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THEORETICAL FOUNDATIONS FOR DESIGN OF REINFORCED CONCTETE UNDER PLANE STRESS STATES

Yu.A. Klimov,
Doctor of Technical Sciences, Professor

A.A. Kozak,
Candidate of technical sciences, associate professor

Kyiv National University of Construction and Architecture, Kyiv

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Theoretical foundations for calculating the strength of reinforced concrete elements are presented based on the theory of concrete plasticity and considering reinforcement as an internal bond that limits the transverse deformations of concrete during compression, causing the emergence of reactive compressive stresses. Calculation dependencies have been obtained for determining the ultimate stresses in a reinforced concrete element under axial compression, plane stress state compression-compression, compression-tension when stresses are transferred directly to concrete or through stretched reinforcement, which take into account the strength characteristics of concrete, content, position and strength characteristics of reinforcement. The graphs are given that allow to estimate the influence of the listed factors on the value of ultimate compressive stresses in concrete. The developed theoretical approach is extended to the design of the strength of compressed elements with welded mesh reinforcement and the possibility of its use for the design of similar elements with confinement reinforcement, as well as pipe-concrete elements, is shown.
Keywords: concrete, compression, transverse reinforcement, theory of concrete plasticity, plane stress state, strength.

1. Introduction

A feature of reinforced concrete is the fact that, under certain conditions, the presence of reinforcement can cause a change in the stress state of the element, compared to the stress state caused by the action of an external load. Thus, transverse reinforcement in the form of welded meshes along the height or transverse reinforcement along the perimeter of the centrally compressed element restrains transverse deformations of concrete, causes the occurrence of reactive compressive stresses and, as a consequence, the transition of the stress state of concrete from axial compression under the action of an external load to triaxial compression and an increase in the ultimate values of concrete strength under compression. A similar phenomenon occurs in pipe-concrete structures, when transverse deformations of concrete under compression are restrained by an external pipe and the concrete operates under conditions of triaxial compression. In beam-walls and column consoles, the concrete in the strip between the support and the load is under conditions of axial compression, and the transverse reinforcement in the strip is located at an angle to the acting force and experiences tensile stresses. Thus, the concrete strip is in the conditions of a plane stress state of compression-tension. These and other cases of stress-strain state of reinforced concrete elements, within the framework of existing design methods, are taken into account very approximately and, as a rule, on the basis of empirical dependencies. This work is devoted to the development of a unified theoretical approach to design the strength of reinforced concrete elements under plane stress state based on the theory of concrete plasticity, taking into account the presence, position, intensity, stress state and strength characteristics of the reinforcement.

2. An overview of literary sources

Most of the studies conducted to date on the influence of reinforcement on the stress-strain state of concrete reinforced concrete elements consist of experimental studies of the strength of compressed elements reinforced with flat welded meshes and confinement reinforcement, as well as pipe-concrete elements.

The experimental studies included tests of axial compression of reinforced concrete elements, in which the intensity of volumetric reinforcement with welded meshes with different bar diameters was varied, the pitch of reinforcement and meshes in height [1-10], the design of confinement reinforcement in the form of hoops of different diameters, shapes, numbers of branches and pitch in height [11 - 15], spirals [16], additional pre-stressed reinforcement rings [17, 18].

As a result of the conducted studies, experimental data were obtained on the nature of the destruction of the strength of elements, changes in longitudinal and transverse deformations of concrete, stresses in transverse reinforcement during loading, including cyclic loading [9, 11].

It has been established that the presence of reinforcement in the form of meshes or confinement reinforcement, due to the limitation of transverse deformations, leads to the occurrence of a triaxial stress state in concrete and an increase in the strength of samples under axial compression by 1.10-1.45 times, which corresponds with the results of concrete tests under triaxial compression at the corresponding values of transverse compressive stresses within 0.1…0.5 of the ultimate under axial compression [19 - 24]. At the same time, with an increase in transverse reinforcement with welded meshes or confinement reinforcement, the strength and plasticity of the elements increase.

