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FINITE ELEMENT MODELING OF THE STATIC BEHAVIOR OF AN ELASTIC-PLASTIC SOIL BASE UNDER ROLLING STOCK LOAD

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A numerical method for studying the static behavior of a soil base in a geometrically and physically nonlinear formulation under the action of rolling stock is proposed. The mathematical model of the boundary-value problem of the ballast prism and soil base statics was constructed using the computational procedures of the finite element analysis program NASTRAN. Both single-layer and multi-layer soil models in the form of a plane half-space were considered. The static action of rolling stock was represented as a concentrated force applied to the ballast prism, corresponding to the weight of empty and loaded freight train cars. To describe the elastic-plastic behavior of the single-layer soil base, the Mohr–Coulomb model was applied, while for the multi-layer base the Drucker–Prager model was used. To evaluate the elastic-plastic behavior of the soil base, results of linear and nonlinear static analyses of both models under two load cases were compared. The Newton–Raphson method was applied to study the static characteristics of the models in a nonlinear formulation. The influence of considering the elastic-plastic models of single-layer and multi-layer soil bases on their stress–strain state was assessed.
Keywords: finite element method, nonlinear static, ground base, rolling stock.

Introduction. The study of nonlinear soil deformation requires the use of a correct computational model that takes into account the properties of soils in individual layers, as well as the hydrogeological conditions of the soil mass. When a weak soil layer is present in the soil medium, the stress distribution pattern changes significantly. In engineering practice and construction, soil models are divided into linear and nonlinear [1, 2]. Each of them has its limitations in application. Linear models, such as the Winkler–Fuss model, consider soil in an elastic state, where deformations are proportional to the applied load. Nonlinear models, namely: the Mohr–Coulomb model, also known as the Mohr–Coulomb strength criterion, which is based on experimental data and describes the conditions under which soil begins to undergo plastic deformations and failure; the hardening soil model; the creep model; the Cam-Clay model; the isotropic linear-elastic perfectly plastic model with the Hoek–Brown strength criterion; as well as the Drucker–Prager model, which depends on hydrostatic pressure, take into account more complex soil behavior, including nonlinear stress–strain relationships. For qualitative analysis, one or a combination of several methods can be used.

The article presents a numerical method for studying the static behavior of a soil base in a geometrically and physically nonlinear formulation under the action of rolling stock. Single-layer and multi-layer soil base models in the form of a plane half-space are considered. To describe the elastic-plastic behavior of the single-layer soil base, the Mohr–Coulomb model is applied, and for the multi-layer base, the Drucker–Prager model is used. The mathematical model of the boundary-value problem of the ballast prism and soil base statics was built using the computational procedures of the finite element analysis program NASTRAN [3]. The influence of elastic-plastic soil behavior on the stress–strain state under the weight of empty and loaded freight cars was assessed.

1. Finite Element Model of the Ballast Prism and Base

The mathematical model of the boundary-value problem of the ballast prism and soil foundation statics is constructed based on Lagrange's principle of virtual displacements in generalized coordinates and is implemented using the finite element method. To obtain the reduced stiffness matrices, geometric stiffness, and generalized external forces, the computational procedures of the NASTRAN program were applied.

The finite element model of the ballast prism and a single-layer soil foundation (sand) in the form of an elastic plane half-space with a length of 200 m and a depth of 30 m was presented by the authors in work [5]. The characteristics of the ballast prism were assumed as follows: sand (ballast): $E_0 = 16,484 \cdot 10^3 \text{kPa}$, $\nu = 0,3$, $\beta = 0,3$, $\rho = 10,0 \text{kN/m}^3$; macadam (ballast): $E = 5 \cdot 10^5 \text{kPa}$, $\nu = 0,27$, $\beta = 0,27$, $\rho = 14,0 \text{kN/m}^3$; reinforced concrete sleepers: $E = 3,8 \cdot 10^7 \text{kPa}$, $\nu = 0,2$, $\beta = 0,05$, $\rho = 24,5 \text{kN/m}^3$.

