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STRENGTH OF WEBS OF I-SHAPED REINFORCED CONCRETE ELEMENTS UNDER SHEAR FORCES

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A method is presented for calculating the strength of the walls of I-shaped reinforced concrete beams during failure along a strip between inclined cracks, based on the provisions of the theory of plasticity of reinforced concrete, considering the concrete of the wall under conditions of flat compression-tension stress state when tensile stresses are transferred to the concrete from shear reinforcement. Within the framework of the developed method, the criterion for failure is taken to be the attainment of limit values by the main compressive stresses in the concrete strip of the web under the corresponding stress state. The general case of the limit equilibrium of a beam within the strip between inclined cracks with arbitrary content and location of shear reinforcement is considered. Calculation dependencies are obtained for calculating the ultimate values of stresses in the strip and shear force during wall failure.
Keywords: I-beam, crack zone, failure, limit state, plasticity theory

1. Introduction

One possible type of failure to reinforced concrete elements under shear forces is the fragmentation of concrete in an inclined strip between inclined cracks. This type of failure occurs in I-beams and I-sections, as well as in thin-webs rectangular beams. In this case, the inclined cracks between which the failure occurs are regular in nature, form in the middle of the section height and, as the load increases, develop along a straight line in the direction of the tension and compression edges. In I-beams, inclined cracks develop within the webs, usually up to the boundaries of the flanges in the tensile and compressive zones.

The concrete of the beam webs between the inclined cracks is under flat stress conditions of compression and tension, which is characterized by axial compression along the axis of the strip and tension transmitted to the concrete by the transverse reinforcement located at an angle. Within the framework of existing calculation methods, the strength of concrete in the strip between inclined cracks, which determines the strength of the element itself, is usually determined very approximately on the basis of empirical dependencies that do not take into account the actual stress-strain state of the concrete strip and, first of all, its shear reinforcement.

This work is a development of the provisions of the theory of plasticity of reinforced concrete [1, 2] as applied to the calculation of the strength of concrete in the strip between inclined cracks in I-beams based on a flat stress state of compression-tension with the transfer of tensile forces to the concrete from the reinforcement.

2. An overview of literary sources

Numerous experimental studies have been devoted to the problem of reinforced concrete resistance to shear forces, as a result of which the main patterns in the processes of crack formation and development, the stress-strain state of concrete, longitudinal and shear reinforcement, possible types of element failure, and the influence of a wide range of factors on their strength have been established [3-19].

The traditional test specimen used in testing was a freely supported beam loaded with two concentrated forces symmetrically located at different distances from the support. It has been established that the main influencing factor determining the nature of crack formation, the stress-strain state of concrete and reinforcement, the type of failure and the strength of the beam is the relative distance from the support to the concentrated force - a/d [4, 5, 9, 12, 13, 17, 18]. Thus, in the range of values a/d =2.0...4.0, a critical inclined crack forms during loading in the middle of the beam cross-section height and subsequently develops in the direction of the compressed and tensile edges along a curved trajectory close to the trajectory of the main tensile stresses. After reaching the critical inclined crack of the cross-section under concentrated force and subsequent increase in load, the beam fails as a result of concrete fragmentation above the critical inclined crack. During failure, the stresses in the transverse reinforcement reach their yield point, while the stresses in the longitudinal reinforcement usually do not reach their yield point. In the range of values a/d =2.0...4.0, failure can also occur as a result of the development of a critical inclined crack along the longitudinal reinforcement towards the support, accompanied by a loss of bond between the reinforcement and the concrete [5, 13, 17, 18].

In the range of values a/d =1.0…2.0, failure occurs along a diagonal critical inclined crack, which runs in a straight line from the support to the concentrated force [5, 9, 10, 12, 13, 18]. The stresses in the shear reinforcement reach the yield strength.

With a decrease in a/d <1.0, failure occurs as a result of concrete fragmentation in the strip between the inclined cracks that run between the support and the concentrated force [12, 17, 18]. In this case, the width of the strip (the distance between the inclined cracks) is determined by the dimensions of the support reaction and concentrated force transfer areas, and the stresses in the shear reinforcement, depending on their intensity, may or may not reach the yield strength.

In general, with a decrease in a/d from 4.0 to 1.0, the value of the ultimate load increases significantly and can be from 2.5 to 3.0 times or more [5, 9, 12, 13].

