific experimental conditions and significant data scatter, these models often lack universal applicability.

Particular attention is given to the classification and analysis of numerical models based on their level of detail. Macroscopic models, from simplified (SDOF, perfect bond model) to advanced approaches (layered section model with equivalent stiffness, models based on systems of differential equations), are evaluated in terms of computational efficiency and accuracy in accounting for the slip effect. Mesoscopic approaches that model reinforcement and concrete as separate bodies are discussed in detail, including models with spring and cohesive zone elements (CZM), frictional-cohesive zone models (FCZM), contact algorithms (1D Slide Line), and lattice models. The advantages and disadvantages of each method, from physical justification to computational complexity, are highlighted. Furthermore, the prospects of applying machine learning methods (e.g., NARX, SSA-ELM) for the rapid and accurate prediction of failure modes and bond-slip relationships are considered.

The paper concludes that accounting for the bond-slip effect is critically important for the adequate modeling of the behavior of reinforced concrete structures, especially after the reinforcement reaches its yield point. The choice of a model should be based on a balance between the required accuracy and available resources. Finally, promising directions for future research are formulated, aimed at creating universal and computationally efficient numerical models.

Keywords: reinforced concrete, bond-slip model, finite-element simulation, 3D cohesive zone model, Pull-out test, Beam-end test, deep learning method.

Introduction. Reinforced concrete structures are currently among the most common building structures, widely used in the construction of buildings and structures for various purposes, roads, and bridges due to their high mechanical properties and durability [1–3]. The key prerequisite for their reliable and effective operation is the joint work between concrete and reinforcement, which is ensured by a bond at the interface between them [1, 2, 4]. It is the bonding mechanism that allows two dissimilar materials to work as a single monolithic body, which is fundamental to ensuring the mechanical properties of reinforced concrete structures [2, 4–6].

The connection at the “reinforcement-concrete” interface ensures such important aspects of structural behavior as crack formation and opening [7, 8], tension stiffening [9, 10], deformability, and load-bearing capacity [2, 11]. In the context of full-scale war in Ukraine, where there is a constant threat of shelling, terrorist acts and sabotage, explosions and impact loads, the reliability of reinforced concrete structures is critical for the preservation of human lives and infrastructure [12, 13]. Accurate prediction of the behavior of civil defense structures and military facilities under explosive loads is impossible without a deep understanding of the bonding mechanism, especially at high deformation rates [12, 14, 53]. Disruption or degradation of bonding, manifested in the form of bond slip, can lead to excessive crack opening, reduced stiffness, and load-bearing capacity, which jeopardizes the safety and serviceability of the structure [1, 10, 15]. This mechanism becomes particularly critical under dynamic and cyclic loads, as well as under the influence of aggressive environmental factors such as corrosion, high temperatures, and freeze-thaw cycles [10, 16].

Despite decades of research, the development of a general universal theory of bond that would meet the needs of design engineers remains an unresolved task [17]. This is due to the extreme complexity of the mechanism, which depends on a large number of factors, including the properties of concrete and reinforcement, the geometry of the specimens, the loading conditions, and the influence of the external environment. Existing analytical and numerical models often have limited scope or are too complex to be implemented in engineering practice [10]. In this regard, a critical analysis of existing approaches to bonding modeling, their systematization, and the identification of gaps is a relevant scientific task of considerable theoretical and engineering value.

Analysis of publications. The problem of modeling the bond between reinforcement and concrete has been the subject of numerous studies, which can be classified into several main areas: experimental studies, development of analytical models, and creation of numerical models.

Experimental studies are the basis for understanding the mechanisms of bond and verifying models [9, 17]. The most common methods are pull-out tests and beam-end tests [5, 10]. Pull-out tests, due to their simplicity and cost-effectiveness, are widely used to study local bond behavior [3]. However, this method has disadvantages: the stress state of concrete (compression) does not correspond to the actual operating conditions in bending elements (tension), which can lead to overestimated stiffness and bond strength [3, 9]. For more realistic modeling of operating conditions, modified schemes have been developed, such as beam-end tests, anchor beam tests, and butt-joint beam tests [1, 10, 19, 20]. Special attention should be paid to studies of the behavior of a bond under dynamic loads, which show that high loading rates can significantly increase the strength and stiffness of a bond [2, 5]. These data are critical for adequate modeling of the response of structures to impact and explosive effects [12, 14]. To obtain more detailed data on local bonding behavior, modern experimental campaigns increasingly use advanced monitoring methods, such as distributed fiber optic sensors (DFOS) for quasi-continuous measurement of deformations along reinforcement and digital image correlation (DIC) for analyzing the kinematics of cracks on the concrete surface [19, 21]. Despite improvements, differences in testing methods and conditions lead to a significant spread of experimental data, which complicates the development of universal models [10, 17].

Analytical models are usually semi-theoretical and semi-empirical, since mathematical dependencies are calibrated based on experimental data. Historically, the first models were obtained through regression analysis of data [10, 22]. Among theoretical approaches, the thick-walled cylinder model, which considers the concrete around the reinforcement as a cylinder under internal pressure, is quite common [10, 16]. To account for concrete cracking, this model is often combined with the fictitious crack model [16, 23, 24]. As a result, a large number of “stress-slip” dependencies have been proposed, which are approximated by polynomial, power, exponential, logarithmic functions, or piecewise models, such as the well-known BPE (Bertero-Popov-Eligehausen) model, which is included in the CEB-FIP standards [3, 5]. However, due to their dependence on experimental conditions, these models are not universally applicable [10].

Numerical models allow for detailed analysis of the complex phenomena accompanying bond failure. Depending on the level of detail, they are divided into macro-, meso-, and microscopic models [9, 14].

Macroscopic models consider reinforced concrete as a homogenized material [14]. In the simplest case, the hypothesis of ideal bond is assumed, which ignores slippage and can lead to inaccurate results, especially under dynamic loads [11, 12, 25]. More sophisticated macromodels, such as the layered section approach, take into account the effect of slippage indirectly through a change in bending stiffness [12]. There are also models that explicitly introduce the dependence of adhesion into the system of differential equilibrium equations [26].

Mesoscopic models simulate reinforcement and concrete as separate bodies, and their interaction is described using special interface elements [9, 16, 28]. These can be bond-link elements or cohesive zone models (CZM), which describe the formation and development of damage at the contact boundary [12, 14]. Improved friction-cohesion models (FCZM) additionally take into account friction on damaged areas. Contact algorithms available in commercial programs such as LS-DYNA are also used [11, 14, 24].

Microscopic models treat concrete as a multiphase material, modeling the cement matrix and aggregate separately. Such models allow the fundamental mechanisms of failure to be investigated, but are extremely resource-intensive [29, 30].

In recent years, artificial intelligence and machine learning (ML) methods have been used to predict bonding behavior, allowing for the creation of fast and accurate predictive models based on large databases [14, 31].

Despite significant progress, existing studies have a number of limitations: the combined effect of various factors has not been sufficiently studied, most of the proposed models are not universal, and detailed numerical modeling remains computationally expensive [10]. This highlights the need for systematic analysis and comparison of existing approaches.

Purpose of the paper. The purpose of this work is to critically analyze, systematize, and compare existing analytical and numerical models describing the bond between reinforcement and concrete in order to identify their advantages, disadvantages, areas of rational application, and promising directions for further research.

Research results. As it is known, the bond between reinforcement and concrete is determined by three main components [10] (Fig. 1):

- *chemical adhesion* (appears in the initial stages of loading, but quickly disappears when the slightest displacement occurs due to slippage of the reinforcement);
- *friction* (occurs due to concrete shrinkage, which creates a tight surface around the reinforcement, as well as due to micro-irregularities) [32];
- *mechanical interlocking* (the main mechanism for periodic profile reinforcement (with ribs), which ensures the transfer of significant forces) [5].

