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NON-STATIONARY BENDING VIBRATIONS OF A HELICOPTER ROTOR BLADE UNDER DISTRIBUTED AERODYNAMIC LOAD. PART 1. BLADE`S EIGENBENDING VIBRATIONS

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The problem of non-stationary eigenbending vibrations of a helicopter rotor blade is solved. The helicopter blade is approximated by a thin rectangular plate. The vibration of the blade is modeled on the basis of the classical theory of bending vibration, the Lagrange-Sophie Germain equation, under the conditions of cantilever fixation. The problem is solved in two ways, both of which reveal the relationship between the eigenvalues. It is shown that the solution of the problem is not unique.
Keywords: helicopter rotor blade, eigenbending vibrations of a rectangular plate, analytical solution.

Introduction. The problem of flight safety is an important part of aircraft flight planning. One of the aspects of helicopter safety is the aerodynamic resistance to changes in the external load on the helicopter rotor [1], since the rotor is responsible for keeping the helicopter in the air in a state of balance.

In different flight regimes, the aerodynamic characteristics of the rotor vary substantially. Variable unsteady loads acting on the helicopter rotor blades cause bending vibrations of the blades, which consequently leads to unsteady changes in aerodynamic characteristics, in particular, thrust. This is a negative impact on flight stability and can lead to unexpected situations, loss of helicopter balance, and subsequent crash. Thus, the relevance and importance of solving the problem of blade eigenvibrations is obvious.

Despite the importance of this issue, a review of existing works indicates that insufficient attention has been paid to the relationship between helicopter rotor blade motion and aerodynamic stability in motion. Most of the existing works are devoted to two areas of research: 1) studying the flow field around the blades, on the basis of which the pressure coefficient and the blade drag coefficient are calculated (using approximate formulas) [2], [3]; 2) studying the oscillation of the blade as an element of a mechanical system using the methods of theoretical mechanics. However, the existing videos of helicopter blade movement in flight (Fig. 1) show that the blade behavior is more similar to small unsteady bending vibrations of a plate. Therefore, in order to study the non-stationary behavior of a helicopter blade, the following model problem is proposed in this paper. Let us assume that the helicopter blade is, to a first approximation, close to a thin rectangular plate, the unsteady vibrations of which are studied in this paper under certain boundary conditions. That is, we will replace the blade in the first approximation with a thin rectangular plate.

Mathematical model of blade vibration. It is known that helicopter rotor blades are manufactured to be sufficiently rigid, which means that the Kirchhoff-Love hypothesis about the absence of transverse deformations of the plate (blade) is fulfilled. So, let's apply the Kirchhoff-Love theory [4] of plate bending.

Consider a rectangular plate (Fig. 2). Let's introduce a rectangular Cartesian coordinate system, so $Oxyz$. Let the deflection of the plate, i.e. its deviation from the initial position $w = 0$, be denoted by $w(x, y, t)$.

(a) (b) (c)

Fig. 1. Instantaneous camera photos of helicopter blade transverse movement: (a) lower position; (b) middle position; (c) upper position

We assume, as a first approximation, that the cylindrical stiffness D of the plate and its thickness h are constant. The equations describing such vibrations are the Lagrange-Sophie Germain equations [5]:

$$D(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4})+\rho h\ddot{w}=q(x,y,t)\,, \tag{1}$$

where $q(x,y,t)$ is the load distributed over the blade surface. To formulate the problem for equation (1), it is necessary to set boundary conditions that maximally meet the conditions for fixing the helicopter rotor blade to the main rotor shaft, as well as initial conditions. However, before formulating and solving the problem, we will present a brief analysis of the already solved problems of plate oscillation under different boundary conditions. This will help us understand the direction and methods of research.

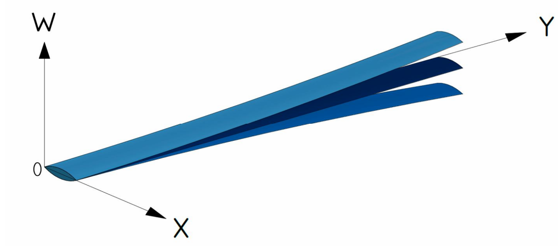

Fig. 2. Different blade positions corresponding to Fig. 1

In [6], the problem of eigenvibrations oscillations of a cantilevered rectangular plate was solved approximately using the Ritz method, [7, p.77, (4.56)]. In [8], [9], the Ritz method was also used to determine an approximate solution for the vibration frequencies and modes. A more detailed analysis of the plate vibration under different boundary conditions is given in monograph [7].