The calculated assessment of the influence of confinement reinforcement on the strength of elements under central compression is reduced to the introduction of empirical coefficients that take into account the intensity, type, configuration and design of confinement reinforcement [25 - 30].

At the same time, despite the fairly large number of reinforced concrete elements, such as beam walls in the zone of action of shear forces, column consoles, shear walls, slabs, the concrete of which is in a plane stress state, similar experimental and theoretical studies of the effect of reinforcement on the stress state of concrete have not been carried out to date. The consequence of this is the fact that the presence of reinforcement is not taken into account at all or is taken into account very approximately, which makes it relevant to conduct targeted, primarily theoretical, research in this area.

The purpose of the work – to develop theoretical foundations for design the strength of reinforced concrete elements under plane stress conditions based on the theory of concrete plasticity, taking into account the presence, position, intensity, stress state and strength characteristics of the reinforcement.

3. Presentation of the main material

Existing theories of the ultimate stressed state of reinforced concrete rely on various conditions of the strength of concrete, proceed from a joint character of deformations of concrete and reinforcement at projecting the forces in the latter on corresponding axes. They ignore the effect of the reinforcement as an internal brace, capable of changing the stressed state of reinforced concrete at a presumed loading.

The theory presented below is based on considering a reinforcing bar, randomly arranged in a concrete bulk, as an internal brace that restrains lateral deformations and changes the stressed state of concrete at presumed loading conditions. As the concrete strength criterion [31, 32] is assumed:

$$\sigma_1^2+\sigma_2^2+\sigma_3^2-(\sigma_1\cdot\sigma_2+\sigma_2\cdot\sigma_3+\sigma_3\cdot\sigma_1)-(f_c-f_{ct})(\sigma_1+\sigma_2+\sigma_3)-f_c\cdot f_{ct}=0. \qquad (1)$$

Based on (1), Fig. 1 shows graphs of changes in ultimate compressive stresses for concrete class C16/20 under two axial and triaxial compression, indicating that under two axial compression, ultimate stresses increase by 1,16...1,97 times, and under triaxial compression, by up to 3,2 times.

In the case of a uniaxial compression with a normal arrangement of the reinforcement (Fig. 2, a), tensile stresses arise in the latter, and reactive compressive stresses, counterbalancing the tensile stresses, in concrete. Thus, at the presence of reinforcement, concrete changes over from the uniaxial stressed state to a biaxial compression - compression stressed state. On the assumption that the ultimate state in the reinforcement and concrete sets up at one and the same time, the concrete strength condition (1) at the corresponding value of the second principal stress take the form of second principal stress $\sigma_2=\rho\cdot f_y$ takes the form

$$\sigma_1^2-[(f_c-f_{ct})+\rho f_y]\sigma_1+\rho^2\cdot f_y^2-(f_c-f_{ct})\cdot\rho\cdot f_y-f_c f_{ct}=0. \qquad (2)$$

Having solved (2) for σ_1 (Fig. 2,a), the relationship for calculation of ultimate stresses at an axial compression with a normal arrangement of the reinforcement:

$$\sigma_{1,u}=\frac{f_c-f_{ct}+\rho\cdot f_y}{2}+\sqrt{\left(\frac{f_c+f_{ct}}{2}\right)^2+0,75\cdot\rho\cdot f_y\left[2(f_c-f_{ct})-\rho\cdot f_y\right]}. \qquad (3)$$