The computational model of the multilayered foundation consisted of five layers of soils from the actual engineering-geological profile (Fig. 1). The first layer (Layer 1) is fine sand of medium density with a low degree of water saturation, 2m thick, $\nu = 0,3 - 0,35$, $\beta = 0,42$; the second layer (Layer 2) is fine sand of medium density, water-saturated, 2m thick, $\nu = 0,3 - 0,35$, $\beta = 0,42$; the third layer (Layer 3) is light silty clay, soft-plastic, 1.5m thick, $\nu = 0,38 - 0,45$, $\beta = 0,35$; the fourth layer (Layer 4) is light and heavy soft-plastic clay, 3m thick, $\nu = 0,38 - 0,45$, $\beta = 0,39$; the fifth layer (Layer 5) is heavy silty clay, 10m thick, $\nu = 0,3 - 0,38$, $\beta = 0,39$. The remaining physical properties of the soil layers are presented in Table 1.

Table 1

Physical properties of the soils in the multilayer foundation model

Layer of soil	Den-sity, g/cm^3	Porosity coefficient	Angle of friction, degrees	Adhesion coefficient, kPa	Modulus of deformation, MPa	Liquidity index	Soil resisti-vity, kPa
1	1.64	0.61	28.2	0.7	17.3	-	300
2	1.95	0.61	30.9	0.7	28.1	-	200
3	1.82	0.85	14.8	15.3	6.7	0.69	120
4	1.86	0.9	16.5	22.0	12.5	0.34	215
5	1.94	0.9	18.3	28.7	18.5	0.18	220

Fig. 1. Finite element model of the ballast prism and multilayer soil foundation

The characteristics of the rolling stock are presented by the authors in work [5]. The static effect of the rolling stock is represented as a concentrated force from the weight of empty and loaded freight train cars applied to the ballast prism. The boundary conditions restrict the movement of the model from the plane of the soil foundation.

2. Consideration of the elasto-plastic properties of soils in the static analysis of the ballast prism and foundation

In modeling the single-layer foundation, the ideal elasto-plastic Mohr-Coulomb model was used, which, due to its simplicity, is widely applied in geotechnical engineering. The Mohr-Coulomb failure criterion states that the soil fails when certain values of normal and shear stresses are reached.

$$\tau = c + \sigma \tan \varphi, \quad (1)$$

where τ – shear stress; c – specific cohesion; σ – normal stress; φ – internal friction angle.

In the analysis of the multilayer foundation, considering the different nonlinear behavior of the soil layers, the Drucker-Prager model was adopted to account for plastic deformations. The Drucker-Prager criterion generalizes the Mohr-Coulomb model by incorporating a continuous yield surface, making it particularly suitable for pressure-sensitive soils and other materials

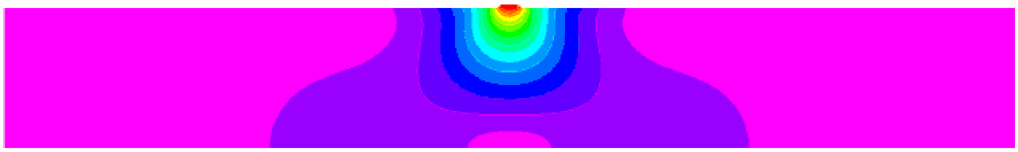
$$F = \tau - P \tan \beta - k = 0, \quad (2)$$

where F – yield function; τ – shear stress; P – mean pressure; β – inclination angle of the yield surface relative to the mean pressure axis; k – constant dependent on cohesion.