Later, the issue of studying the free oscillations of a rectangular plate is returned to again [10], [11], where the eigenvalues and modes of oscillations are studied under the condition of asymptotic approximations. Thus, we can see that not for all cases of loading the boundary of a rectangular plate, it was possible to find an analytical solution without resorting to asymptotic and approximate methods of solving problems.

Since we are interested in finding an analytical solution of the problem of oscillations of a cantilevered plate, we consider two variants of the problem statement: 1) eigenvibrations of the plate, 2) forced vibrations of the plate.

Eigenbending vibrations of a cantilevered plate. Formulation of the problem

First, we solve the problem of eigenvibrations of a plate (blade), i.e. when $q(x,y,t)=0$. We set the following boundary conditions:

1) clamped edge, $y=0$:

$$W(x,0,t)=\frac{\partial W}{\partial x}(x,0,t)=0, \tag{2}$$

2) free edge, $y=R$:

$$M_x=-D(\frac{\partial^2 W}{\partial x^2}+\nu\frac{\partial^2 W}{\partial y^2})=0, \quad Q_x^*=-D\left[\frac{\partial^3 W}{\partial x^3}+(2-\nu)\frac{\partial^3 W}{\partial x\partial y^2}\right]=0, \tag{3}$$

3) free edge, $x=0;\ x=c$:

$$M_x=-D(\frac{\partial^2 W}{\partial x^2}+\nu\frac{\partial^2 W}{\partial y^2})=0,\ Q_X^*=-D\left[\frac{\partial^3 W}{\partial x^3}+(2-\nu)\frac{\partial^3 W}{\partial x\partial y^2}\right]=0. \tag{4}$$

In the boundary conditions (2)-(4), the stationary part of the blade displacement $w(x,y,t)$ is denoted by $W(x,y)$. To solve problems (1)-(4), we apply the well-known Navier approach.

Determination of eigenvalues and eigenmodes of a plate by the Navier method. To find the eigenvalues and natural oscillation shapes of the plate, we need to put in equation (1) $q(x,y,t)=0$. That is, consider the natural oscillations of a helicopter blade. The boundary conditions (2)-(4) are those of a cantilevered plate. According to Navier's approach [5], [12], the solution of the stationary part of the problem should be sought in the form:

$$W(x,y)=\sin\lambda_1 x\cdot\sin\lambda_2 y, \tag{5}$$

where λ_1,λ_2 are the eigenvalues determined from the boundary conditions. Boundary condition (2) is exactly satisfied, since at $y=0$ we have $\sin\lambda_2\cdot 0=0$. Therefore, we will not find eigenvalues λ_1,λ_2 from it. Instead, the boundary condition (3), consisting of two equations, is the determining one.

Indeed, the boundary condition $M_x=-D(\frac{\partial^2 W}{\partial x^2}+\nu\frac{\partial^2 W}{\partial y^2})_{|y=R}=0$, after differentiation and substitution $y=R$, yields:

$$-\lambda_1^{\ 2}\sin\lambda_1 x\cdot\sin\lambda_2 R-\nu\lambda_2^2\sin\lambda_1 x\cdot\sin\lambda_2 R=0\,. \tag{6}$$

Since $\sin\lambda_1 x$ can take on different values, we assume that $\sin\lambda_1 x\neq 0$. So, dividing (6) by $\sin\lambda_1 x$, we have:

$$(\lambda_1^{\ 2}+\nu\lambda_2^2)\sin\lambda_2 R=0\,. \tag{7}$$

The expression $\lambda_1^{\ 2}+\nu\lambda_2^2>0$, since both eigenvalues do not turn to zero simultaneously (we are not interested in a trivial solution), then we have:

$$\sin\lambda_2 R=0\,. \tag{8}$$

Equation (8) defines a set of eigenvalues: $\lambda_2=\dfrac{\pi n}{R},n\in Z$.

Fulfilment of the boundary condition

$$Q^*_{X|y=R}=-D\left[\frac{\partial^3 W}{\partial x^3}+(2-\nu)\frac{\partial^3 W}{\partial x\partial y^2}\right]_{|y=R}=0$$

leads to the following relation:

$$(-\lambda_1^{\ 3}\cdot\sin\lambda_2 R-(2-\nu)\lambda_1\lambda_2^2)\cdot\cos\lambda_1 x\cdot\sin\lambda_2 R=0\quad . \tag{9}$$

Since $\cos\lambda_1 x\neq 0$ for most values of x, therefore, relation (9) is satisfied for $\sin\lambda_2 R=0$. Thus, both free edge boundary conditions give the same equation for the eigenvalues of λ_2.