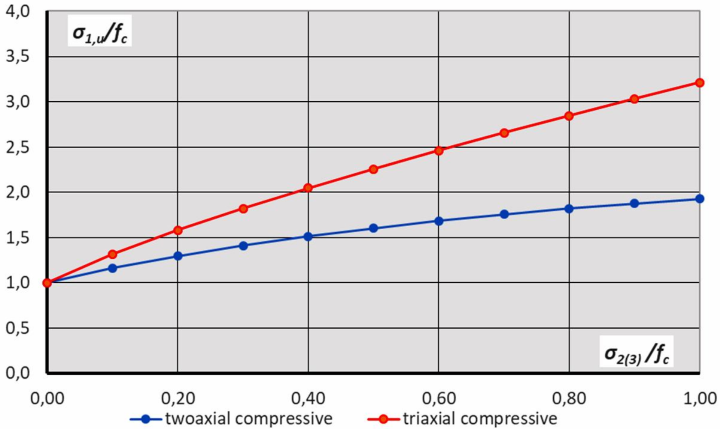
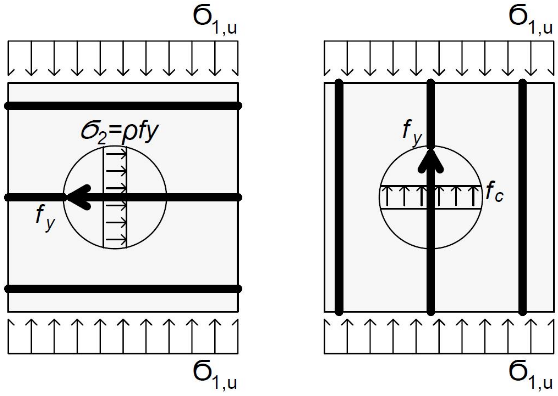


Fig. 1. Dependence of relative ultimate compressive stresses in concrete under biaxial and triaxial compression

When the reinforcement is arranged parallel to the action of the compressive force (Fig. 2,b), concrete is under conditions of a uniaxial compression, and the magnitude of ultimate stresses is found from (1) with the well-known formula, serving as a basis for analysis of centrally compressed elements:

$$\sigma_{1,u} = f_c + \rho \cdot f_{yc}. \quad (4)$$



(a) (b)

Fig. 2. Stress state of concrete under axial compression with the reinforcement normally positioned (a) and along the compression direction (b)

Figure 3 shows graphs of changes in the relative ultimate stresses of concrete under axial compression for concrete classes C16/20 and C25/30 with normal placement of reinforcement depending on the reinforcement ratio, indicating that the relative increase in strength increases with an increase in the reinforcement ratio to a greater extent for concretes of lower strength.

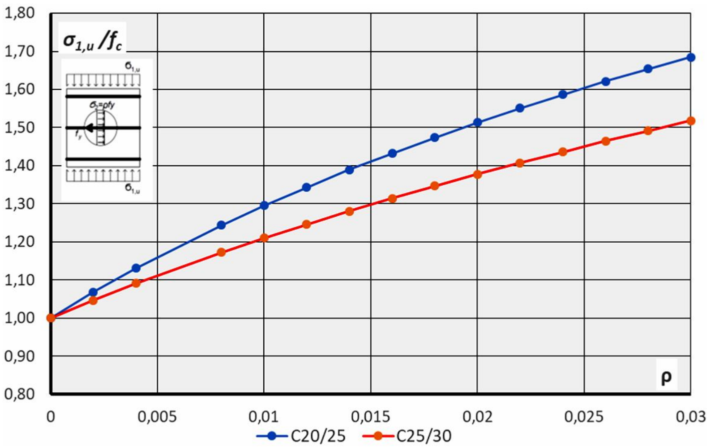


Fig. 3. Dependence of the relative ultimate compressive stresses of concrete under axial compression on the reinforcement coefficient and the class of concrete

When the reinforcement is arranged in the concrete bulk at a random angle θ (Fig. 4), with θ increasing from 0 to 90º the stresses in it vary from ultimate in compression, f_{yc} (Fig. 2,b) to ultimate