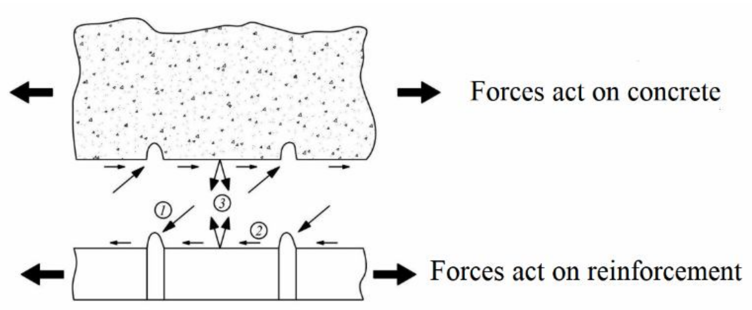
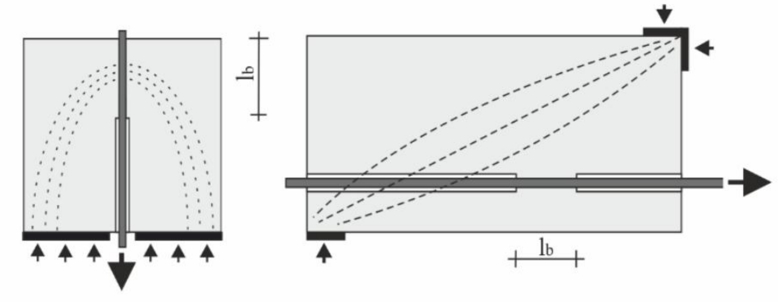


Fig. 1. Scheme of force factors, the combined action of which characterizes the phenomenon of reinforcement bonding with concrete: 1 – forces of resistance to bending and shearing due to the presence of reinforcement protrusions; 2 – friction forces; 3 – forces of adhesive interaction

Numerous factors affect the strength of the bond, including: the type and strength of concrete, the thickness of the protective cover, the diameter and geometry of the reinforcement ribs, the elasticity modules of concrete and reinforcement, the presence of transverse reinforcement (clamps), the type of stress-strain state, as well as the type and rate of loading [1, 13, 26].

Due to the large number of factors that influence the nature of the interaction between reinforcement and concrete under load and stress, various scientists have developed a large number of bond models of varying computational complexity. These models reflect the behavior of reinforcement with concrete only under the conditions of specific experiments, which may differ significantly from each other, both in methodology and in the results obtained, and, unfortunately, cannot be used as a generalized bonding model [17]. Mathematical models are usually combined with experimental data, which are used to adjust the parameters in these models, resulting in the transformation of analytical models into semi-theoretical and semi-empirical ones [10].

The most common experimental methods for studying bonding are Pull-out tests [1, 10, 34] and Beam-end tests [14, 35] (Fig. 2). Over time, new schemes have emerged and been refined, such as eccentric Pull-out, anchor beam tests, and beam tests with butt joints [9, 10]. Pull-out tests are one of the simplest and most inexpensive methods for studying the local behavior of a bond [3].



(a) (b)

Fig. 2. Schemes of experimental studies of reinforcement bonding with concrete with compressive stress trajectories [6]: (a) – RILEM [34] pull-out test method; (b) – ASTM [35] beam-end test method

According to the method of mathematical modeling of the bond between reinforcement and concrete in reinforced concrete structures, bond models can be divided into **analytical** and **numerical** models.

The most common method of modeling the bond between reinforcement and concrete, both in analytical and numerical models, is the "laws" or relationships between the bond stress τ_{bond} on the slip of the reinforcement s along the contact are with concrete (bond-slip relationships). They are key in modeling this process, as they describe the mechanism of force transfer at the contact area between two materials [3]. These relationships are most often developed and calibrated based on the results of pull-out tests [6].

When pulling a reinforcing bar out of a concrete block, a characteristic feature of the bond stress diagram is that it varies along the length of contact between the reinforcement and the concrete depending on the position of the cross-section in the experimental specimen (Fig. 3). This complicates the modeling of the dependence of bond stresses on the mutual displacements of the reinforcement relative to the concrete using a single continuous function.

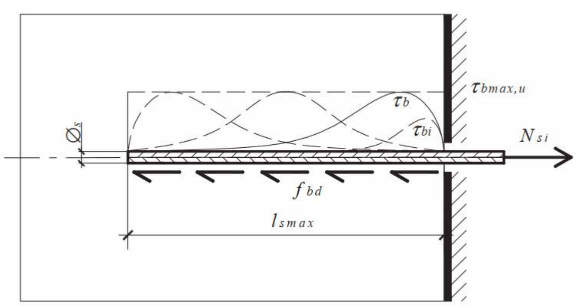
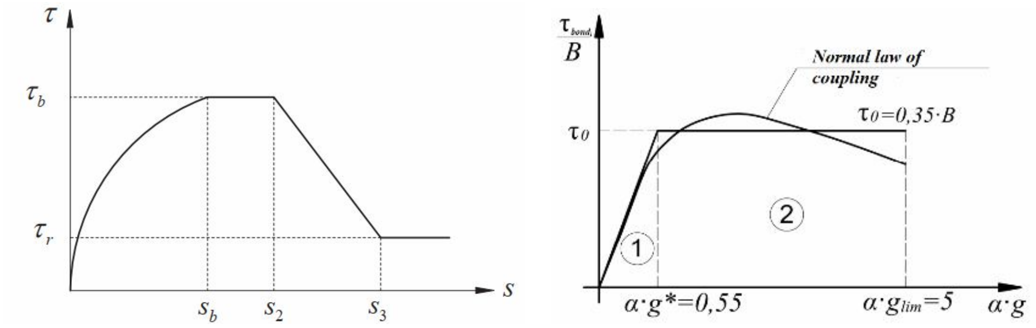


Fig. 3. Variation of bond stresses along the length when pulling a reinforcing bar out of concrete

Bond-slip relationships and analytical models. One of the most well-known relationships between "bond stress" and "slip" is the **BPE (Bertero-Popov-Eligekhausen) model** (Fig. 4 (a)) and its modifications in fib Model Code 2010 [3, 16].



(a) (b)

Fig. 4. Some of the most common dependencies between "bond stress" and "slip":
(a) – BPE $\tau_{\text{bond}} - s$ model; (b) – Kholmiansky's normal law and its elastic-plastic approximation [36]

The model is a piecewise-defined function that contains four sections and describes different stages of bond behavior depending on the slip value (1). The ascending branch of this model is characterized by a nonlinear increase in bond stresses to peak values $\tau_{\max}$, after which a plateau zone with a constant stress value $\tau_{\max}$ occurs in the slip value range from s_b to s_2. After the horizontal section, the model assumes a linear decrease in bond stress to residual stresses τ_f, which are taken to be 0.4 $\tau_{\max}$. Residual stresses remain constant at high slip values $s \geq s_3$

$$\tau=\begin{cases}\tau_{\max}\left(s/s_1\right)^{\alpha},\ 0\le s\le s_1;\\ \tau_{\max},\ s_1\le s\le s_2;\\ \tau_{\max}+\left(\tau_f-\tau_{\max}\right)\times\left((s-s_2)/(s_3-s_2)\right),\ s_2\le s\le s_3;\\ \tau_f,\ s>s_3.\end{cases} \tag{1}$$

s_1, s_2, s_3 – characteristic slip values that define the limits of each stage. For conditions of good adhesion and sufficient compression, s_1 = 1.0 mm, s_2 = 2.0 mm, and s_3 is equal to the distance between the ribs of the reinforcement.