Fulfilment of the boundary condition $M_{X|x=0}=0$ leads to the relation:

$$-\lambda_1^{\ 2}\sin\lambda_1 0\cdot\sin\lambda_2 y-\nu\lambda_2^2\sin\lambda_1 0\cdot\sin\lambda_2 y\equiv 0\,, \tag{10}$$

because $\sin\lambda_1 0\equiv 0$.

Fulfilling the boundary condition $M_{x|x=c}=0$ gives the following relationship:

$$-\lambda_1^{\ 2}\sin\lambda_1 C\cdot\sin\lambda_2 y-\nu\lambda_2^2\sin\lambda_1 C\cdot\sin\lambda_2 y=0\,. \tag{11}$$

Since $\sin\lambda_2 y$ it is different from zero for virtually the entire region of change in the values of y, except for a counted number of points, the relation (11) is fulfilled under the condition:

$$\sin\lambda_1 c = 0 \text{ , that is } \lambda_1 = \frac{\pi m}{c},\ m \in Z \text{ .} \quad (12)$$

In the end, we obtained a set of eigenforms that are the same as in the Navier solution. But the fulfilment of the condition $Q_{x|x=0} = 0$ leads to the following relation:

$$(-\lambda_1{}^3 - (2-\nu)\lambda_1\lambda_2^2)\cos\lambda_1 0\cdot\sin\lambda_2 y = 0 \text{ .} \quad (13)$$

Since $\cos\lambda_1 0 = 1$ and $\sin\lambda_2 y \neq 0$, then the relation (13) is satisfied under the condition:

$$-\lambda_1{}^3 - (2-\nu)\lambda_1\lambda_2^2 = 0 \text{ ,} \quad (14)$$

which is only valid when

$$\lambda_1 = \pm i(2-\nu)\lambda_2 \text{ .} \quad (15)$$

Relationship (15) indicates the dependence of the two forms of fluctuations on each other. Fulfilment of the condition $Q_{x=c} = 0$ leads to the following equality:

$$(-\lambda_1{}^3 - (2-\nu)\lambda_1\lambda_2^2)\cos\lambda_1 C\cdot\sin\lambda_2 y = 0 \text{ ,} \quad (16)$$

which is satisfied by condition (15), or

$$\cos\lambda_1 c = 0 \text{ .} \quad (17)$$

If condition (17) is fulfilled, condition (12) is not fulfilled, i.e., we assume that $\cos\lambda_1 c \neq 0$. Thus, for the conditions of the free edge of the plate, we have obtained the dependence of the eigenvalues on each other. Why is this so? The answer to this question is given by another method of solving this problem.

Solving problems (1)-(4) by the Levy method. Indeed, this problem can be solved by another method after we have found $\lambda_2 = \frac{\pi n}{R},\ n \in Z$. Let's apply the approach used by Levy when solving the problem for a rectangular plate with two parallel edges hinged (see [4]).

Consider the case of the homogeneous equation (1). The values found above will not be affected by the zero equality of the right-hand side of equation (1), since we found these values only from satisfying the boundary conditions without solving equation (1).

Since the helicopter rotor rotates around the helicopter main shaft with a frequency of ω, we assume that the load is harmonically distributed in time: $w(x,y,t) = W(x,y)\cos(\omega t - \varphi)$. Therefore, we have:

$$D\left(\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4}\right) - \rho h\omega^2 W = 0 \text{ .} \quad (18)$$

According to [4], the general solution $W_{nm}(x,y)$ of equation (18) is presented in the form:

$$W_{nm}(x,y) = X_m(x)\cdot\sin\frac{\pi n}{R}y \text{ .} \quad (19)$$

Substituting (19) into (18) results in the usual linear homogeneous differential equation of the fourth order for $X_m(x)$:

$$X_m'''' - 2\left(\frac{\pi n}{R}\right)^2 X_m'' + \left[\left(\frac{\pi n}{R}\right)^4 - \frac{\rho h}{D}\omega^2\right]X_m = 0 \text{ .} \quad (20)$$

The characteristic equation for a given differential equation is:

$$s^4 - 2(\pi n/R)^2 s^2 + \left[(\pi n/R)^4 - \frac{\rho h}{D}\omega^2\right] = 0 \text{ .} \quad (21)$$