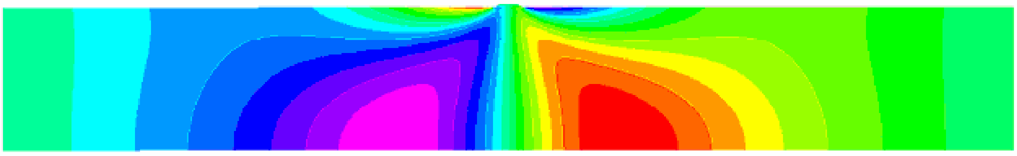
3. Static analysis of the ballast prism and soil foundation in linear and nonlinear formulations

Linear and nonlinear analyses of the computational model of the ballast prism and soil foundation, presented as a plane half-space, were performed under the load from the weight of empty and loaded freight train cars. The solution of the boundary-value statics problem was carried out using the Newton-Raphson method, implemented in the NASTRAN program.

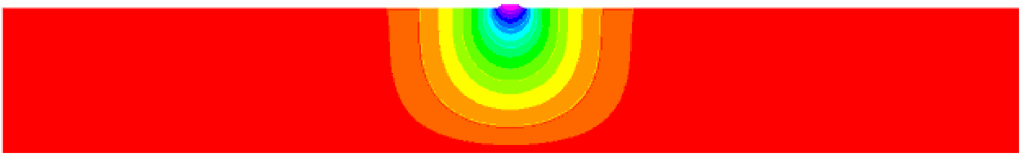
In the nonlinear static analysis of the single-layer soil foundation model, the elasto-plastic behavior of the soil (sand) was taken into account, represented in the study by the Mohr-Coulomb model. The results of the calculations for the single-layer model under the load from an empty car in linear and nonlinear formulations are shown in Figs. 2 and 3.



(a)

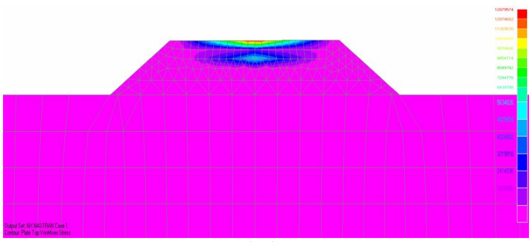


(b)

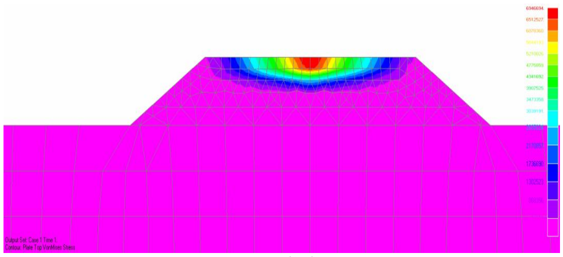


(c)

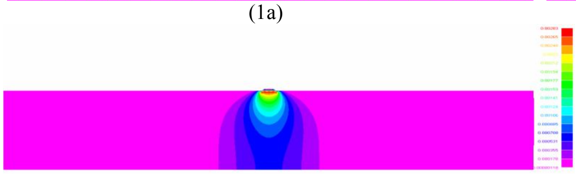
Fig. 2. Isolines of nodal displacements of the single-layer foundation under the weight of an empty car (P = 157 kN) in linear and nonlinear static analyses: (a) total displacements; (b) horizontal displacements; (c) vertical displacements



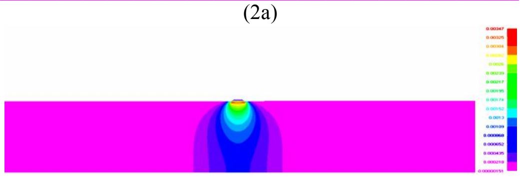
(1a)



(2a)



(1b)



(2b)

Fig.. 3. Isolines of equivalent (von Mises) stresses (a) and von Mises strains (b) in linear (1) and nonlinear (2) static analyses of the single-layer foundation under the weight of an empty car (P = 157 kN)

The values of maximum nodal displacements, equivalent stresses, and von Mises strains in linear (1) and nonlinear (2) static analyses of the single-layer foundation under the weight of an empty car are summarized in Table 2.

