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RATIONAL TOPOLOGY OF STEEL I-BEAMS WITH VARIOUS GRADIENTS OF CHANGING WALL HEIGHT AND SHELF WIDTH AT SPECIFIED SECTIONS ALONG THE LENGTH OF THE BEAM

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Abstract. The behavior of a steel beam with variable flange width and web height under a uniformly distributed load is considered. It is established that in certain steel I-beams with a simultaneous smooth decrease of flange width and web height in the direction of decreasing bending moment, the maximum stresses occur in cross-sections where the maximum bending moment does not act. Analytical dependencies for determining the optimal I-beam web height as a function of the rate-of-change parameter for a linear law of flange width and web height variation were obtained by the Lagrange multipliers method. An improved approach was developed for determining the regularities of linear variation of flange width and web height during the search for a new topology of rational steel I-beam structures. Examples are provided.

Keywords: steel beam structures, modeling, steel I-beams of variable cross-section, optimal topology, objective function, Kuhn–Tucker conditions, Lagrange multipliers method, steel beams with different web height and flange width variation rates on separate segments

Introduction. The problem of finding a rational topology of steel beams with constant and variable cross-sections has always been a relevant and interesting scientific challenge [1, 4, 5, 6, 8, 9, 10, 11, 12, 15, 16]. Such studies are important today from the standpoint of the historical development of the theory and the advancement of optimal design. There is also an opportunity to unlock the potential of well-known general theoretical approaches for solving new types of optimal design problems for I-section steel beams [4, 5, 6, 7, 8]. Known scientific results exist for determining the optimal height of steel I-sections with constant cross-section [1, 7, 8,10, 20], for beams with corrugated webs, as well as for steel beams with variable web height [5, 6, 12, 16, 20, 21, 23, 25, 27]. A general approach of finding the optimal height via an extremum of the steel consumption objective function has been used to determine the optimal height of steel I-beams of constant and variable cross-section, taking into account the development of limited plastic deformations [7]. The generalized problem of finding optimal parameters of constant cross-section beams using the Lagrange multipliers method is presented in the fundamental work [8].

Consideration of dynamic characteristics of beam systems [13, 14, 18, 19, 22] is also employed in the search for a rational topology. Improvements and modifications of first-order gradient methods are used to solve problems of multi-dimensional unconstrained optimization for various structural systems [24, 26]. The need to design steel frame and bar structures for survivability and exposure to temperature effects [18] has also introduced new challenges in researching and finding rational structural systems in steel. For some beam systems, it becomes necessary to find rational cross-sectional parameters considering the loss of stability of the flat bending shape; these problems are solved by gradient methods and their modifications [12, 16, 21, 26]. On the other hand, given the complexity of the stress–strain state of steel variable-section beams, an approach is applied in which at the first stage the rational parameters are determined using the Lagrange multipliers method, and at the second stage the design is refined with numerical studies by the finite element method [3, 8, 9, 13, 14]. Thus, the choice of approach for searching the optimal topology of steel beams is influenced by the convexity (flatness) of the feasible solution region, the discreteness of the change of the design parameters, the multimodality of the problem, the multiplicity of optimal solutions, and the complexity of writing analytical approximation equations for structural, technological, economic and operational requirements [2, 25, 26].

At the preliminary design stage for variable cross-section beams, it is necessary to conduct additional studies to find rational parameters of steel I-beams in bending, given that the sections with maximum stresses do not coincide with the sections where the maximum bending moments occur [4, 5].

Research Aim. To determine the patterns of influence of cross-section variability (web height and flange width) of steel I-beams on the optimal parameters from the standpoint of minimizing steel consumption.