In this case $\sqrt{\frac{\rho h}{D}\omega^2} > (\pi n/a)^2$, we have the following solutions:

$$s_{1,2} = \pm i\sqrt{\sqrt{\frac{\rho h}{D}\omega^2} - (\pi n/R)^2},\ \ s_{3,4} = \pm\sqrt{\sqrt{\frac{\rho h}{D}\omega^2} + (\pi n/R)^2} \text{ .} \quad (22)$$

So, then

$$X_n(x) = C_1 \sin k_1 x + C_2 \cos k_1 x + C_3 shk_2 x + C_4 chk_2 x\,, \tag{23}$$

where

$$k_1 = \sqrt{\sqrt{\frac{\rho h}{D}\omega^2} - (\pi n/R)^2}\,,\;\; k_2 = \sqrt{\sqrt{\frac{\rho h}{D}\omega^2} + (\pi n/R)^2}\,. \tag{24}$$

Taking into account (23), the solution of the general form of equation (18) is written:

$$W(x,y) = \sum_{n=1}^{\infty} W_n(x,y) = \sum_{n=1}^{\infty} (C_1 \sin k_1 x + C_2 \cos k_1 x + C_3 shk_2 x + C_4 chk_2 x)\cdot \sin\frac{\pi n}{R} y\,. \tag{25}$$

It is easy to see from (24), (25) that the general solution has only one sum over n, since the expressions k_1, k_2 depend only on the numbers n and the rotation frequency of ω.

Next, we satisfy the boundary conditions for $M_{x|x=0;c} = 0,\; Q_{x|x=0;c} = 0$:

a) $M_{x|x=0} = \dfrac{\partial^2 W}{\partial x^2} + \nu \dfrac{\partial^2 W}{\partial y^2} = 0$, or after substitution (25):

$$(-C_1 k_1^{\,2} \sin(k_1\cdot 0) - C_2 k_1^{\,2} \cos(k_1 \cdot 0) + C_3 k_2^{\,2} sh(k_2\cdot 0) + C_4 k_2^{\,2} ch(k_2 \cdot 0)\sin\frac{\pi n}{R} y -$$

$$-\nu(C_1 \sin(k_1\cdot 0) + C_2 \cos(k_1\cdot 0) + C_3 sh(k_2\cdot 0) + C_4 ch(k_2\cdot 0)(\frac{\pi n}{R})^2 \sin\frac{\pi n}{R} y = 0\,. \tag{26}$$

b) $Q_{x|x=0} = \dfrac{\partial^3 W}{\partial x^3} + (2-\nu)\dfrac{\partial^3 W}{\partial x \partial y^2} = 0$, or after substitution (25):

$$(-C_1 k_1^{\,3} \cos(k_1\cdot 0) - C_2 k_1^{\,3} \sin(k_1\cdot 0) + C_3 k_2^{\,3} ch(k_2\cdot 0) + C_4 k_2^{\,3} sh(k_2\cdot 0)\sin\frac{\pi n}{R} y -$$

$$-(2-\nu)(C_1 k_1 \cos(k_1\cdot 0) - C_2 k_1 \sin(k_1\cdot 0) + C_3 k_2 ch(k_2\cdot 0) + k_2 C_4 sh(k_2\cdot 0)(\frac{\pi n}{R})^2 \sin\frac{\pi n}{R} y = 0\,. \tag{27}$$

c) $M_{x|x=c} = \dfrac{\partial^2 W}{\partial x^2} + \nu \dfrac{\partial^2 W}{\partial y^2} = 0$, or after substitution (25):

$$(-C_1 k_1^{\,2} \sin(k_1\cdot c) - C_2 k_1^{\,2} \cos(k_1 \cdot c) + C_3 k_2^{\,2} sh(k_2\cdot c) + C_4 k_2^{\,2} ch(k_2 \cdot c)\sin\frac{\pi n}{R} y -$$

$$-\nu(C_1 \sin(k_1\cdot c) + C_2 \cos(k_1\cdot c) + C_3 sh(k_2\cdot c) + C_4 ch(k_2\cdot c)(\frac{\pi n}{R})^2 \sin\frac{\pi n}{R} y = 0\,. \tag{28}$$

d) $Q_{x|x=c} = \dfrac{\partial^3 W}{\partial x^3} + (2-\nu)\dfrac{\partial^3 W}{\partial x \partial y^2} = 0$, or after substitution (25):