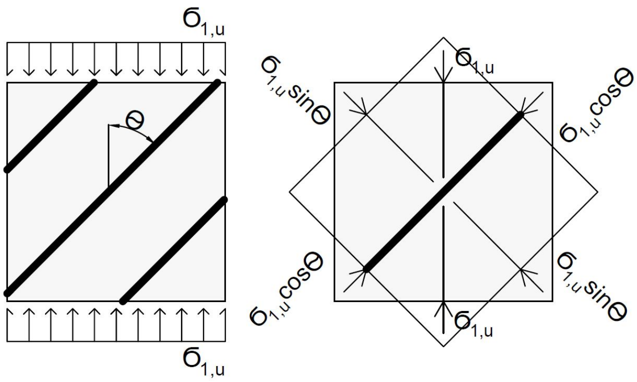


Fig. 4. To determine stresses in concrete with inclined reinforcement

in tension, f_y (Fig. 2,a). Considering stresses in the reinforcement as a resultant of compressive and tensile stresses and assuming that the latter vary in proportion to respective components of the external force, we derive the relation for calculating the stresses in the reinforcement in the ultimate state is derived:

$$\sigma_s = \sigma_{sc} - \sigma_{st} = f_{yc}\cos\theta - f_y \sin\theta. \quad (5)$$

As follows from (5), reactive compressive stresses in concrete σ_2 and ultimate stresses $\sigma_{1,u}$ (Fig. 3) are:

$$\sigma_2 = \rho \cdot \sigma_s \cdot \sin^2\theta = \rho \cdot f_y \cdot \sin^3\theta, \quad (6)$$

$$\sigma_{1,u} = \sigma_1 + \rho \cdot \sigma_s \cdot \cos^2\theta = \sigma_1 + \rho(f_{yc}\cos\theta - f_y \sin\theta)\cos^2\theta. \quad (7)$$

Having substituted the values of σ_2 and σ_1 from (6) and (7) into (1) and solved the latter for $\sigma_{1,u}$, we obtain the following expression for ultimate compressive stresses, which may be considered as the condition of strength of reinforced concrete at a uniaxial compression:

$$\sigma_{1,u} = \rho \cdot f_y \left(\frac{f_{yc}}{f_y}\cos\theta - \sin\theta \right)\cos^2\theta + \frac{f_c - f_{ct} + \rho \cdot f_y \sin^3\theta}{2} +$$
$$+ \sqrt{\left(\frac{f_c + f_{ct}}{2} \right)^2 + 0{,}75 \cdot \rho \cdot f_y \cdot \sin^3\theta \cdot \left[2(f_c - f_{ct}) - \rho \cdot f_y \sin^3\theta \right]}. \quad (8)$$

Based on (8), Fig. 5 shows graphs of changes in relative ultimate compressive stresses in concrete depending on the angle of inclination of the reinforcement for concrete classes C16/20 and C25/30 with a reinforcement ratio of 0.01, according to which the minimum stress value occurs at inclination angles of 30º...45º, and the maximum when the reinforcement is located normally or along the direction of the force.

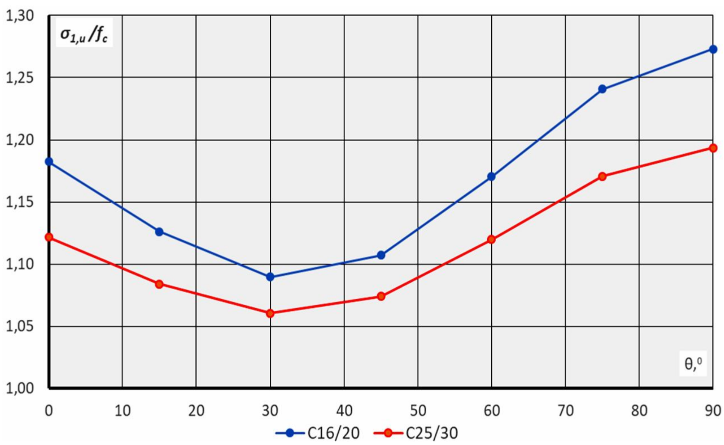


Fig. 5. Dependence of relative ultimate compressive stresses in concrete under axial compression on the angle of inclination of reinforcement and the class of concrete