Table 2

Results of linear and nonlinear static analyses of the single-layer soil model under the weight of an empty car (P = 157 kN)

Problem formulation	Total nodal displacements, m	Horizontal displacements, m	Vertical displacements, m	Von Mises equivalent stresses, kPa	Von Mises strains	Plastic deformations
Linear	0,0386	0,0040	-0,0386	12879	0,0028	-
Nonlinear	0,0461	0,0051	-0,0461	6947	0,0035	0,0007

It can be observed that due to the physical nonlinearity of the soil, the von Mises strains in the elements of the single-layer foundation model under the weight of an empty car increased by 15.9%, while the plastic deformation accounted for 20% of the total deformation, and the von Mises stresses decreased by 53.94%.

Table 3 presents the values of maximum displacements, stresses, and strains obtained in the analysis of the single-layer model under the weight of a loaded car (P = 230.5 kN). The study showed that the isolines of displacements, strains, and stresses in the elements of the single-layer foundation model under the weight of the loaded car are similar to the corresponding isolines in the elements of this model under the weight of the empty car, as shown in Figs. 2 and 3.

Table 3

Results of linear and nonlinear static analyses of the single-layer soil model under the weight of a loaded car (P = 230.5 kN)

Problem formulation	Total nodal displacements, m	Horizontal displacements, m	Vertical displacements, m	Von Mises equivalent stresses, kPa	Von Mises strains	Plastic deformations
Linear	0,0564	0,0059	0,0564	18981	0,0043	-
Nonlinear	0,0674	0,0075	0,0674	10254	0,0035	0,0011

The comparison of results showed an increase in deformations by 16.7% due to the physical nonlinearity of the single-layer soil under the weight of the loaded car, with plastic deformation accounting for 31.4% of the total deformation, and the von Mises stresses decreasing by 54.02%.

The results of linear and nonlinear static analyses of the multilayer model under the weight of empty and loaded cars, presented as isolines of displacements, stresses, and strains, are shown in Figs. 4 and 5. In the nonlinear static analysis of the multilayer soil foundation model, the elasto-plastic properties of the foundation layers were taken into account, represented in the study using the Drucker-Prager model.

Table 4 presents the values of maximum displacements, stresses, and strains obtained in the analysis of the multilayer model under the weight of an empty car.

Table 4

Results of linear and nonlinear static analyses of the multilayer soil model under the weight of an empty car (P = 157 kN)

Problem formulation	Total nodal displacements, m	Horizontal displacements, m	Vertical displacements, m	Von Mises equivalent stresses, kPa	Von Mises strains	Plastic deformations
Linear	0,0364	0,0042	-0,0364	12759	0,0045	-
Nonlinear	0,0435	0,0043	-0,0435	7216	0,0048	0,0026