The resistance to shear forces of thin web of I-beams has its own fundamental characteristics and practically does not depend on the distance from the support to the concentrated force [15,18,19]. Inclined cracks in such beams form in the middle of the section height and then develop along the compressed and tensile edges in a straight line, reaching the flanges. As a result, a system of regular inclined cracks forms in the web of the beams, and as the load increases, failure occurs as a result of the concrete between the inclined cracks breaking up.

The experimental data obtained were used as the basis for the developed calculation models of the strength of reinforced concrete elements under the action of shear forces, oriented towards possible forms of failure - truss model [20-26], strut-and-tie model [27-28], compressive-force path model [29-30], nonlinear finite-element model [31], mechanical model [32, 33]. At the same time, while the calculation models [20-26, 29-33] primarily considered failures due to concrete fragmentation above an inclined crack and along a diagonal crack, the strut-and-tie model [27, 28] – failure in the strip between the support and the concentrated force at a/d <1.0. The results of theoretical developments, in one form or another, were subsequently used in the relevant documents on structural design [34-40].

The failure of concrete webs of I-beams between inclined cracks was described by computational models [40-42]. However, neither these nor other calculation methods, including indirect assessment based on a truss model [34], were able to provide the necessary calculation accuracy for this type of failure [43], leading in some cases to underestimation of strength and in others to overuse of transverse reinforcement.

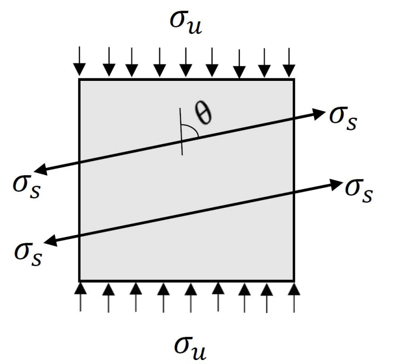

Fig. 1. Diagram of the flat stress state of reinforced concrete under compression and tension when tensile stresses are transferred to the concrete from the reinforcement

The purpose of the work – to develop a method for calculating the strength of I-beam walls under transverse forces during failure as a result of concrete fragmentation in the strip between inclined cracks based on the provisions of the theory of reinforced concrete plasticity [1, 2].

3. Presentation of the main material

In [2], based on the theory of reinforced concrete plasticity [1], various cases of flat stress state of reinforced concrete are considered, including the case that occurs in the webs of I-beams under the action of shear forces, namely, the stress state of compression-tension with the transfer of tensile stresses to concrete from reinforcement located at an angle to the direction of compressive stresses (Fig. 1).

The value of the ultimate compressive stress σ_u in the case under consideration is calculated using the formula:

$$\sigma_u = \rho \cdot \sigma_s (\cos\theta - \sin\theta)\cos^2\theta + \frac{f_c - f_{ct} + \rho \cdot \sigma_s \cdot \sin^2\theta \cdot (\sin\theta - \sigma_s / f_y)}{2} +$$

$$+\sqrt{\left(\frac{f_c + f_{ct}}{2}\right)^2 + 0{,}75 \cdot \rho \cdot f_y \cdot \sin^2\theta \cdot (\sin\theta - \sigma_s / f_y) \cdot \left[2\left(f_c - f_{ct}\right) - \rho \cdot f_y \cdot \sin^2\theta \cdot (\sin\theta - \sigma_s / f_y)\right]}. \quad (1)$$

As an example, Fig. 2 shows graphs of changes in ultimate compressive stresses in concrete (f_c=20MPa), constructed according to (1) depending on the content (ρ =0,002), angle of inclination and stresses (f_y=400MPa) in reinforcement

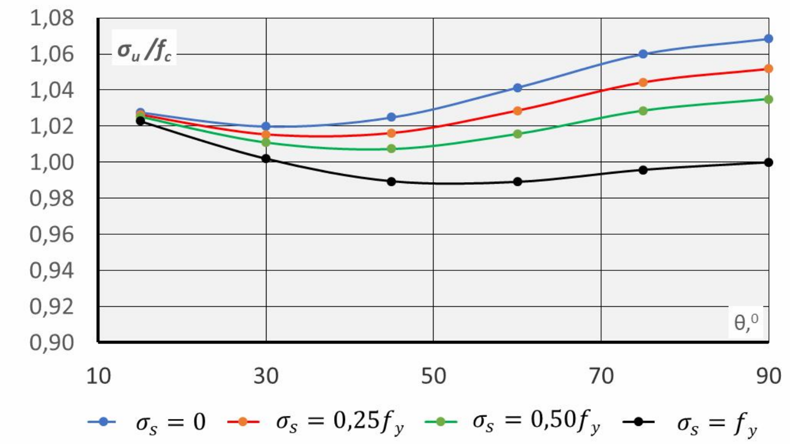

Fig. 2. Dependence of ultimate compressive stresses on content, angle of inclination and stresses in reinforcement

The design model of an I-beam element failing as a result of concrete fragmentation between inclined cracks is shown in Fig. 3.