Investigation. A steel I-beam with variable stiffness along its length (1, 2) is considered. Notations are adopted as follows: h_z, h_0 – the height of the steel I-beam at a current coordinate z, and the initial height of the I-beam at z=0, respectively; $b_{f,z}$, $b_{f,0}$ – the flange width of the variable cross-section at a current coordinate along the length of the structure and at the cross-section at z=0, respectively; l– the span of the beam. The slenderness of the I-beam web is denoted according to standard documents as $\lambda_\omega \approx h_0 / t_\omega$. Local stability and local strength of the flanges and web are assumed to be ensured. The stability of the flat bending shape is provided by horizontal bracings of the structure. It is assumed that the maximum bending moment $M_{x,0}$ acts at the initial cross-section, which has the maximum web height. The initial cross-sectional area of the beam at z = 0 is given by expression: $2A_f h_0 + t_w h_0 = A_b$. The flange cross-sectional area varies along the beam length according to: $2A_{f,z} = 2b_{f,z}t_f$, while the web thickness t_w is constant

$$h_z = h_0\left(1-\gamma_h \frac{z}{l}\right),\; z = l \to h_{z=l} = h_n \to \gamma_h = \left(1-\frac{h_n}{h_0}\right),\; A_{f,z} = t_f b_{f,0}\left(1 \mp \gamma_b \frac{z}{l}\right),$$

$$b_{f,z} = b_{f,0}\left(1 \mp \gamma_b \frac{z}{l_b}\right),\; \gamma_b = \left(1 \mp \frac{b_{f,n}}{b_{f,0}}\right),\; A_{w,z} = t_w b_{f,0}\left(1-\gamma_h \frac{z}{l}\right). \tag{1}$$

A feature of the formulation and solution of this problem is that the flange width change parameter accounts for both a decrease and an increase of the flange width. It is also possible to vary the ratio of flange area to web area. In equations (1), the web variability exponent (rate) coefficient: γ_h, and a flange variability coefficient γ_b are introduced for a linear law of change.

The moment of inertia and the section modulus of the I-beam cross-section have the traditional form, taking into account the variability of the web height and flange width by formulas (2). In generalized form, the variation of the flange and web can be described by any power-law dependence

$$h_z = h_0(1-\gamma_h \frac{z}{l_b}) = h_0 f_{hz},\; b_{f,0}(1-\gamma_b \frac{z}{l_b}) = b_{f,0} f_{bz},\; t_w = \frac{h_0}{\lambda_{w0}},$$

$$W_{x,z} = \frac{2I_{x,z}}{h_z} \to W_{x,z} = \frac{\left(h_0 f_{hz}\right)^3}{\lambda_w f_{hz}}\left(\frac{t_f b_{f,0}\lambda_w}{h_0^2}\frac{f_{bz}}{f_{hz}} + \frac{1}{6}\right). \tag{2}$$

The problem of finding the optimal geometric dimensions of variable cross-section steel I-beams is a nonlinear mathematical programming problem (3), [1]

$$\Delta m_{b,z} = 2\rho \Delta l b_{f,0} t_f f_{bz} + \rho \Delta l h_{w,0} t_w f_{hz} \to \min. \tag{3}$$

In the strength constraints (4), a coefficient k_{MW}=1/k_{W0} is also used – it defines the need to increase the maximum section modulus of the I-beam at the section where the maximum bending moment occurs, due to the occurrence of maximum stresses in another section

$$u_1\left(h_z, b_{f,z}\right) = \frac{k_{MW} M_{x,z} h_z}{2I_{x,z}} - R_y\gamma_c \ge 0,\quad u_2\left(h_z, b_{f,z}\right) = \frac{Q_z}{t_w h_z} - R_s\gamma_c \ge 0, \tag{4}$$

$$\frac{k_{MW} M_{x,z} h_z}{2\left(2\frac{A_{f,z}h_z^2}{4} + \frac{h_z^3}{12}\frac{h_0}{\lambda_{w0}}\right)} - R_y\gamma_c,\; A_{f,z} \ge 0,\; I_{x,z} - I_{x,n} \ge 0. \tag{5}$$

Equations (4) and (5) are the conditions of complementary flexibility. The optimization problem (3) is written in the space of variables h_z, $b_{f,z}$ (1, 2, 3) and with constraint functions in the form of inequalities (4, 5, 6). The first inequality is the strength constraint in bending for each cross-section along the beam; the second inequality is the shear strength constraint for the I-beam web at each section along the beam. It is assumed that the flange cross-sectional area values are positive. It is also assumed that the objective function (3) is continuous, twice differentiable, and convex at the extremum point. The discreteness of standard profile sizes is not considered; a continuous variation of cross-sections is assumed. A condition of linear change in web height and flange width of the steel beam is adopted. This can be achieved in fabrication by cutting steel plate elements on CNC machines. The minimization problem (3) is solved under the assumption that the web strength constraints are inactive.