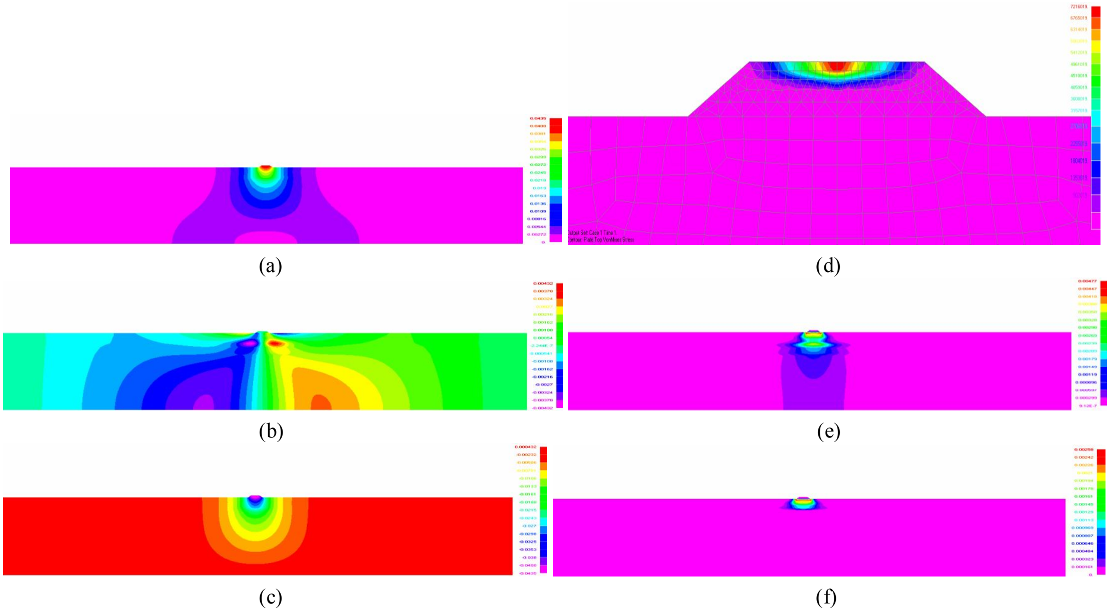


Fig. 4. Nonlinear static analysis of the multilayer foundation under the weight of an empty car (P = 157 kN): total nodal displacements (a), horizontal nodal displacements (b), vertical nodal displacements (c), von Mises equivalent stresses (d), von Mises strains (e), plastic deformations (f)

It can be observed that in the elements of the multilayer foundation model under the weight of an empty car, due to the physical nonlinearity of the soils, the von Mises strains increased by 5.8%, with plastic deformation accounting for 54.1% of the total deformation, while the von Mises stresses decreased by 43.45%.

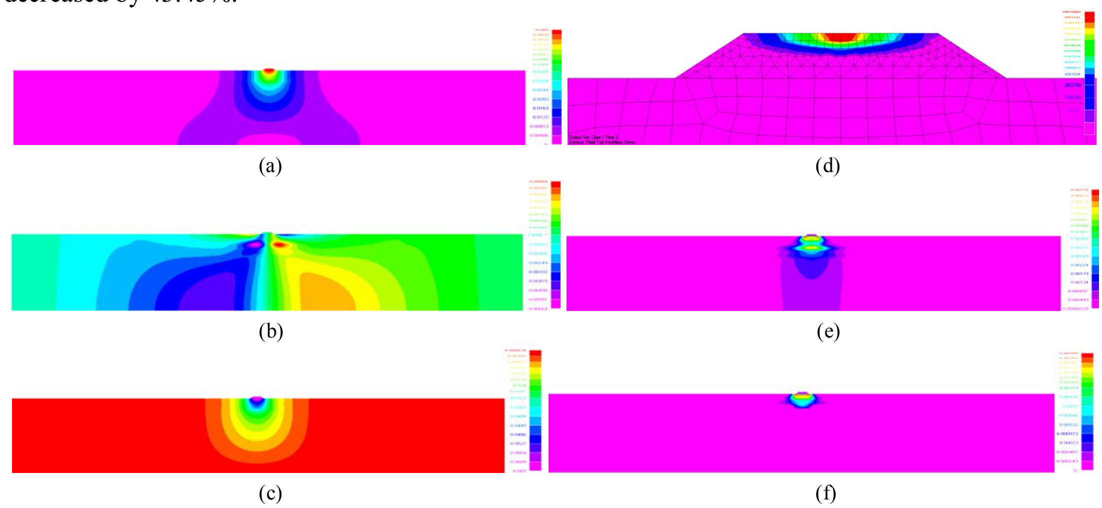


Fig. 5. Nonlinear static analysis of the multilayer foundation under the weight of a loaded car (P = 230.5 kN): total nodal displacements (a), horizontal nodal displacements (b), vertical nodal displacements (c), von Mises equivalent stresses (d), von Mises strains (e), plastic deformations (f)

Table 5 presents the values of maximum displacements, stresses, and strains obtained in the analysis of the multilayer model under the weight of a loaded car.