$$(-C_1 k_1^{\,3} \cos(k_1\cdot c) - C_2 k_1^{\,3} \sin(k_1\cdot c) + C_3 k_2^{\,3} ch(k_2\cdot c) + C_4 k_2^{\,3} sh(k_2\cdot c)\sin\frac{\pi n}{R} y -$$

$$-(2-\nu)(C_1 k_1 \cos(k_1\cdot c) - C_2 k_1 \sin(k_1\cdot c) + C_3 k_2 ch(k_2\cdot c) + k_2 C_4 sh(k_2\cdot c)(\frac{\pi n}{R})^2 \sin\frac{\pi n}{R} y = 0\,. \tag{29}$$

The multiplier $\sin\dfrac{\pi n}{R} y$ remains unchanged and can be reduced by almost everywhere in equations (26)-(29) (except for the points $y = 0; R$). As a result, we have a homogeneous system of four linear algebraic equations with respect to unknown constants C_1, C_2, C_3, C_4, which has a nontrivial solution only if its determinant is zero. If we fulfil this condition, then our system of linear algebraic equations will have a rank less than the number of unknowns. This means that we actually have a linear relationship among the unknown constants C_1, C_2, C_3, C_4. From the condition of equality of the zero determinant of the matrix, we also get the dependence of the eigenvalues. Thus, as in the above Navier solution, we also find a linear dependence of the eigenvalues in this approach. This means that in the case of one boundary being clamped and the remaining three free, the solution to the homogeneous equation (18) is not unique.

Further direction of research. In a real situation, a helicopter blade modelled by a plate is actually under the influence of a distributed aerodynamic load caused by the lift. Therefore, in Part 2 of this paper, we propose a solution to the problem of eigenoscillations of the plate under the specified boundary conditions and an example of numerical calculation. A general solution to the problem of forced non-stationary oscillations of a plate and a helicopter blade is also obtained.

Conclusions. In this paper, the problem of small bending eigenoscillations of a rectangular thin plate, which is used to model a helicopter blade, is formulated and analytically solved, provided that one edge is cantilevered and the other three edges are free. To solve this problem, two approaches were used - Navier and Levy. Both approaches showed the dependence of the eigenvalues, i.e., the existence of more than one solution to the problem. Therefore, in Part 2 of this paper, we propose an example of solving the problem under specified boundary conditions that allow us to distinguish from the entire set of solutions a single solution to the problem of free oscillations of a plate, as well as to write down a general solution to the problem of forced bending oscillations of a plate and a helicopter blade.

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НЕСТАЦІОНАРНІ ЗГИНАЛЬНІ КОЛИВАННЯ ЛОПАТІ РОТОРА ВЕРТОЛЬОТУ ПІД ДІЄЮ РОЗПОДІЛЕНОГО АЕРОДИНАМІЧНОГО НАВАНТАЖЕННЯ. ЧАСТИНА 1. ВЛАСНІ ЗГИНАЛЬНІ КОЛИВАННЯ ЛОПАТІ

Стабільність поведінки вертольоту під час польоту залежить від багатьох факторів. Один з них безпосередньо пов'язаний з нестаціонарними пружними згинальними коливаннями лопатей ротора. Тиск повітря на поверхню лопаті призводить до появи в даному елементі згинальних деформацій. Оскільки рух ротора вертольоту є періодичним, нестаціонарне навантаження на лопать змінюється за гармонічним законом. В результаті маємо змінний тиск та зміну підйомної сили. Все перелічене призводить до зміни аеродинамічних параметрів, що спричиняє нестійкість руху вертольоту. В даній роботі дослідження згинальних коливань лопаті виконується на основі її моделювання у вигляді тонкої прямокутної пластини.

Наявні експериментальні дані фотозйомок зафіксували рух лопаті вертольоту у вигляді коливань, які є подібними до коливань консольно закріпленої пластини. Відомо, що лопаті гелікоптера виготовляються досить жорсткими за рахунок наявних ребер жорсткості - нервюр.

Отже, у даній роботі використано гіпотези Кірхгофа-Лява, рівняння Лагранжа-Софі Жермен для моделювання нестаціонарних згинальних коливань лопаті вертольоту як тонкої пластини.

Огляд існуючих робіт показав, що вивченню згинальних коливань прямокутної пластини, що описується рівнянням Лагранжа-Софі Жермен, присвячено чимало досліджень. Але, точного розв'язку задачі про вільні коливання пластини у випадку її консольного закріплення немає, існують лише наближені розв'язки з використанням методу Ріца.