One of the modifications of model (1) is the relationship defined in [37], which takes into account the nonlinear behavior of materials on the descending branch:

$$\tau=\begin{cases}\tau_{\max}\left(s/s_{\max}\right)^{\alpha},\ s\le s_{\max};\\ \tau_{\max}\left(s/s_{\max}\right)^{-\alpha},\ s>s_{\max}.\end{cases} \tag{2}$$

Equally well known are M.M. Kholmiansky's "normal law" of reinforcement-concrete bonding (3) and its elastic-plastic approximation [36]. (Fig. 4 (b))

$$\tau=B\cdot\frac{\ln(1+\alpha\cdot S)}{1+\alpha\cdot S}., \tag{3}$$

The parameters B and α can be determined based on the results of experimental studies.

In models based on the **thick-walled cylinder theory** (Fig. 5), the concrete surrounding the reinforcement is modeled as a thick-walled cylinder subjected to internal pressure from the reinforcement ribs [12, 16, 38]. The application of elasticity theory solutions for thick-walled cylinders [23] allows for the analytical determination of stresses and deformations in concrete that lead to splitting failure. To account for concrete cracking, a fictitious crack model is used, which takes into account the softening of concrete after it reaches its tensile strength limit. Radial stresses σ_r at the contact boundary are related to bond stress τ through equilibrium conditions on the surface of the reinforcement ribs, and slip s is related to radial deformations through the geometry of the ribs (modeled as a truncated cone).

A distinctive feature of these models is that they require accurate determination of the mechanical properties of concrete, in particular its behavior after cracking. The characteristic assumption of uniform pressure and axisymmetry is a simplification of the real three-dimensional stress picture [1]. Analytical solutions for such models can be complex, so simplified formulas obtained by regression analysis are often used [23].

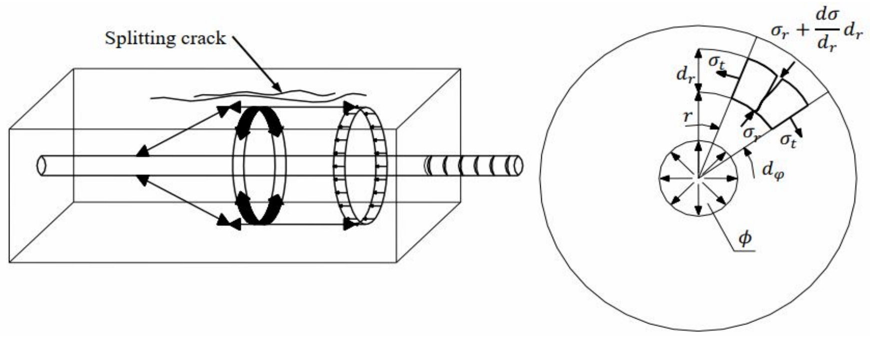


Fig. 5. Model based on the thick-walled cylinder theory [1]: (a) – Tepfers' thick-walled cylinder bond model [38]; (b) – Radial and tangential force distribution acting on a differential element inside the cylinder

Modified analytical models of "bond stress" – "slip" for various load conditions are also being actively developed and used. For example, the Long et al. model [4] for static and dynamic loads, which is essentially a simplified version of the BPE model (Fig. 6) and is more convenient for engineering calculations and takes into account the influence of loading speed on bond stress – see formulas (4–6).

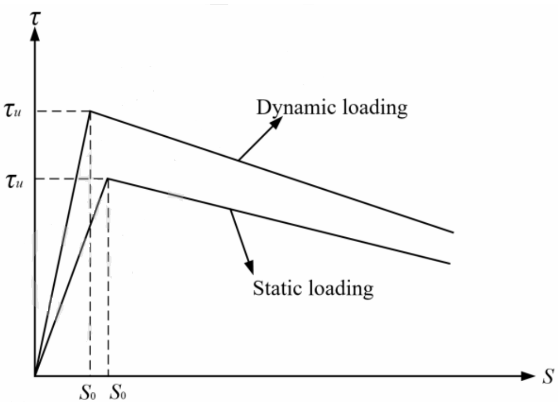


Fig. 6. The $\tau_{\text{bond}} - s$ model for different types of loads by Long et al. [4]

Expression for the ascending branch:

$$\tau = \tau_u \cdot \left(S/S_0\right)^a,\ 0 \le S \le S_0. \qquad (4)$$

The same, but for the descending branch:

$$\tau = \tau_u \cdot \left\{ k_r + (1-k_r)\exp\left[\beta \cdot \left(\frac{S}{S_0} - 1\right)^{0.5}\right] \right\}, S > S_0, \qquad (5)$$

where τ_u and S_0 are the maximum shear stress and corresponding displacement; a is a nonlinearity parameter that depends on the loading rate v:

$$a = 0.007 \times \ln(v) + 0.116\ ; \qquad (6)$$

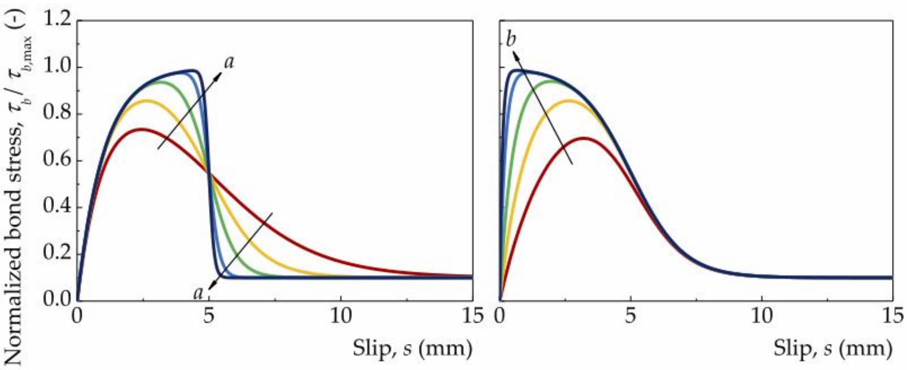
β – parameter describing the rate of decline (suggested value is -0.43); $k_r = \tau_r / \tau_u$ – relative residual friction coefficient.

The Biscaia & Carmo model [39] is single-functional and offers a single continuous function (7) to describe all three stages of the bond diagram (elastic work stage, softening stage, and residual friction stage at the contact between reinforcement and concrete), which makes it convenient for use in numerical models (Fig. 7)

$$\frac{\tau_b(s)}{\tau_{b,\max}} = \left(1 - e^{-b \times s}\right) \cdot \frac{\alpha + e^{-a\cdot(s-s_t)}}{1 + e^{-a\cdot(s-s_t)}}\ . \qquad (7)$$

Parameters a, b, s_t are calibrated according to experimental data and determine the shape of the curve. The main parameters of the model are: maximum bond stress $\tau_{b,\max}$, $\alpha = \tau_{bf} / \tau_{b,\max}$ – the ratio of residual stress to maximum stress.

The model by X. Lv et al. [40] based on micromechanics considers the contact zone as a parallel system of microelements (spring, friction element, and switcher). In particular, elastic deformation, adhesion failure, and crack formation are characterized by an elastic element, while friction and sliding between reinforcement and concrete are represented by a friction element. In addition, to control the operating sequence of the two elements based on a number of physical properties of the contact surface, a switching element was used, which opened at the stage of elastic deformation and remained closed after the destruction of the spring element. This approach allows for a physical justification of macroscopic behavior (Fig. 8).