Shear forces act on the strip of web between the inclined cracks, caused by tangential stresses τ_c acting at the contact between the web and the flange in the compressed and tensile zones, and tensile forces transmitted to the strip from the shear reinforcement (Fig. 4).

Within the accepted model, the criterion for element failure is the attainment of the ultimate values of the main compressive stresses in the concrete wall $\sigma_{cm,u}$ according to (1), which in the case under consideration takes the form:

$$\sigma_{cm,u} = \rho \cdot \sigma_{sw} (\cos\theta - \sin\theta)\cos^2\theta + \frac{f_c - f_{ct} + \rho \cdot \sigma_{cm} \cdot \sin^2\theta \cdot (\sin\theta - \sigma_{sw} / f_y)}{2} +$$

$$+\sqrt{\left(\frac{f_c + f_{ct}}{2}\right)^2 + 0{,}75 \cdot \rho \cdot f_y \cdot \sin^2\theta \cdot (\sin\theta - \sigma_{sw} / f_y) \cdot \left[2\left(f_c - f_{ct}\right) - \rho \cdot f_y \cdot \sin^2\theta \cdot (\sin\theta - \sigma_{sw} / f_y)\right]}, \quad (2)$$

where θ – acute angle between the direction of action of the main compressive stresses θ_m and the transverse reinforcement θ_s (Fig. 3 (a)):

at $\theta_m \geq \theta_s$

$$\theta = \theta_m - \theta_s, \quad (3)$$

at $\theta_m < \theta_s$

$$\theta = \pi / 2 - \theta_m + \theta_s. \quad (4)$$

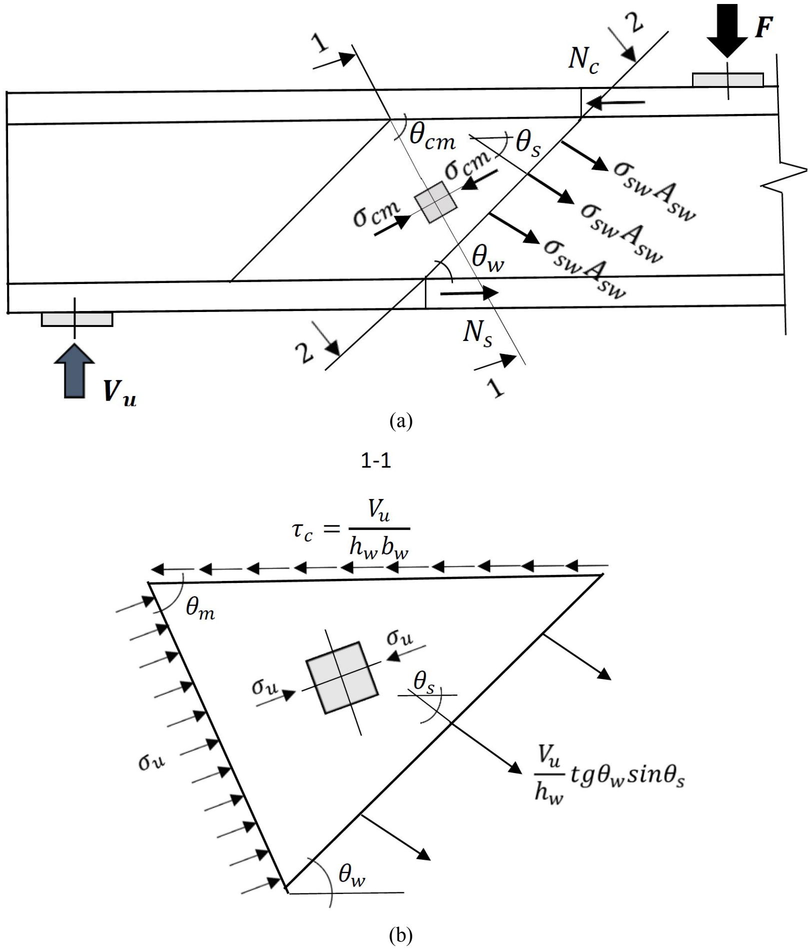

Fig. 3. Design model of an I-beam element failure due to concrete fragmentation in the zone between inclined cracks: (a) - distribution of external and internal forces; (b) - distribution of forces in the strip