The study showed that due to the physical nonlinearity of the soils, the von Mises strains in the elements of the multilayer foundation model under the weight of a loaded car increased by 7.4%, with plastic deformation accounting for 54.33% of the total deformation, while the von Mises stresses decreased by 43.34%.

Table 5

Results of linear and nonlinear static analyses of the multilayer soil model under the weight of a loaded car (P = 230.5 kN)

Problem formulation	Total nodal displaceme nts, m	Horizontal displaceme nts, m	Vertical displaceme nts, m	Von Mises equivalent stresses, kPa	Von Mises strains	Plastic deformatio ns
Linear	0,0534	0,0061	-0,0534	18736	0,0066	-
Nonlinear	0,0650	0,0066	-0,0650	10615	0,0072	0,0039

Conclusion. The proposed methodology allowed for the development of a computational model of the ballast prism and foundation and for the investigation of nonlinear soil deformation, taking into account the elasto-plastic properties of its individual layers under the static load of rolling stock.

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Лук'янченко О.О., Козак А.А., Костін Д.Є.

СКІНЧЕННО-ЕЛЕМЕНТНЕ МОДЕЛЮВАННЯ СТАТИЧНОЇ ПОВЕДІНКИ ПРУЖНО-ПЛАСТИЧНОЇ ҐРУНТОВОЇ ОСНОВИ ПРИ ДІЇ РУХОМОГО СКЛАДУ

Запропонована чисельна методика дослідження статичної поведінки ґрунтової основи у геометрично і фізично нелінійній постановці від дії рухомого складу. Математична модель крайової задачі статики баластової призми і ґрунтової основи побудована із застосуванням обчислювальних процедур програми скінченно-елементного аналізу NASTRAN. Розглянуто одношарова і багатошарова моделі основи у вигляді плоского напівпростору. Статична дія рухомого складу подана у вигляді зосередженої сили, прикладеної до баластової призми, від ваги порожнього і завантаженого вагонів вантажного потягу. Для описання пружно-пластичної поведінки одношарової ґрунтової основи застосована модель Мора-Кулона, для багатошарової − модель Друкера-Прагера. Для оцінки пружно-пластичної поведінки ґрунтової основи виконано порівняння результатів лінійного і нелінійного статичних розрахунків двох моделей від двох навантажень. Застосовано метод Н'ютона-Рафсона для дослідження статичних характеристик моделей в нелінійній постановці. Оцінено вплив врахування пружно-пластичних моделей одношарової і багатошарової ґрунтових основ на їх напружено-деформований стан. За рахунок фізичної нелінійності ґрунту в елементах одношарової моделі основи при дії ваги порожнього і завантаженого вагону спостерігалось збільшення деформації за Мізесом відповідно на 15,9% і 16,7%, при цьому пластична деформація склала 20% і 31,4% від загальної деформації, напруження за Мізесом зменшилось на 53,94% і 54,02%. Результати дослідження багатошарової основи при дії ваги порожнього і завантаженого вагону показали, що за рахунок фізичної нелінійності ґрунтів деформації елементів за Мізесом збільшились відповідно на 5,8% і 7,4 %, при цьому пластична деформація склала 54,1% і 54,33% від загальної деформації, напруження за Мізесом зменшилось на 43,45% і 43,34%. Запропонована методика дозволила сформувати розрахункову модель баластової призми і основи та дослідити нелінійне деформування ґрунтів із урахуванням їх пружно-пластичних властивостей при статичній дії рухомого складу.

Ключові слова: метод скінченних елементів, нелінійна статика, ґрунтова основа, рухомий склад.

Lukianchenko O.O., Kozak A.A., Kostin D.Ye.