Further, by analogy, the cases of compression - compression loading with transmission of respective forces to concrete as well as the case of the compression - tension loading with transmission of the tensile forces to concrete via the reinforcement have been analysed. The value of principal stress σ_2 is calculated as an algebraic sum of reactive compressive stresses and stresses transmitted directly to concrete σ_2, or via the stretched reinforcement σ_s. Corresponding relations had the following forms:

- at transmission of the force directly to concrete (the plus sign corresponds to compressive, and the minus sign, to tensile stresses):

$$\sigma_{1,u} = \rho \cdot f_y \left(\frac{f_{yc}}{f_y} \cos\theta - \sin\theta \right) \cos^2\theta + \frac{f_c - f_{ct} + \rho \cdot (f_y \cdot \sin^3\theta \pm \sigma_2)}{2} + \\ + \sqrt{\left(\frac{f_c + f_{ct}}{2} \right)^2 + 0,75 \cdot (\rho \cdot f_y \cdot \sin^3\theta \pm \sigma_2) \cdot \left[2(f_c - f_{ct}) - \rho \cdot (f_y \cdot \sin^3\theta \pm \sigma_2) \right]}; \quad (9)$$

- at transmission of the tensile force via the reinforcement:

$$\sigma_{1,u} = \rho \cdot f_y \left(\frac{f_{yc}}{f_y} \cos\theta - \sin\theta \right) \cos^2\theta + \frac{f_c - f_{ct} + \rho \cdot f_y \cdot \sin^2\theta \cdot (\sin\theta - \sigma_s / f_y)}{2} + \\ + \sqrt{\left(\frac{f_c + f_{ct}}{2} \right)^2 + 0,75 \cdot \rho \cdot f_y \cdot \sin^2\theta \cdot (\sin\theta - 2 \cdot \sigma_s / f_y) \cdot \left[2(f_c - f_{ct}) - \rho \cdot f_y \cdot \sin^2\theta \cdot (\sin\theta - 2 \cdot \sigma_s / f_y) \right]}. \quad (10)$$

Based on the developed theory, for the strength analysis of reinforced concrete structures with an oblique reinforcement, acted upon by lateral forces, have been elaborated.

In centrally compressed elements with an oblique reinforcement in the form of mesh, concrete is under conditions of triaxial compression stressed state of a no uniform cubic compression. In this case the reactive compressive stresses transmitted to concrete from the reinforcement are $\sigma_2 = \rho_2 \cdot f_y$, $\sigma_3 = \rho_3 \cdot f_y$ (ρ_2, ρ_3 are reinforcement ratios in respective directions). Having substituted the values of σ_2 and σ_3 into (1) and solved the latter for $\sigma_{1,u}$, the expression for calculating ultimate compressive stresses in elements with an oblique reinforcement:

$$\sigma_{1,u} = \frac{f_c - f_{ct} + f_y(\rho_2 + \rho_3)}{2} + \sqrt{\left(\frac{f_c + f_{ct}}{2} \right)^2 + 0,75 \cdot f_y \cdot (\rho_2 + \rho_3) \cdot \left[2(f_c - f_{ct}) - f_y \cdot (\rho_2 - \rho_3)/(\rho_2 + \rho_3) \right]}. \quad (11)$$

The developed theoretical approach based on the theory of plasticity of concrete [31, 32] can also be extended to the calculation of compressed elements reinforced with welded meshes, confinement reinforcement in the form of stirrup or spirals, as well as pipe-concrete elements [33] in which the concrete is in triaxial compression due to the limitation of transverse deformations by the reinforcement or pipe and the occurrence of reactive compressive stresses in the concrete. In this case, the value of the reactive compressive stresses is based on ultimate limit state in the mesh reinforcement, the confinement reinforcement or the pipe and on the co-existence of their transverse deformations of the reinforcement and concrete elements.

4. Conclusions

Theoretical principles for calculating the strength of reinforced concrete elements under plane stress conditions have been developed based on the theory of concrete plasticity (1) and consideration of reinforcement as an internal connection that limits transverse deformations of concrete under compression, causing the occurrence of reactive compressive stresses.