Problem (3) is solved for a constant web thickness in a symmetric I-beam cross-section using the Lagrange multipliers method [4]. It is assumed that the web thickness is unchanged along the beam's length (see condition (6))

$$t_w = t_{w,z} = \frac{h_0}{\lambda_{w0}}\left(\frac{h_0}{h_z}\right)^{m=0} \to \frac{h_0}{\lambda_{w0}} = const. \quad (6)$$

The Lagrange function has an analytical form (7) taking into account condition (4)

$$F(\lambda_m, A_{f,z}, h_z) = \rho\Delta l\left(2A_{f,z} + t_w h_z\right) + \lambda_{m0} u_1\left(h_z, t_w, A_{f,z}\right),$$

$$F(\lambda_m, A_{f,z}, h_z) = \rho\Delta l\left(2t_f b_{f,0} f_{bz} + \frac{h_0^2 f_{hz}}{\lambda_w}\right) + \lambda_{m0}\left[\frac{k_{MW} M_{x,z}}{\left(t_f b_{f,0} f_{bz}\left(h_0 f_{hz}\right) + \frac{\left(h_0 f_{hz}\right)^3}{6\lambda_w f_{hz}}\right)} - R_y \gamma_c\right]. \quad (7)$$

By solving the optimization problem, the extremum points must satisfy the Kuhn–Tucker conditions [4, 12]. Differentiating the system of equations (7) with consideration of (5) leads to a system of three algebraic equations (8) with three unknowns: h_z, A_{fz}, λ_{m0}

$$\begin{cases} \dfrac{\partial F(\lambda_{m1}, A_{f,zk}, h_{zk})}{\partial\left(b_{f,0} f_{bz}\right)} = 0 \\ \dfrac{\partial F(\lambda_{m1}, A_{f,zk}, h_z)}{\partial\left(h_0 f_{hz}\right)} = 0 \\ \dfrac{\partial F(\lambda_{m1}, A_{f,zk}, h_{zk})}{\partial \lambda_{m0}} = 0. \end{cases} \quad (8)$$

Further differentiation of conditions (8) yields a system of three nonlinear algebraic equations with three unknowns: h_z, A_{fz}, λ_{m0}. The first equation provides a formula for determining the unknown coefficient λ_{m0} as a function of the changes in the flange and web dimensions

$$2 - \lambda_{m0}\frac{k_{MW} M_{x,z}}{\left(h_0 f_{hz}\right)\left(t_f b_{f,0} f_{bz} + \frac{\left(h_0 f_{hz}\right)^2}{6\lambda_w f_{hz}}\right)^2} = 0 \to \lambda_{m0} = \frac{2\left(t_f b_{f,0} f_{bz} + \frac{\left(h_0 f_{hz}\right)^2}{6\lambda_w f_{hz}}\right)}{R_y \gamma_c}. \quad (9)$$

The second equation, after differentiation, also takes a simplified analytical form

$$2\left(h_0 f_{hz}\right)^3\left(t_f b_{f,0} f_{bz} + \frac{\left(h_0 f_{hz}\right)^2}{6\lambda_w f_{hz}}\right)^2 - \lambda_{m0} k_{MW} \lambda_w f_{hz} M_{x,z}\left(t_f b_{f,0} f_{bz} + \frac{1}{2}\frac{\left(h_0 f_{hz}\right)^2}{\lambda_w f_{hz}}\right) = 0. \quad (10)$$