В даній роботі двома різними способами було здійснено спробу аналітично розв'язати задачу про вільні коливання малої амплітуди тонкої пластини за класичних граничних умов консольного закріплення пластини. Знайдений аналітичний розв'язок методом Леві, як виявилось, не є єдиним. Система рівнянь для визначення коефіцієнтів загального розв'язку має ранг, менший за кількість невідомих. Це вказує на залежність власних чисел задачі, тобто зв'язаність коливань, яка раніше

Леві була знайдена за інших граничних умов. Оскільки у даній роботі за мету поставлено знаходження єдиного розв'язку, то у Частині 2 даної роботи буде наведено приклад уточнених граничних умов, які дозволили знайти єдиний розв'язок задачі.

Ключові слова: лопать ротора вертольоту, власні згинальні коливання прямокутної пластини, аналітичний розв'язок.

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NON-STATIONARY BENDING VIBRATION OF A HELICOPTER ROTOR BLADE UNDER DISTRIBUTED AERODYNAMIC LOAD. PART 1. BLADE`S EIGENBENDING VIBRATIONS

The stability of helicopter behaviour during flight depends on many factors. One of them is directly related to the non-stationary elastic bending vibrations of the rotor blades. Air pressure on the blade surface causes bending deformations in this element. Since the movement of the helicopter rotor is periodic, the non-stationary load on the blade varies according to a harmonic law. As a result, we have a variable pressure and a change in lift. All of the above leads to a change in aerodynamic parameters, which causes helicopter motion instability. In this paper, the study of the blade's bending vibrations is based on its modelling as a thin rectangular plate.

The available experimental data from photographs recorded the movement of the helicopter blade in the form of oscillations similar to those of a cantilevered plate. It is known that helicopter blades are made quite rigid due to the existing stiffeners - nerves.

Therefore, in this work, the Kirchof-Love hypothesis and the Lagrange-Sophie Germain equation are used to simulate the non-stationary bending vibrations of a helicopter blade as a thin plate.

A review of existing works has shown that many studies have been dedicated to the study of bending vibrations of a rectangular plate described by the Lagrange-Sophie Germain equation. However, there is no exact solution to the problem of free vibrations of a plate in the case of its cantilever mounting. There are only approximate solutions using the Ritz method.

In this paper, an attempt was made to solve analytically the problem of free oscillations of small amplitude of a thin plate under classical boundary conditions of cantilevered plate fixation by two different methods. As it turned out, the analytical solution obtained by the Levy method is not unique. The system of equations for determining the coefficients of the general solution has a rank less than the number of unknowns. This indicates the dependence of the eigenvalues of the problem, i.e. the coupling of oscillations, which was previously found by Levy under other boundary conditions. Since the goal of this paper is to find a single solution, Part 2 of this paper presents an example of refined boundary conditions that allowed us to find a single solution to the problem.

Keywords: helicopter rotor blade, bending eigen vibrations of a rectangular plate, analytical solution.

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Лук'янов П.В., Лукан А.В., Шкриль О.О. **Нестаціонарні згинальні коливання лопаті ротора вертольота. Частина 1. Власні згинальні коливання лопаті** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2025. – Вип. 115. – С. 114-120.

Розв'язано задачу про нестаціонарні власні згинальні коливання лопаті ротора вертольота. Лопать вертольоту наближено моделюється тонкою прямокутною пластиною. Досліджено власні коливання пластини на основі рівняння Лагранжа - Софі Жермен за умов консольного закріплення одного кінця та трьох вільних кінців. Показано, що розв'язок не є єдиним.

Іл. 2. Бібліогр. 12 назв.

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Lukianov P.V., Lukan A.V., Shkryl O.O. **Non-stationary bending vibrations of helicopter rotor blade under distributed aerodynamic load. Part 1. Blade`s eigenbending vibrations** // Strength of Materials and Theory of Structures: Scientific-&-Technical collected articles – Kyiv: KNUBA, 2025. – Issue 115. – P. 114-120.

The problem of non-stationary eigenbending vibrations of a helicopter rotor blade is solved. The helicopter blade is approximated by a thin rectangular plate. The eigenvibrations of the plate are investigated on the basis of the Lagrange-Sophie Germain equation under the conditions of cantilevered fixation of one end and three free ends. It is shown that the solution is not unique.

Fig. 2. Ref. 12.

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