The corresponding calculation dependencies were obtained for determining the ultimate stresses in a reinforced concrete element with different reinforcement arrangements under axial compression (3), (4), (8), plane compression-compression stress state, compression-tension when stresses are transferred directly to concrete (9) or through stretched reinforcement (10), which take into account the strength characteristics of concrete, the content, position and strength characteristics of reinforcement.

As examples, graphs are constructed: changes in the relative ultimate compressive stresses of concrete under axial compression for concrete classes C16/20 and C25/30 with normal placement of reinforcement depending on the reinforcement coefficient, indicating that the relative increase in strength increases with the increase in the reinforcement ratio to a greater extent for concretes of lower strength (Fig. 3); changes in relative ultimate compressive stresses in concrete depending on the angle of inclination of the reinforcement for concrete classes C16/20 and C25/30 with a reinforcement ratio of 0.01, according to which the minimum stress value occurs at inclination angles of 30º...45º, and the maximum when the reinforcement is located normally or along the direction of the force (Fig. 5).

The developed theoretical approach is extended to the calculation of the strength of compressed elements with welded mesh reinforcement (11), and the possibility of its use for the calculation of similar elements with protective reinforcement, as well as pipe-concrete elements, is shown.

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Климов Ю.А., Козак А.А.

ТЕОРЕТИЧНІ ОСНОВИ РОЗРАХУНКУ МІЦНОСТІ ЗАЛАЗОБЕТОННИХ ЕЛЕМЕНТІВ ПРИ ПЛОСКОМУ НАПРУЖЕНОМУ СТАНІ

Плоский напружений стан має місце у стінках балок у зоні дії поперечних сил, консолях колон, діафрагмах жорсткості, плитах, оболонках та багатьох інших залізобетонних конструкціях. При розрахунку міцності таких конструкцій використовуються різні, що іноді відрізняються один від одного, теоретичні підходи і методи розрахунку, засновані, в цілому ряді випадків на емпіричних залежностях. У той же час, існують реальні передумови побудови методів розрахунку міцності залізобетонних елементів при плоскому напруженому стані на основі єдиного підходу виходячи з теорії пластичності бетону з врахуванням наявності, положення, інтенсивності, напруженого стану та характеристик міцності арматури. У цій роботі наведено теоретичні основи розрахунку міцності залізобетонних елементів на основі теорії пластичності бетону та розгляду арматури як внутрішнього в'язі, яка обмежує поперечні деформації бетону при стисканні, викликаючи виникнення реактивних стискаючих напружень. Отримані розрахункові залежності для визначення граничних напружень в залізобетонному елементі при осьовому стиску, плоскому напруженому стані стиснення-стиснення, стиснення розтягування при передачі напружень безпосередньо на бетон або через розтягнуту арматуру, які враховують характеристики міцності бетону, вміст, положення і характеристики міцності арматури. Наведено графіки, які дозволяють оцінити вплив наведених факторів на величину граничних стискаючих напружень у бетоні. Розроблений теоретичний підхід поширений на розрахунок міцності стиснутих елементів з армуванням зварними сітками і показано можливість його використання для розрахунку аналогічних елементів з поперечною арматурою, що стримує поперечні деформації бетону, а також трубобетонних елементів.

Ключові слова: бетон, стиск, поперечна арматура, теорія пластичності, плоский напружений стан, міцність.

Klimov Yu.A., Kozak A.A.

THEORETICAL FOUNDATIONS FOR DESIGN OF REINFORCED CONCRETE UNDER PLANE STRESS STATES

The plane stress state occurs in beam walls in the zone of action of shear forces, column consoles, shear walls, slabs, shells and many other reinforced concrete structures. When calculating the strength of such structures, various, sometimes different from each other, theoretical approaches and design methods are used, based, in a number of cases, on empirical dependencies. At the same time, there are real prerequisites for the creation of methods for design the strength of reinforced concrete elements under a plane stress state based on a unified approach based on the theory of concrete plasticity and taking into account the presence, position, intensity, stress state and strength characteristics of the reinforcement.