(a) (b)

Рис. 7. Bond-slip model by Biscaia & Carmo [39] with $\tau_{bf} / \tau_{b,\max} = 0.1$ and $s_t = 5$ mm under influence of: (a) – parameter a; and (b) –parameter b

The key idea of the model is to take into account the stochastic nature of the destruction of friction bonds at the micro level, with the assumption that the destruction threshold of spring elements is a random variable that defines by the Weibull distribution.

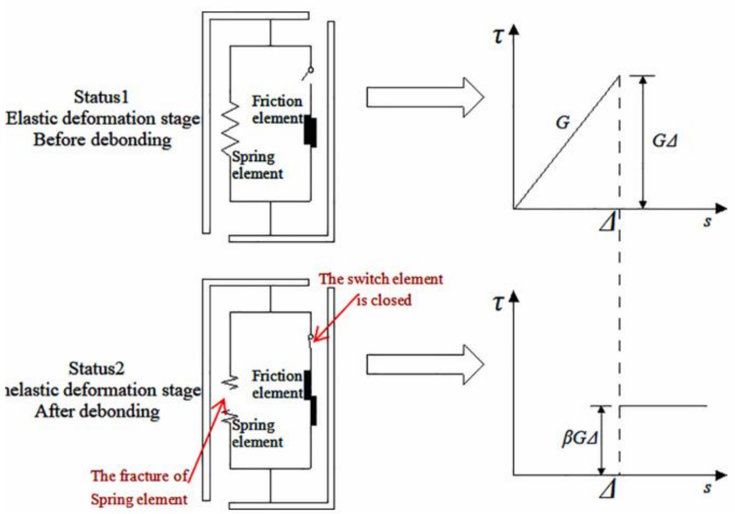


Fig. 8. Scheme of two stages of microelements operation under load in the model X. Lv et al. [40]

Formula (8) for determining the average bond stresses of this model:

$$\tau_{s=s_0} = (1-\beta)\cdot G\cdot s_0\cdot e^{-(s_0^m/a)} + \beta\cdot G\int_0^{s_0} e^{-(x^m/a)}dx = \tau_0, \quad (8)$$

where G is the initial bond stiffness, β is the friction coefficient, and parameters a and m are determined from the conditions of reaching peak strength τ_0 and zero slope of the curve at this point.

There are many other bond relationships between reinforcement and concrete that have been proposed by various scientists, such as power, polynomial, exponential, fractional, and others [17].

Due to the complexity of the bond mechanism, differences in the properties of sample materials, and experimental testing methods, there are significant discrepancies in the characteristics of the proposed ‘bond-slip’ relationships [10, 16]. Thus, the work of Benin A.V. et al. presents a diagram comparing various bond-slip relationships (Fig. 9), which demonstrates how different analytical models (polynomial, power, logarithmic, spline functions, etc.) give significantly different curves even when their parameters are selected to coincide at the maximum point. This visual comparison clearly

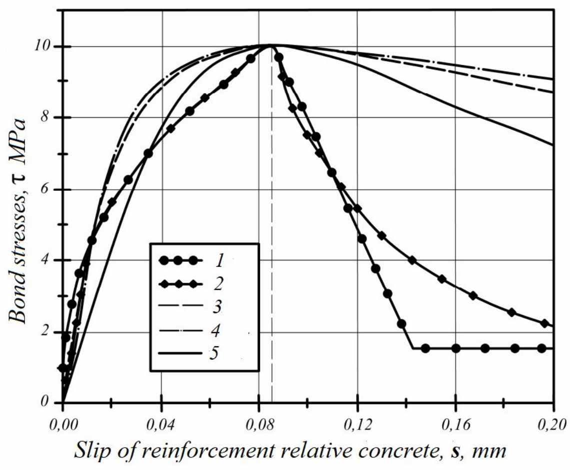


Fig. 9 Diagram of comparative analysis of the different bond-slip relationships: 1 – BPE relationship $\tau_{bond} - s$; 2 – modified BPE relationship [37]; 3 – normal law of Kholmiansky's M. [40]; 4 – modification of the normal law by Shima H. [40]; 5 – approximation of the bond relationship of Balázs G. [42]

shows the significant spread of results, which is a consequence of the diversity of experimental data on which these models were based.

Due to these differences, the proposed constitutive relationships are often not universally applicable.

Thus, classical coupling models, such as the BPE/fib Model Code and its numerous modifications, are convenient for engineering practice but have limitations, especially for new types of concrete and low compression conditions.

Thus, the BPE model was developed primarily for conventional heavy concrete and reinforcement with sufficient compression in experimental samples, in which failure occurs due to pull-out failure [5]. For cases of splitting failure of concrete, the model requires parameter adjustments, especially for the descending branch, which becomes nonlinear. The model does not always accurately reflect the behavior of modern ultra-high performance concretes, such as UHPC, as it may underestimate the strength of the bond [32].

Models based on thick-walled cylinder theory provide a deeper physical understanding of the mechanism of splitting failure, but require accurate input data and are often difficult to apply directly in engineering calculations.

Modern single-function and micromechanical models offer a more accurate and physically adequate description of behavior, which is promising for numerical modeling but requires careful calibration of parameters based on experimental data.

Each model has its advantages and disadvantages, and the choice of a specific model depends on the task at hand: whether it is a practical engineering calculation or a detailed scientific study of failure mechanisms.

Numerical models. Finite element method (FEM) numerical modeling is a powerful tool for analyzing the complex behavior of reinforced concrete (RC) structures, in particular the bond-slip mechanism between reinforcement and concrete. With the development of computing technology and software, a number of approaches have been developed that can be classified according to the level of detail and the method of accounting for interaction at the contact boundary [4, 12]. According to various sources, there are **macroscopic**, **mesoscopic**, and **microscopic** models, each of which has its own scope of application, advantages, and disadvantages [9, 14].

Macroscopic models (Structural Element Scale). At the macro level, reinforced concrete elements are modeled using one-dimensional (beam) or two-dimensional (shell) finite elements [9, 12, 43]. In such models, the interaction between reinforcement and concrete is usually taken into account indirectly, through modified material properties or specialized element formulations.

Модель Perfect Bond model. This is the simplest approach, where the hypothesis of perfect bonding is assumed, i.e., no slippage between the reinforcement and concrete [2, 11]. This assumption greatly simplifies calculations, but can lead to an overestimation of the stiffness and load-bearing capacity of the structure, especially under dynamic or cyclic loads, where the displacement of the reinforcement relative to the concrete plays a significant role [12, 14].

Single degree of freedom (SDOF) model. This simplified model is popular in engineering practice because it is easy to use, especially for dynamic load calculations such as explosions or impacts, but it has limitations in accurately modeling the nonlinear behavior of reinforced concrete beams. It uses many approximations and cannot accurately account for the nonlinearity that arises from concrete cracking and reinforcement yielding. The model may be ineffective for beams with large deformations caused by reinforcement yielding [12].

Model based on a system of differential equations (improved macroscopic model). This approach explicitly takes into account the "bond-slip" effect at the structural element level [44]. The interaction between reinforcement and concrete is described by a system of equilibrium differential equations (9), into which the adhesion relationship is directly introduced, in particular, the bilinear "stress-slip" diagram [26]. The model considers a reinforced concrete element as a single rod and describes its behavior through generalized parameters (force, displacement) (Fig. 10), which is characteristic of the macroscopic level. Unlike classical beam models, this approach rejects the ideal coupling hypothesis and allows for a more accurate simulation of the nonlinear nature of the connection crossing the crack, while maintaining computational efficiency.