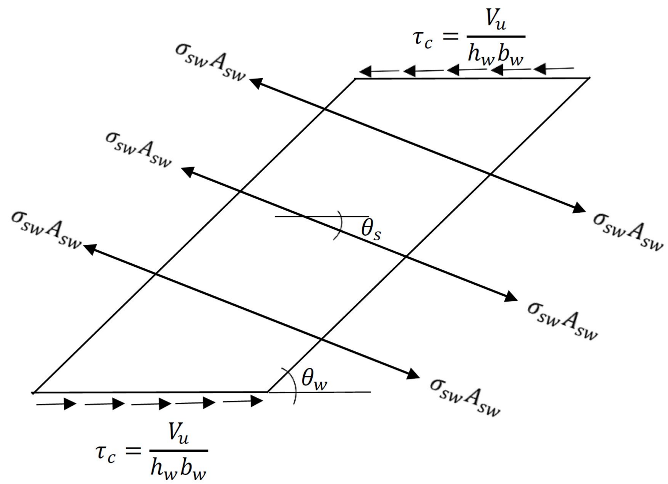

Fig. 4. Forces in the strip of web between inclined cracks

The ultimate shear force experienced by the element is determined from the equilibrium of the strip, selected by section 1-1 (Fig. 3 (b)) by:

$$V_u = \sigma_{cm,u} \cdot b_w \cdot h_w \frac{\sin\theta_w}{\cos(\theta_m - \theta_w)[\sin\theta_m - \cos\theta_m \cdot tg(\theta_m - \theta_s)}. \quad (5)$$

Angle of inclination of the main compressive stress action areas in the concrete strip θ_m is found from the equilibrium condition, assuming that the tangential stresses in section 1-1 (Fig. 3 (b)) are zero according to:

$$\frac{\cos\theta_m}{\cos(\theta_m - \theta_s)} = \sin\theta_w \cdot \sin\theta_s \cdot [\cos_w + \sin_w \cdot tg(\theta_m - \theta_w). \quad (6)$$

Solution of equation (3) with respect to θ_m depending on the angle of the crack θ_W and the angle of inclination of the shear reinforcement θ_s are listed in Table 1.

Table 1

Values θ_m of the angle of inclination of the main compressive stresses in an inclined strip

θ_W ,°	θ_m ,°			
	θ_s =30°	θ_s =45°	θ_s =60°	θ_s =90°
22	79,5	71,0	65,5	64,5
24	78,0	69,6	63,8	63,0
26	77,5	68,5	62,4	61,9
28	77,0	67,2	61,1	60,9
30	76,5	66,0	60,0	59,9
32	75,6	65,1	59,1	59,2
34	75,0	64,3	58,4	58,6
36	74,5	63,7	57,8	58,1
38	74,0	63,4	57,5	58,1
40	73,9	63,4	57,5	58,0
42	73,8	63,3	57,5	58,0
45	73,7	63,3	57,5	58,0

In general, the sequence of calculations for determining the ultimate shear force at failure of the web of I-beam elements as a result of concrete fragmentation in the strip between inclined cracks under a given loading scheme, element geometry, transverse reinforcement, and strength characteristics of concrete and reinforcement can be represented as follows:

- the calculation is performed by sequentially considering a series of crack inclination angles in the range $1,0 \le \cot\theta_w \le 2,5$;

- for each inclination angle, according to Table 1, the areas of main compressive stresses θ_m are taken;

- according to (2), the value of the main compressive stresses $\sigma_{cm,u}$ is calculated within the framework of a conservative approach, assuming that the stresses in the shear reinforcement are equal to the yield strength (f_y); otherwise, the stresses in the shear reinforcement can be determined from the element equilibrium equation in section 2-2 (Fig. 3 (a)), assuming;

- according to (3), the ultimate shear force perceived by the element at failure V_u is calculated.