This paper presents the theoretical foundations for design the strength of reinforced concrete elements based on the theory of concrete plasticity and consideration of reinforcement as an internal connection that ultimate transverse deformations of concrete under compression, causing the occurrence of reactive compressive stresses. Calculation dependencies have been obtained for determining the ultimate stresses in a reinforced concrete element under axial compression, plane stress state compression-compression, compression-tension when stresses are transferred directly to concrete or through stretched reinforcement, which take into account the strength characteristics of concrete, content, position and strength characteristics of reinforcement. The graphs are given that allow to estimate the influence of the listed factors on the value of ultimate compressive stresses in concrete. The developed theoretical approach is extended to the design of the strength of compressed elements with welded mesh reinforcement and the possibility of its use for the design of similar elements with confinement reinforcement, as well as pipe-concrete elements, is shown.

Keywords: concrete, compression, transverse reinforcement, theory of concrete plasticity, plane stress state, strength.

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Наведено теоретичні основи розрахунку міцності залізобетонних елементів на основі теорії пластичності бетону та розгляду арматури як внутрішнього в'язі, яка обмежує поперечні деформації бетону при стисканні, викликаючи виникнення реактивних стискаючих напружень. Отримані розрахункові залежності для визначення граничних напружень в залізобетонному елементі при осьовому стиску, плоскому напруженому стані стиснення-стиснення, стиснення розтягування при передачі напружень безпосередньо на бетон або через розтягнуту арматуру, які враховують характеристики міцності бетону, вміст, положення і характеристики міцності арматури. Наведено графіки, які дозволяють оцінити вплив наведених факторів на величину граничних стискаючих напружень у бетоні. Розроблений теоретичний підхід поширений на розрахунок міцності стиснутих елементів з армуванням зварними сітками і показано можливість його використання для розрахунку аналогічних елементів з поперечною арматурою, що стримує поперечні деформації бетону, а також трубобетонних елементів.
Іл. 5. Бібліогр. 33 назв.

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Klimov Yu.A., Kozak A.A. **Theoretical foundations for design of reinforced concrete under plane stress states** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – K.: KNUBA. 2025. – Issue 114. – P. 311-318.
Theoretical foundations for calculating the strength of reinforced concrete elements are presented based on the theory of concrete plasticity and considering reinforcement as an internal bond that limits the transverse deformations of concrete during compression, causing the emergence of reactive compressive stresses. Calculation dependencies have been obtained for determining the ultimate stresses in a reinforced concrete element under axial compression, plane stress state compression-compression, compression-tension when stresses are transferred directly to concrete or through stretched reinforcement, which take into account the strength characteristics of concrete, content, position and strength characteristics of reinforcement. The graphs are given that allow to estimate the influence of the listed factors on the value of ultimate compressive stresses in concrete. The developed theoretical approach is extended to the design of the strength of compressed elements with welded mesh reinforcement and the possibility of its use for the design of similar elements with confinement reinforcement, as well as pipe-concrete elements, is shown.
Fig. 5. Ref. 33.

Автор (науковий ступінь, вчене звання, посада): доктор технічних наук, професор, професор кафедри залізобетонних і кам'яних конструкцій КНУБА, КЛІМОВ Юлій Анатолійович
Адреса робоча: 03680 Україна, м. Київ, пр. Повітряних Сил 31, Київський національний університет будівництва і архітектури
Робочий тел.: +38(044) 241-54-33
E-mail: yuliiklymov@gmail.com
ORCID ID: https://orcid.org/0000-0002-4275-7058

Автор (науковий ступінь, вчене звання, посада): кандидат технічних наук, доцент, доцент кафедри будівельної механіки КНУБА, КОЗАК Андрій Анатолійович
Адреса робоча: 03680 Україна, м. Київ, пр. Повітряних Сил 31, Київський національний університет будівництва і архітектури
Робочий тел.: +38(044) 248-32-37
E-mail: kozak.aa@knuba.edu.ua
ORCID ID: https://orcid.org/0000-0002-3192-1430