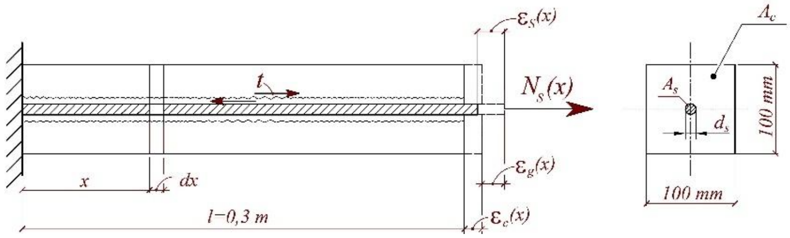


Fig. 10. Calculation scheme of a model element for modeling the behavior of links crossing the crack [26]

$$\begin{cases} \varepsilon_s(x) = \dfrac{1}{E_s \cdot A_S} \cdot N_s(x); \\ \varepsilon_c(x) = \begin{cases} \dfrac{N_c(x)}{E_{cm} \cdot A_c}, \; if \; \dfrac{N_c(x)}{A_c} \le 0{,}9 \cdot f_{ctm}, \\ \dfrac{18 N_c(x)}{E_{cm} \cdot A_c} - 15{,}3 \dfrac{f_{ctm}}{E_{cm}}, if \; \dfrac{N_c(x)}{A_c} > f_{ctm}; \end{cases} \\ \dfrac{dN_s(x)}{dx} = \begin{cases} \pi d_s 0{,}4 E_{cm}\left[\varepsilon_s(x) - \varepsilon_c(x)\right], \; if \; \varepsilon_g(x) \le \varepsilon_g^*(x) = 4{,}95 \dfrac{f_{ctm}}{E_{cm}}, \\ \pi d_s \left\{0{,}0232 E_{cm}\left[\varepsilon_s(x) - \varepsilon_c(x)\right] + 1{,}866 f_{ctm}\right\}, if \; \varepsilon_g(x) > \varepsilon_g^*(x) = 4{,}95 \dfrac{f_{ctm}}{E_{cm}}; \end{cases} \\ \dfrac{dN_c(x)}{dx} = \begin{cases} -\pi d_s 0{,}4 E_{cm}\left[\varepsilon_s(x) - \varepsilon_c(x)\right], \; if \; \varepsilon_g(x) \le \varepsilon_g^*(x) = 4{,}95 \dfrac{f_{ctm}}{E_{cm}}, \\ -\left\{0{,}0232 E_{cm}\left[\varepsilon_s(x) - \varepsilon_c(x)\right] + 1{,}866 f_{ctm}\right\}, if \; \varepsilon_g(x) > \varepsilon_g^*(x) = 4{,}95 \dfrac{f_{ctm}}{E_{cm}}. \end{cases} \end{cases} \tag{9}$$

Fiber Model and Layered Section Approach. These approaches allow for the nonlinear behavior of materials by dividing the cross-section of an element into separate "fibers" or layers [12]. Classic implementations of these models are based on the hypothesis of ideal bond and cannot take into account the effect of slippage and the associated rotation in fixed-end sections (fixed-end rotation). However, there are also improved versions that attempt to implement this effect.

Improved model of a layered cross-section with equivalent stiffness. To account for the effect of slippage, which is dominant after the yield of the reinforcement, some models propose to change the bending stiffness (EI) of the elements within the length of the plastic hinge [12]. The equivalent bending stiffness is calculated based on the condition of deformation compatibility, taking into account the rotation in the support section caused by the slippage of the reinforcement. This approach allows the bond-slip effect in beam elements to be taken into account without significantly complicating the calculation scheme.

Mesoscopic models (Bar Scale). At this level, reinforcement and concrete are modeled as separate bodies, and their interaction at the contact boundary is described using special interface elements or contact algorithms [9, 11, 27, 29]. These models provide high accuracy but are more resource-consuming.

Models with interface elements (Bond-Link / Bond-Zone Elements).

Spring elements (Spring Elements). The interaction is modeled using a set of nonlinear springs connecting the reinforcement and concrete nodes [4, 12, 14, 45]. The stiffness of these springs is determined based on an experimentally obtained "bond-slip" diagram. This method allows nonlinearity and damage at the contact area to be explicitly taken into account [4] (Fig. 11).

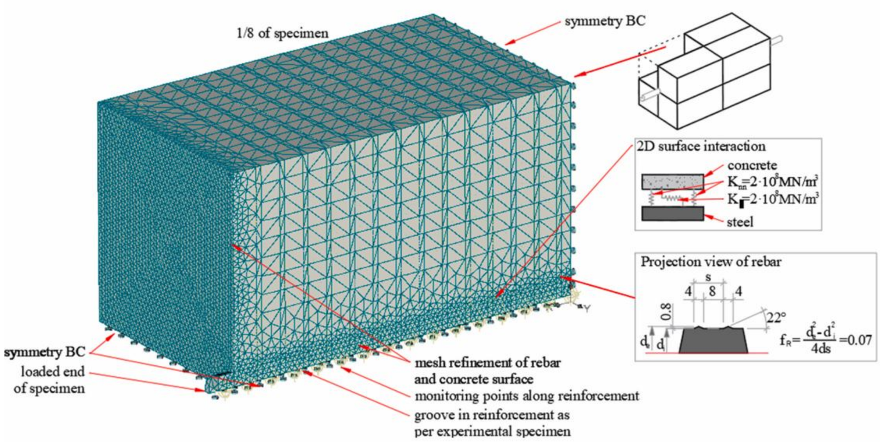


Fig. 11. Macroscopic MCE 3D model of a concrete block fragment with reinforcement working in pull-out and description of the main parameters of the model [9]

Zero-thickness cohesive elements (Cohesive Zone Models, CZM). This approach uses special zero-thickness elements placed at the "reinforcement-concrete" contact boundary [4, 12, 27]. The behavior of these elements is described by the traction-separation law, which takes into account the appearance and growth of damage [4]. Bilinear cohesive models are common due to their simplicity and fewer parameters. Thus, the «bond-slip» curve obtained from the results of calculations using the modified Tvergaard cohesion zone model [46, 47], accurately reproduces the initial stiffness and peak strength of the bond and successfully models nonlinear softening (stress reduction after the peak) because it takes into account damage at the contact boundary (Fig. 12 (a)). However, this model is not capable of reproducing residual stress, as it does not take into account the friction effect that occurs after the destruction of the adhesive bond.

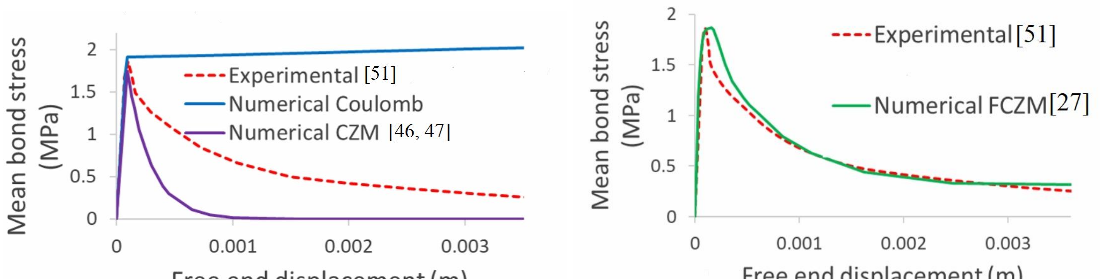


Fig. 12. "Mean bond stress"-"displacement" curves obtained using numerical models [27, 46, 47] and based on the results of Anwar Hossai's experiment [51]: (a) – results of numerical modeling using the CZM model [46, 47]; (b) – results of numerical modeling using the FCZM model [27]

Frictional Cohesive Zone Models (FCZM). This is an improved version of CZM, which combines damage and friction in a single thermodynamically consistent basis [27]. The zone of interaction between reinforcement and concrete is considered as a combination of an undamaged part (described by the cohesive model [46]) and a damaged part (described by Coulomb's friction law) (Fig. 13).