Substitution of the expression in equation (10) leads to a relationship for the rational web height of the beam, which can be used in approximate calculations

$$\left(h_0 f_{hz}\right)^3 = \frac{\lambda_w f_{hz} k_{MW} M_{x,z}}{R_y \gamma_c} \frac{\left(t_f b_{f,0} f_{bz} + \frac{1}{2}\frac{\left(h_0 f_{hz}\right)^2}{\lambda_w f_{hz}}\right)}{\left(t_f b_{f,0} f_{bz} + \frac{\left(h_0 f_{hz}\right)^2}{6\lambda_w f_{hz}}\right)} = \frac{\lambda_w f_{hz} k_{MW} M_{x,z}}{R_y \gamma_c} \frac{\left(\frac{t_f b_{f,0} f_{bz} \lambda_w f_{hz}}{\left(h_0 f_{hz}\right)^2} + \frac{1}{2}\right)}{\left(\frac{t_f b_{f,0} f_{bz} \lambda_w f_{hz}}{\left(h_0 f_{hz}\right)^2} + \frac{1}{6}\right)}, \quad (11)$$

$$\left(h_0 f_{hz}\right) = \sqrt[3]{\frac{\lambda_w f_{hz} k_{MW} M_{x,z}}{R_y \gamma_c} \frac{\left(\frac{t_f b_{f,0} f_{bz} \lambda_w f_{hz}}{\left(h_0 f_{hz}\right)^2} + \frac{1}{2}\right)}{\left(\frac{t_f b_{f,0} f_{bz} \lambda_w f_{hz}}{\left(h_0 f_{hz}\right)^2} + \frac{1}{6}\right)}}. \quad (12)$$

Repeated substitution based on the strength conditions (4) in equation (11) provides sufficient conditions for a rational cross-section of a steel I-beam

$$\frac{k_{MW} M_{x,z}}{\left(h_0 f_{hz}\right)\left(t_f b_{f,0} f_{bz} + \frac{\left(h_0 f_{hz}\right)^2}{6\lambda_w f_{hz}}\right)} = R_y \gamma_c \rightarrow \frac{\left(h_0 f_{hz}\right)^2}{\lambda_w f_{hz}} = \left(t_f b_{f,0} f_{bz} + \frac{1}{2}\frac{\left(h_0 f_{hz}\right)^2}{\lambda_w f_{hz}}\right) \rightarrow \frac{\left(h_0 f_{hz}\right)^2}{\lambda_w f_{hz}} = 2 t_f b_{f,0} f_{bz}. \quad (13)$$

Thus, a sufficient condition for the rationality (optimality) of a steel I-beam is the equality of the web area and the total area of the flanges, that is $A_{zh}=2A_{f,z}$ for each cross-section. This condition, in turn, has been proven for constant cross-section beams [12]. Therefore, the sufficient condition for optimality is the equality of the web area and flange areas in each current cross-section (13). The optimal web height of the beam, taking into account condition (13), is determined by the analytical formula (14)

$$\left(h_0 f_{hz}\right) = \sqrt{\frac{3}{2}\frac{k_{MW} M_{x,z}}{t_w R_y \gamma_c}}. \quad (14)$$

Thus, the degree of variability of the optimal height of a steel I-beam depends on the bending moment distribution and, in general, does not exhibit a linear variation pattern (17). Therefore, it is necessary to investigate the feasibility of using a linear variation of web height and flange width and its influence on the occurrence of maximum stresses along the beam length.

Example 1. If the beam has a constant cross-section, then there is an immediate transition to the well-known formula [6, 12]:

$$f_{hz} = 1{,}0 \rightarrow h_0 = \sqrt{\frac{3}{2} k_{MW}} \sqrt{\frac{M_{x,\max}}{t_w R_y \gamma_c}}. \quad (15)$$

This confirms the validity of the analytical approaches by which the relationships [13,14] were obtained for determining the regularities of the optimal web height and flange width in variable-stiffness I-beams.