Fig. 6 shows an example of the application of the developed calculation method for an I-beam with concrete strength f_c=20 MPa, with transverse reinforcement ρ =0,001 with strength f_y=400 MPa in the vertical position of the ties, and graphs comparing the ultimate shear force as a function of the angle of inclination of the strip between the inclined cracks are constructed for calculations using the developed method and EN 1992-1-1:2023 [34]. The analysis of Fig. 6 shows that the developed method generally correctly estimates the trends in the change in the ultimate shear force from the angle of inclination of the strip, while the calculated values exceed the corresponding values according to EN 1992-1-1:2023 [34]. The latter corresponds to the conclusions [43], according to which the experimental value of the ultimate shear force perceived by the element at an angle of inclination of the transverse reinforcement of 45^0 can exceed the calculated value according to [34] by 2...3.5 times.

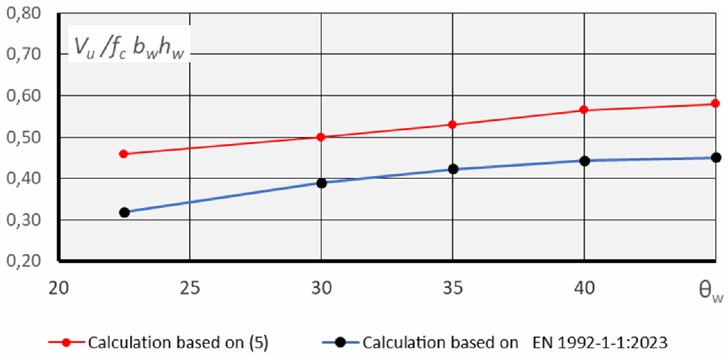

Fig. 6. Dependence of the calculated ultimate shear force on the angle of inclination of the strip

4. Conclusions

Concrete crushing in the strip between inclined cracks is one of the possible forms of failure of thin-webs reinforced concrete I-beams. Existing calculation methods, including indirect assessment based on a truss model [34], have been unable to provide the necessary calculation accuracy for this type of failure [43], leading in some cases to underestimation of strength and in others to overuse of shear reinforcement.

This paper presents a developed method for design strength of webs of reinforced concrete I-beams in case of failure along a strip between inclined cracks, based on the provisions of the theory of plasticity of reinforced concrete [1, 2], considering the concrete of the web under conditions of plane stress compression-tension when tensile stresses are transferred to the concrete from shear reinforcement. Within the framework of the developed method, the criterion for failure is taken to be the attainment of the main compressive stresses in the concrete strip of the web to their ultimate values under the corresponding stress state.

The general case of the ultimate equilibrium of a beam within the strip between inclined cracks with arbitrary content and location of shear reinforcement is considered. Calculated dependencies for determining the ultimate stress values in the strip and the shear force at web failure are obtained.

A comparison was made between the calculations using the developed method and EN 1992-1-1:2023 [34], establishing the correctness of the developed method in reflecting the main trends in the change in the ultimate shear force perceived by the element, which allows the developed method to be further extended to the calculation of thin-walled T-shaped and rectangular beams.

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МІЦНІСТЬ СТІНОК ДВОТАВРОВИХ ЗАЛІЗОБЕТОННИХ ЕЛЕМЕНТІВ ПРИ ДІЇ ПОПЕРЕЧНИХ СИЛ

Всі існуючі методи розрахунку міцності залізобетонних елементів засновані на встановлених експериментально можливих видах руйнування елементів при певних зусиллях, викликаних зовнішніми силовими впливами. Так, при дії згинальних моментів або спільній дії моментів і поздовжніх сил руйнування відбувається в результаті роздроблення бетону стиснутої зони при досягненні або не досягненні напруженнями в поздовжній арматурі граничних значень при розтягуванні (межа плинності) і стисненні, при дії крутних моментів – по критичній спіральній тріщині, при прикладенні стискаючої сили через майданчик малих розмірів – від роздроблення (зминання) бетону під майданчиком. Опір залізобетонних елементів дії поперечних сил істотно відрізняється від інших силових впливів, оскільки характеризується різними видами руйнувань – роздробленням бетону над критичною похилою тріщиною, зрізом бетону над критичною похилою тріщиною, роздробленням бетону в похилій смузі між похилими тріщинами або опорою і вантажем, втратою зчеплення поздовжньої арматури з бетоном в розтягнутій зоні за критичною похилою тріщиною, руйнуванні по похилих тріщинах при продавлюванні що пояснюється впливом схеми навантаження, геометрією та конструктивними особливостями елемента. Відповідно, для оцінки міцності при можливих видах руйнування застосовується відповідний розрахунковий апарат, який не завжди реалізується в рамках єдиної розрахункової моделі елементів. Тому на практиці застосовуються різні методи розрахунку стосовно кожного з можливих видів руйнування.