Such models are universal and capable of reproducing the behavior of both smooth and ribbed reinforcement [18, 27].

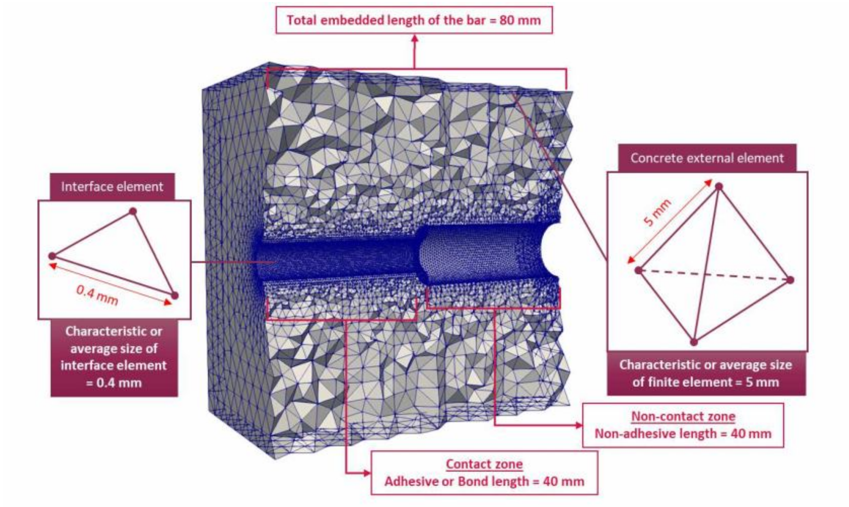


Fig. 13. Fragment of the FEM model of the concrete part of the experimental specimen, showing the total length of the reinforcement embedment, as well as the average size of the element sides, which models the contact between the reinforcement and concrete and the external tetrahedral elements of concrete in the FCZM model [27]

Models based on contact algorithms.

One-dimensional slide line contact model (1D Slide Line Contact Model). This approach, available in programs such as ANSYS LS-DYNA, simulates the sliding of reinforcement nodes along a line of concrete nodes (Fig. 14). The interaction is implemented through imaginary springs. The model is described by three parameters: the shear modulus of adhesion, the maximum elastic slip, and the damage curve exponent coefficient. This method is effective for modeling dynamic problems such as explosions or impacts [14].

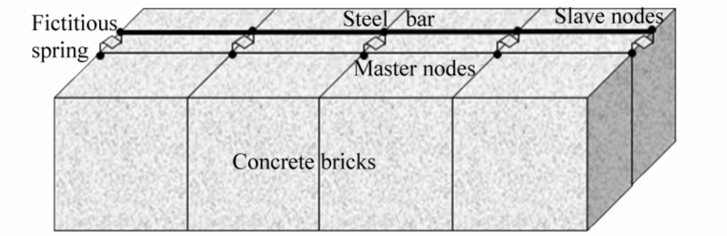


Fig. 14. Sketch of an imaginary spring between the master and slave nodes in a one-dimensional sliding element model [14]

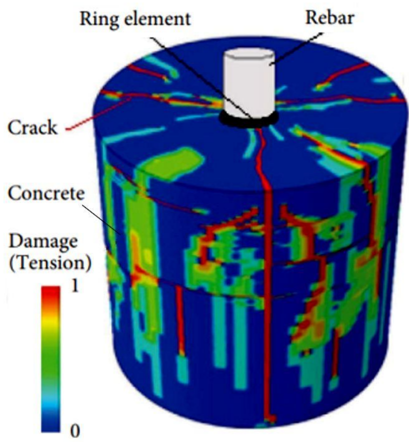


Fig. 15. Nature of sample destruction when simulating a pull-out test using a CDP model [29]

Concrete Damage Plasticity (CDP) model. This is a common material model in ABAQUS that accounts for concrete failure due to tension and compression [12, 29, 48]. Although the model itself does not describe adhesion (Fig. 15), it is often used in combination with cohesive or contact elements for comprehensive analysis [29].

Discrete models

Lattice models. In these models, concrete, reinforcement, and their interaction are modeled as a set of beam elements (Fig. 16). The interaction between reinforcement and concrete is modeled using special interface elements, whose properties are determined based on analytical bond-slip models and stochastic analysis of grid randomness [51]. These models accurately reproduce the cone-shaped stress transfer mechanism in concrete and can predict spalling failure without directly specifying its criteria [30].

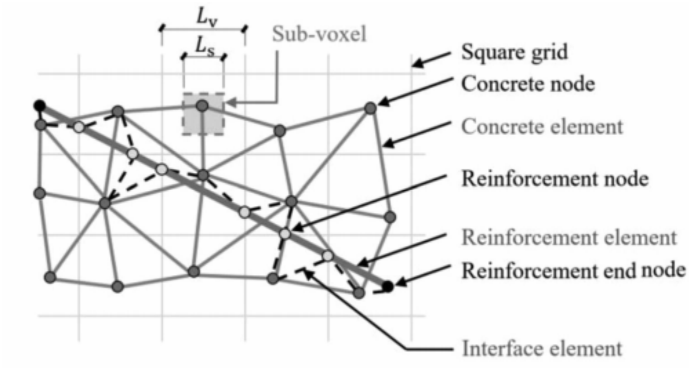


Fig. 16. 2D image of a lattice grid in a lattice model for reinforced concrete [30]

Machine learning (ML) models. In recent years, machine learning methods such as neural networks have been used to predict bond-slip behavior [4, 12].

Dynamic neural network NARX (Nonlinear Autoregressive with Exogenous Inputs). This recurrent neural network can be trained on data obtained from MCE simulations or experiments to quickly predict “bond-slip” relationships under both static and dynamic loads. The model uses previous values of input and output sequences to predict the current output value.

Instead of conducting hundreds of expensive experiments, the authors first created and verified an accurate FEM model based on cohesive elements [4]. Then, using this FEM model, they conducted parametric studies by varying the diameter of the reinforcement, the anchorage length, and the loading rate, and generated 42 data sets. The resulting data (bond-slip curves) are used as a training set for the NARX network. The input data is time and slip magnitude, and the output is bond stress. After training, the NARX model demonstrated extremely high accuracy. The Pearson correlation coefficient (R) between the MCE and NARX predictions reached 0.97. This means that the neural network can replace resource-intensive MCE modeling with virtually no loss of accuracy, but with a huge gain in speed.

SSA-ELM (Sparrow Search Algorithm - Extreme Learning Machine) model. In source [2], AI is used to solve another important problem — predicting the mode of bond failure: pull-out or splitting. This is a typical classification problem in machine learning [9]. The authors propose a model based on Extreme Learning Machine (ELM), optimized using the Sparrow Search Algorithm (SSA). ELM is a type of single-layer neural network that is characterized by a very high learning rate, since the weights and thresholds of the hidden layer are generated randomly and do not require iterative tuning. The SSA algorithm is used to optimize these random parameters in order to improve the accuracy and stability of the ELM model [31].