Example 2. A cantilever beam fixed at one end and subjected to a uniformly distributed load is considered. The optimal height of the beam at each cross-section varies according to equation (14)

$$M_{x,z} = q_b \frac{l^2}{2}\left(1 - \frac{z^n}{l^n}\right)^m \rightarrow t_w = \frac{h_0}{\lambda_w} \rightarrow \left(h_0 f_{hz}\right)^2 \geq \frac{3}{2}\frac{k_{MW} M_{x,z}}{t_w R_y \gamma_c} = \frac{3}{2}\frac{q_b \frac{l^2}{2}}{t_w R_y \gamma_c}\left(1 - \frac{z^n}{l^n}\right)^m,$$

$$\left(h_0 f_{hz}\right) = \sqrt{\frac{3}{4}\frac{q_b l^2}{t_w R_y \gamma_c}} \sqrt{\left(1 - \frac{z^n}{l^n}\right)^m}. \quad (16)$$

If a nonlinear variation of the web height and flange width is assumed, a formula is obtained for their change along the beam's length that ensures an optimal structural topology

$$f_{hz}=\left(B_h-\gamma_h\frac{z^n}{l^n}\right)^m=\sqrt{\frac{3}{4}\frac{l^2}{h_0^3}\frac{q_b\lambda_w}{R_y\gamma_c}\left(1-\frac{z^n}{l^n}\right)^m},\ B_h=\gamma_h=\frac{3}{4}\frac{q_b\lambda_w}{R_y\gamma_c}\frac{l^2}{h_0^3}=\frac{3}{4}\frac{q_b}{t_wR_y\gamma_c}\frac{l^2}{h_0^2}. \quad (17)$$

Example 3. Based on the results obtained in Example 2, and previous findings by the authors, it follows that under linear variation of web height and flange width in a steel I-beam, the maximum stresses occur not in the section where the maximum bending moment acts.The normal stresses in the current cross-section, taking into account the derived relationships, are determined by

$$\frac{\lambda_w t_f b_{f,0}}{(h_0)^2}=\frac{1}{2},M_{x,0}=q_b\frac{l^2}{2}\to M_{x,z}=M_{x,0}\left(1-\frac{z^n}{l^n}\right)^m,\sigma_{x,z}=\left(1-\frac{z^n}{l^n}\right)^m\frac{k_{MW}M_{x,0}}{\frac{(h_0)^3}{\lambda_w}f_{hz}^2\left(\frac{1}{2}\frac{f_{bz}}{f_{hz}}+\frac{1}{6}\right)}. \quad (18)$$

If we accept the optimality condition for each cross-section (13), which requires that the total area of the flanges equals the web area, then for the section with the maximum bending moment, the condition k_b=1,0 holds true

$$W_{x,0}=\frac{(h_0)^3}{\lambda_w}\left(\frac{1}{2}+\frac{1}{6}\right)=\frac{2}{3}\frac{(h_0)^3}{\lambda_w},\ \sigma_{x,0}=\frac{k_{MW}M_{x,0}}{W_{x,0}}=\frac{3}{2}\frac{k_{MW}M_{x,0}\lambda_w}{(h_0)^3},\ \sigma_{x,z}=\left(1-\frac{z^n}{l^n}\right)^m\frac{\sigma_{x,0}}{f_{hz}^2\left(\frac{1}{2}\frac{f_{bz}}{f_{hz}}+\frac{1}{6}\right)}. \quad (19)$$

Thus, for cantilever structures, the ratio of normal stresses in the flange of a steel I-beam at a given section with coordinate z to the stress at the section with z=0 reflects the pattern of stress variation along the length of the beam

$$\frac{\sigma_{x,z}}{\sigma_{x,0}}=\frac{1}{\frac{3}{2}f_{hz}^2\left(\frac{1}{2}\frac{f_{bz}}{f_{hz}}+\frac{1}{6}\right)}\left(1-\frac{z^n}{l^n}\right)^m. \quad (20)$$

Relationship (20) also identifies the cross-sections where maximum stresses occur.

Numerical studies were conducted using analytical expression (20) to evaluate the variation of stresses in variable cross-section beams, depending on the changes in web height and flange width, under the condition of maintaining the optimal ratio.