У даній роботі наведено розроблений метод розрахунку міцності стінок двотаврових залізобетонних балок при руйнуванні по смузі між похилими тріщинами, заснований на положеннях теорії пластичності залізобетону, розглядаючи бетон стінки в умовах плоского напруженого стану стиснення-розтягнення при передачі розтягуючих напружень на бетон з поперечної арматури. В рамках розробленого методу, в якості критерію руйнування прийнято досягнення головними стискаючими напруженнями в смузі бетону стінки граничних значень при відповідному напруженому стані. Розглянуто загальний випадок граничної рівноваги балки в межах смуги між похилими тріщинами при довільному вмісті і розташуванні поперечної арматури. Отримано розрахункові залежності для обчислення граничних значень напружень в смузі і поперечної сили при руйнуванні стінки.

Ключові слова: двотаврова балка, смуга між тріщинами, руйнування, граничний стан, теорія пластичності.

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STRENGTH OF WEBS OF I-SHAPED REINFORCED CONCRETE ELEMENTS UNDER SHEAR FORCES

All existing methods for calculating the strength of reinforced concrete elements are based on experimentally established possible types of element failure under certain forces caused by external forces. Thus, under the action of bending moments or the combined action of moments and longitudinal forces, failure occurs as a result of the crushing of the concrete in the compressed zone when the stresses in the longitudinal reinforcement reach or do not reach the limit values for tension (yield strength) and compression. under the action of torsional moments – along a critical spiral crack, when a compressive force is applied over a small area – from the crushing of concrete under the area. The resistance of reinforced concrete elements to shear forces differs significantly from other forces of action, as it is characterized by different types of failure – concrete fragmentation above a critical inclined crack, concrete shear above a critical inclined crack, crushing of concrete in an inclined strip between inclined cracks or a support and a load, loss of adhesion of longitudinal reinforcement to concrete in the tension zone beyond a critical inclined crack, and failure along inclined cracks during punching, which is explained by the influence of the loading pattern, the geometry and design features of the element. Accordingly, to assess strength in the event of possible types of failure, the appropriate calculation tool is used, which is not always implemented within a single calculation model for elements. Therefore, in practice, different calculation methods are used for each of the possible types of failure.

This paper presents a method for calculating the strength of I-beam reinforced concrete webs when they fail along a strip between inclined cracks, based on the principles of reinforced concrete plasticity theory, considering the concrete of the web under conditions of plane stress compression-tension when tensile stresses are transferred to the concrete from shear reinforcement. Within the framework of the developed method, the criterion for failure is taken to be the attainment of the main compressive stresses in the concrete strip of the web at the corresponding stress state. The general case of the ultimate equilibrium of a beam within the strip between inclined cracks with arbitrary content and location of transverse reinforcement is considered. Calculated dependencies are obtained for calculating the limit values of stresses in the strip and shear force at wall failure.

Keywords: I-beam, crack zone, failure, limit state, plasticity theory.

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Наведено розроблений метод розрахунку міцності стінок двотаврових залізобетонних балок при руйнуванні по смузі між похилими тріщинами, заснований на положеннях теорії пластичності залізобетону, розглядаючи бетон стінки в умовах плоского напруженого стану стиснення-розтягнення при передачі розтягуючих напружень на бетон з поперечної арматури. В рамках розробленого методу, в якості критерію руйнування прийнято досягнення головними стискаючими напруженнями в смузі бетону стінки граничних значень при відповідному напруженому стані. Розглянуто загальний випадок граничної рівноваги балки в межах смуги між похилими тріщинами при довільному вмісті і розташуванні поперечної арматури. Отримано розрахункові залежності для обчислення граничних значень напружень в смузі і поперечної сили при руйнуванні стінки.

Іл. 6. Бібліогр. 43 назв.

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A method is presented for calculating the strength of the walls of I-shaped reinforced concrete beams during failure along a strip between inclined cracks, based on the provisions of the theory of plasticity of reinforced concrete, considering the concrete of the wall under conditions of flat compression-tension stress state when tensile stresses are transferred to the concrete from shear reinforcement. Within the framework of the developed method, the criterion for failure is taken to be the attainment of limit values by the main compressive stresses in the concrete strip of the web under the corresponding stress state. The general case of the limit equilibrium of a beam within the strip between inclined cracks with arbitrary content and location of shear reinforcement is considered. Calculation dependencies are obtained for calculating the ultimate values of stresses in the strip and shear force during wall failure.
Fig. 6. Ref. 43.

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