A database of 399 pull-out test results was compiled from 15 published scientific papers. The database included 16 geometric and material parameters of the samples, as well as the recorded failure mode [4]. It was found that the ratio of the protective layer thickness to the reinforcement diameter (*c/d*) is the most important factor affecting the failure mode. Concrete strength, anchorage length, and the presence of transverse reinforcement were also found to be significant [31].

The SSA-ELM model was trained on the selected data. Its accuracy was compared with nine other popular classification algorithms (logistic regression, k-nearest neighbors, decision tree, support vector method, backpropagation neural network, etc.). As a result, the SSA-ELM model showed the best results, achieving a prediction accuracy of 95.8% on the test data set [31]. This is significantly higher than other ML algorithms considered and confirms that this approach can be a reliable tool for engineers in assessing the safety and design of reinforced concrete structures [9].

Thus, it can be argued that the application of AI and ML in bonding modeling is a very promising direction. It allows the creation of fast, accurate, and versatile predictive models that can complement or even replace traditional approaches in certain tasks [4, 9]. The main advantages are the ability to process large data sets to identify complex relationships and a significant reduction in computational costs compared to MCE.

Microscopic models. Unlike macro- and mesoscopic models, where concrete is considered a homogeneous material, microscopic models are the most detailed and physically sound, as they model concrete as a multiphase material [27]. This approach aims to investigate the fundamental mechanisms of failure that occur at the level of individual concrete components and at their contact boundaries.

A key feature of microscopic models is that they explicitly model individual phases of concrete, such as the cement matrix and coarse aggregate. This allows us to study how microcracks form and propagate in the transition zone between the aggregate and the matrix, which is one of the weakest links in the concrete structure [49].

At this level of detail, it becomes possible to directly study the three main components that form bond: chemical adhesion, friction, and mechanical interlocking [5].

Due to the extremely high computational costs and complexity of determining material parameters at this level, microscopic models are not used to analyze full-scale structures [11]. Their main area of application is fundamental scientific research aimed at a deeper understanding of the physics of failure at the "reinforcement-concrete" interface [27, 49], studying the effect of concrete structure heterogeneity on bond characteristics [50], and justifying and calibrating parameters for less detailed but more practical mesoscopic models.

In this analytical review and analysis of numerical models, the main focus was on research devoted to the development of practically applicable mesoscopic and macroscopic models for the analysis of elements and structures as a whole. Mesoscopic models that simulate the geometry of reinforcement ribs but consider concrete as a homogeneous material already provide a sufficient level of accuracy for many engineering and scientific tasks and are a compromise between accuracy and computational costs.

The results of a comparative analysis of the numerical models considered are presented in Table 1.

Table 1

Comparative analysis of the numerical models reviewed

Model type	**Description**	**Scope of application**	**Advantages**	**Disadvantages**
1	**2**	**3**	**4**	**5**
Macroscopic models				
Perfect bonding	hypothesis of complete compatibility of deformations	preliminary calculations, analysis of structures where bond-slip is not dominant	simplicity, low computational costs.	does not accurately account for stiffness and deformation, may overestimate load-bearing capacity
SDOF	a simplified model with one degree of freedom	practical engineering calculations, preliminary assessment of dynamic reactions	easy to use	cannot accurately simulate nonlinear behavior
Model based on a system of differential equations (improved macro model)	takes into account the bond-slip effect through a system of four differential equilibrium equations, where the coupling law is explicitly introduced. Operates with generalized parameters at the element level	nonlinear analysis of reinforced concrete elements, where it is necessary to accurately account for the redistribution of forces and deformations due to reinforcement slippage	a more fundamental approach than other macro models. It explicitly accounts for nonlinear coupling at the equation level while maintaining computational efficiency	requires solving a nonlinear boundary value problem, which is more complex than standard beam models; accuracy depends on the adequacy of the adopted coupling law
Layered cross-section with equivalent stiffness	takes into account bond slip due to changes in bending stiffness EI in the plastic hinge zone	analysis of beams under explosive and impact loads, where rotation in supports is important	allows the bond-slip effect to be implemented in beam models, improving accuracy compared to classical models	it is an approximate method, the accuracy of which depends on the correct determination of the length of the plastic hinge and the equivalent stiffness
Mesoscopic models				
Spring elements	nonlinear springs between reinforcement and concrete nodes	detailed nonlinear analysis of individual elements, especially under cyclic loads	allows you to explicitly specify any "bond-slip" relationship	the complexity of determining spring stiffness can be computationally costly

1	2	3	4	5
Cohesion models (CZM, FCZM)	zero-thickness elements with the law of destruction	analysis of initiation and propagation of damage at the contact boundary; investigation of delamination failure	physically based, taking into account the energy of destruction; FCZM are universal for different types of reinforcement and modes of destruction	sensitive to cohesion law parameters (strength, fracture energy, stiffness); require careful calibration
1D Slide Line	specialized contact algorithm for sliding.	dynamic analysis (impact, explosion) in programs that support this type of contact (e.g., LS-DYNA)	effective for modeling reinforcement slipping along concrete	it is difficult to determine the parameters of the model; the parameters vary significantly depending on the conditions
Lattice models	discretization of concrete, reinforcement, and interface with beam elements	research into the mechanisms of destruction and the impact of concrete heterogeneity	they take into account the stochasticity and heterogeneity of the material and model cracking well	require a special approach to calibrate the properties of interface elements
Machine learning models				
NARX	recurrent neural network	rapid bond-slip prediction for parametric studies and engineering calculations	high prediction speed after training, high accuracy (up to 97%)	requires large amounts of data for training, is a "black box," does not reveal physical mechanisms

Conclusions. Based on the results of theoretical analysis of existing studies of analytical and numerical models of reinforcement-concrete bond, as well as the results of certain experimental studies, the following conclusions can be drawn:

1. Most analytical models of "bond-slip" are semi-empirical, as they are based on regression analysis of data obtained from experiments, mainly from pull-out tests and beam-end tests.

2. There is considerable variability between the developed analytical "bond-slip" models and the values of their parameters. This is due to differences in experimental techniques, specimen geometry, material properties (concrete and reinforcement), and loading conditions.

3. Due to this dependence on experimental conditions, many existing models are not universal and may be unsuitable for conditions different from those for which they were developed. This highlights the importance of selecting an appropriate model for a specific case or conducting additional research to calibrate it.

In this regard, experimental studies of the bond between reinforcement and concrete using modern measurement and monitoring technologies, such as distributed fiber optic sensors (DFOS) and digital image correlation (DIC), are becoming particularly relevant. This will allow simultaneous investigation of internal reinforcement deformations, crack formation on the concrete surface, and local distribution of bond stresses, which will contribute to the development of more accurate and universal mechanical models.

4. Macroscopic numerical models are computationally efficient but have limitations in accuracy. In particular, classical beam models based on the ideal bond hypothesis cannot account for the effect of reinforcement slippage. However, advanced macroscopic approaches that indirectly implement this effect by modifying the bending stiffness in the plastic hinge zone allow for more accurate calculations, particularly dynamic ones. Mesoscopic models provide a higher level of detail by modeling reinforcement and concrete as separate bodies and their interaction using special contact elements. These methods allow for in-depth analysis of local phenomena but require significant computational resources.

5. Taking into account the bond-slip effect is critical for accurately assessing the response of reinforced concrete elements, especially under dynamic and impact loads. Sources show that ignoring slip leads to inaccurate results while including it significantly improves the correspondence of numerical results to experimental data. This becomes significant after reaching the yield strength of the reinforcement, when slip becomes the dominant factor in deformations.