Figure 1 presents the results of calculations for cantilever beams with variable cross-section under a uniformly distributed load, showing the dependence of the ratio of normal stresses in the I-beam flanges ($\sigma_{x,z}/\sigma_{x,0}$) on the relative position of the section along the beam (z/l). Graph 1 shows that with both web height and flange width varying (γ_h= γ_b =0,4) the maximum stress occurs at z/l=0,4 and exceeds the stress in the fixed (clamped) section—where the maximum bending moment acts — by 19,05% ($\sigma_{x,z}/\sigma_{x,0}$=1,1905). This stress exceedance occurs over the segment (0<z/l<0,77). Graph 2 shows the variation in ($\sigma_{x,z}/\sigma_{x,0}$) along the beam length for a case with web height variation γ_h=0,2; and flange width variation γ_b =0,4. For such I-beams, stress exceedance is observed in the range (0<z/l<0,4), with a maximum stress ratio of ($\sigma_{x,z=0,2}/\sigma_{x,0}$=1,04167 Graph 3 corresponds to a beam with variable web height and constant flange width (γ_h= 0,2; γ_b =0). Here, stress exceedance occurs within 0<z/l<0,25, and the maximum stress ratio is $\sigma_{x,z=0,125}/\sigma_{x,0}$=1,016. Thus, it is established that for any variation in cross-sectional geometry, there are segments along the beamwhere the normal stress exceeds that in the section with the maximum bending moment. Graph 4 illustrates the stress variation in a beam with a constant web height and slightly varying flange width. In this case, stress exceedance is observed in the interval (0<z/l≤0,075, where $\sigma_{x,z=0,05}/\sigma_{x,0}$=1,0013).

Figure 2 presents the results of studying the variation of stress ratios in current beam sections depending on the parameters of geometric variability. Here, a "variability parameter" refers to the rate of change of a linear dimension (web height or flange width) per unit length of the beam. Graph 1 (Fig. 2) illustrates the variation of relative stresses for a beam with piecewise variability of web height and flange width: γ_h= 0,2 on segment (0<z/l≤0,275) and γ_h= 0,45 on (0,275<z/l≤1,0) the flange width increases with γ_b = -0,5 along 0≤z/l≤0,7 and then decreases with: γ_b = 0,3 along 0,7<z/l≤1. Graph 2 shows a steel I-beam with variable web height along the entire span: γ_h= 0,2 for 0≤z/l≤1,0; the flange width increases over 0≤z/l≤0,575 with γ_b =-0,4; and then decreases on 0,575<z/l≤1,0 with γ_b =0,5.

Graph 3 represents a beam with constant web variability γ_h= 0,3(0≤z/l≤1,0) over the full length; flange width changes with γ_b =-0,5 (0≤z/l≤0,5); and γ_b = 0,25(0,5<z/l≤1,0). Graph 4 corresponds to a beam with constant web height (γ_h=0; 0≤z/l≤1,0), and variable flange width: γ_b= -0,5(0≤z/l≤0,225); γ_b =0,3 (0,225<z/l≤1,0).