FINITE ELEMENT MODELING OF THE STATIC BEHAVIOR OF AN ELASTIC-PLASTIC SOIL BASE UNDER THE ACTION OF ROLLING STOCK

A numerical method for studying the static behavior of the soil base in a geometrically and physically nonlinear formulation from the action of rolling stock is proposed. A mathematical model of the boundary value problem of the statics of the ballast prism and the soil base is constructed using the computational procedures of the finite element analysis program NASTRAN. Single-layer and multilayer base models in the form of a flat half-space are considered. The static action of the rolling stock is presented in the form of a concentrated force applied to the ballast prism from the weight of empty and loaded freight train cars. To describe the elastic-plastic behavior of a single-layer soil base, the Mohr-Coulomb model is used, and for a multilayer one, the Drucker-Prager model. To assess the elastic-plastic behavior of the soil base, a comparison of the results of linear and nonlinear static calculations of two models from two loads is performed. The Newton-Raphson method is used to study the static characteristics of the models in a nonlinear formulation. The influence of taking into account the elastic-plastic models of single-layer and multi-layer soil bases on their stress-strain state was assessed. Due to the physical nonlinearity of the soil in the elements of the single-layer base model under the action of the weight of an empty and loaded car, an increase in the Mises deformation was observed by 15.9% and 16.7%, respectively, while the plastic deformation was 20% and 31.4% of the total deformation, the Mises stress decreased by 53.94% and 54.02%. The results of the study of the multilayer base under the action of the weight of an empty and loaded car showed that due to the physical nonlinearity of the soils, the Mises deformation of the elements increased by 5.8% and 7.4%, respectively, while the plastic deformation was 54.1% and 54.33% of the total deformation, the Mises stress decreased by 43.45% and 43.34%. The proposed methodology allowed us to form a computational model of the ballast prism and base and to investigate the nonlinear deformation of soils taking into account their elastic-plastic properties under the static action of rolling stock.

Keywords: finite element method, nonlinear static, ground base, rolling stock.

УДК 539.3
Лук'янченко О.О., Козак А.А. Костін Д.Є. **Скінченно-елементне моделювання статичної поведінки пружно-пластичної ґрунтової основи при дії рухомого складу** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2025. – Вип. 115. – С. 107-113.

Запропонована чисельна методика дослідження статичної поведінки ґрунтової основи у геометрічно і фізичнo нелінійній постановці від дії рухомого складу. Математична модель крайової задачі статики баластової призми і грунтової основи побудована із застосуванням обчислювальних процедур програми скінченно-елементного аналізу NASTRAN. Розглянуто одношарова і багатошарова моделі основи у вигляді плоского напівпростору. Для описання пружно-пластичної поведінки одношарової грунтової основи застосована модель Мора-Кулона, для багатошарової − модель Друкера-Прагера. Оцінено вплив пружно-пластичної поведінки грунтів на їх напружено-деформований стан при дії ваги порожнього і завантаженого вагонів вантажного потягу.
Табл. 5. Іл. 5. Бібліогр. 5 назв.

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Lukianchenko O.O., Kozak A.A., Kostin D.Ye. **Finite element modeling of the static behavior of an elastic-plastic soil base under the action of rolling stock** // Strength of Materials and Structural Theory: Sci.-Tech. Coll. – Kyiv: KNUBA, 2025. – Issue 115. – P. 107-113.

A numerical methodology is proposed for investigating the static behavior of a soil foundation under geometrically and physically nonlinear conditions due to the action of rolling stock. The mathematical model of the boundary-value statics problem of the ballast prism and soil foundation was constructed using the computational procedures of the NASTRAN finite element analysis program. Single-layer and multilayer foundation models in the form of a plane half-space were considered. The Mohr-Coulomb model was applied to describe the elasto-plastic behavior of the single-layer soil foundation, while the Drucker-Prager model was used for the multilayer foundation. The influence of the elasto-plastic behavior of soils on their stress-strain state under the weight of empty and loaded freight train cars was assessed.
Table 5. Fig. 5. Ref. 5.

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