Thus, a promising direction for further research is the improvement of existing numerical models of reinforcement-concrete bond, namely the development of effective numerical algorithms that accurately account for the effect of slip without significant computational costs. It is also relevant to create unified mesomechanical models that would combine friction, damage, and adhesion in a single consistent thermodynamic basis (for example, a model of a cohesive zone with friction).

6. A promising direction is the application of machine learning (ML) methods, in particular dynamic neural networks (e.g., NARX). Such models, trained on large databases obtained from experiments and detailed MCE simulations, can quickly and accurately (over 97%) predict “bond-slip” curves. This avoids the significant computational and labor costs associated with detailed mesoscopic modeling and provides engineers with an effective and accurate tool for parametric studies.

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КРИТИЧНИЙ АНАЛІЗ АНАЛІТИЧНИХ ТА ЧИСЕЛЬНИХ МОДЕЛЕЙ ЗЧЕПЛЕННЯ АРМАТУРИ З БЕТОНОМ

У статті представлено комплексний критичний аналіз існуючих підходів до моделювання зчеплення арматури з бетоном, що є фундаментальним фактором для забезпечення надійності та довговічності залізобетонних конструкцій. Актуальність дослідження посилюється в контексті сучасних викликів, пов'язаних із повномасштабною війною в Україні, що вимагає точного прогнозування поведінки захисних споруд під дією динамічних, ударних та вибухових навантажень.

Систематизовано та проаналізовано ключові напрямки досліджень: експериментальні методи, аналітичні моделі та чисельні симуляції. Розглянуто обмеження класичних експериментальних методів, таких як випробування на висмикування (pull-out test) та балочні випробування (beam-end test), і висвітлено переваги сучасних технологій моніторингу, зокрема розподілених волоконно-оптичних датчиків (DFOS) та цифрової кореляції зображень (DIC), які дозволяють отримати деталізовані дані про локальну поведінку зчеплення.

Проведено критичний огляд аналітичних моделей "напруження зчеплення-проковзування", від напівемпіричних залежностей, як-от модель BPE, до більш теоретично обґрунтованих підходів на основі теорії товстостінного циліндра та моделі фіктивних тріщин. Продемонстровано, що через залежність від умов експерименту та значний розкид даних ці моделі часто не є універсальними.

Особливу увагу приділено класифікації та аналізу чисельних моделей за рівнем деталізації. Макроскопічні моделі, від спрощених (SDOF, модель ідеального зчеплення) до вдосконалених (модель багатошарового перерізу з еквівалентною жорсткістю, моделі на основі систем диференціальних рівнянь), оцінено з точки зору обчислювальної ефективності та точності врахування ефекту проковзування. Детально розглянуто мезоскопічні підходи, які моделюють арматуру та бетон як окремі тіла, включаючи моделі з пружинними та когезійними елементами (CZM), фрикційно-когезійні моделі (FCZM), контактні алгоритми (1D Slide Line) та решітчасті (Lattice) моделі. Висвітлено переваги та недоліки кожного підходу, від фізичної обґрунтованості до обчислювальної складності. Також розглянуто перспективи застосування методів машинного навчання (напр., NARX, SSA-ELM) для швидкого та точного прогнозування режимів руйнування та залежностей "зчеплення-проковзування".

У статті зроблено висновок про критичну важливість врахування ефекту проковзування для адекватного моделювання поведінки залізобетонних конструкцій, особливо після досягнення межі плинності арматури. Визначено, що вибір моделі має ґрунтуватися на балансі між необхідною точністю та наявними ресурсами, і сформульовано перспективні напрямки подальших досліджень, спрямовані на створення універсальних та обчислювально ефективних моделей.

Ключові слова: залізобетон, модель зчеплення-ковзання, моделювання методом скінченних елементів, 3D-модель когезивної зони, випробування на висмикування, балково-кінцеве випробування, метод глибокого навчання.

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CRITICAL ANALYSIS OF ANALYTICAL AND NUMERICAL MODELS OF BOND BETWEEN REINFORCEMENT AND CONCRETE

This paper presents a comprehensive critical analysis of existing approaches to modeling the bond between reinforcement and concrete, which is a fundamental factor in ensuring the reliability and durability of reinforced concrete structures. The relevance of this research is amplified in the context of the current challenges posed by the full-scale war in Ukraine, which demands accurate prediction of the behavior of protective structures under dynamic, impact, and blast loadings.

The key research areas have been systematized and analyzed: experimental methods, analytical models, and numerical simulations. The limitations of classical experimental methods, such as pull-out tests and beam-end tests, are reviewed, and the advantages of modern monitoring technologies are highlighted. These include distributed fiber optic sensing (DFOS) for quasi-continuous measurement of reinforcement strains and digital image correlation (DIC) for analyzing crack kinematics, both of which provide detailed data on local bond behavior.

A critical review of analytical bond-slip models is conducted, ranging from semi-empirical relationships like the BPE model to more theoretically grounded approaches based on the thick-walled cylinder theory and the fictitious crack model. It is demonstrated that due to their dependence on specific experimental conditions and significant data scatter, these models often lack universal applicability.

Particular attention is given to the classification and analysis of numerical models based on their level of detail. Macroscopic models, from simplified (SDOF, perfect bond model) to advanced approaches (layered section model with equivalent stiffness, models based on systems of differential equations), are evaluated in terms of computational efficiency and accuracy in accounting for the slip effect. Mesoscopic approaches that model reinforcement and concrete as separate bodies are discussed in detail, including models with spring and cohesive zone elements (CZM), frictional-cohesive zone models (FCZM), contact algorithms (1D Slide Line), and lattice models. The advantages and disadvantages of each method, from physical justification to computational complexity, are highlighted. Furthermore, the prospects of applying machine learning methods (e.g., NARX, SSA-ELM) for the rapid and accurate prediction of failure modes and bond-slip relationships are considered.

The paper concludes that accounting for the bond-slip effect is critically important for the adequate modeling of the behavior of reinforced concrete structures, especially after the reinforcement reaches its yield point. The choice of a model should be based on a balance between the required accuracy and available resources. Finally, promising directions for future research are formulated, aimed at creating universal and computationally efficient numerical models.

Keywords: reinforced concrete, bond-slip model, finite-element simulation, 3D cohesive zone model, Pull-out test, Beam-end test, deep learning method.

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Представлено комплексний критичний аналіз існуючих аналітичних та чисельних моделей зчеплення арматури з бетоном, актуальність якого підкреслена потребою проєктування надійних захисних споруд в умовах війни в Україні. Систематизовано підходи до моделювання за рівнями деталізації — від макроскопічних моделей (ідеальне зчеплення, SDOF, багатошаровий переріз) до мезоскопічних (CZM, FCZM, Lattice models) та мікроскопічних, а також розглянуто сучасні методи на основі машинного навчання (NARX, SSA-ELM). На основі аналізу переваг та недоліків кожного методу зроблено висновок про критичну важливість врахування ефекту проковзування для точної оцінки поведінки залізобетонних конструкцій та визначено перспективні напрями досліджень.
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The comprehensive critical analysis of existing analytical and numerical models for rebar-concrete bond, the relevance of which is heightened by the need to design reliable protective structures under the conditions of war in Ukraine were presented. Modeling approaches are systematized by levels of detail — from macroscopic models (perfect bond, SDOF, layered section) to mesoscopic (CZM, FCZM, Lattice models) and microscopic, as well as modern machine learning methods (NARX, SSA-ELM). Based on an analysis of the advantages and disadvantages of each method, it is concluded that considering the bond-slip effect is critical for accurately assessing the behavior of reinforced concrete structures, and promising directions for future research are identified
Fig. 16. Ref. 53.

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