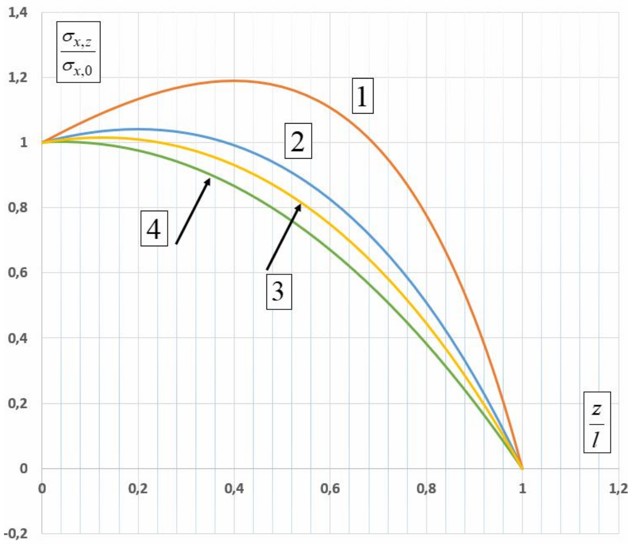

Fig. 1. Variation of normal stresses in cantilever beams with variable cross-section under a uniformly distributed load. Graph 1 – variation parameters:γ_h= γ_b =0,4. Graph 2 - γ_h=0,2; γ_b =0,4. Graph 3 - γ_h= 0,2; γ_b =0 (flange width is constant). Graph 4 - γ_h=0 (web height is constant); γ_b =0,1

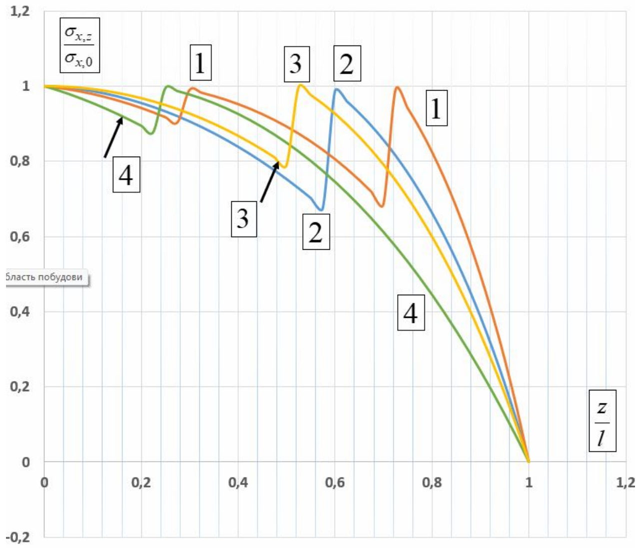

Fig. 2. Variation of normal stresses in cantilever beams with variable cross-section under a uniformly distributed load. Graph 1 – variation parameters: γ_h= 0,2 (0<z/l≤0,2275), γ_h= 0,45(0,275<z/l≤1,0); γ_b = -0,5 (0≤z/l≤0,7), γ_b = 0,38 (0,7<z/l≤1). Graph 2: γ_h= 0,2; (0≤z/l≤1,0); γ_b =-0,4(0≤z/l≤0,575); γ_b =0,5 (0,575<z/l≤1,0). Graph 3 - γ_h= 0,3(0≤z/l≤1,0); γ_b =-0,5 (0≤z/l≤0,5); γ_b = 0,25(0,5<z/l≤1,0). Graph 4 - γ_h=0 (0≤z/l≤1,0), γ_b= -0,5(0≤z/l≤0,225); γ_b =0,3 (0,225<z/l≤1,0)

The developed methodological approach for modifying the cross-section parameters of steel beams allows for the determination of optimal structural configurations of I-beams, including the application of a reverse variation law for flange width: γ_b= (1 - b_{fn}/ b_{f0}), b_{fz}= b_{f0} (1 + γ_b z/ l). However, this approach is applied only to specific beam segments, while on other parts the flange width decreases traditionally along the span: b_{fz}= b_{f0} (1 - γ_b z/ l).

Based on the conducted numerical studies, a comparison of the normalized weight of the beams was also performed, as presented in Table 1.

Table 1

Normalized steel consumption for cantilever steel I-beams with variable cross-section

Graph (from Figure 2)	Cross-section variation	Normalized steel consumption $\frac{m_{b,z}}{\rho l} = 2\sum A_{f,k} + \sum A_{w,k}$.	Percentage
1	γ_h= 0,2 (0<z/l≤0,2275), γ_h= 0,45(0,275<z/l≤1,0), γ_b = -0,5 (0≤z/l≤0,7), γ_b = 0,38 (0,7<z/l≤1)	0,0167	89,19
2	γ_h= 0,2; (0≤z/l≤1,0), γ_b =-0,4(0≤z/l≤0,575), γ_b =0,5 (0,575<z/l≤1,0)	0,1712	91,78
3	γ_h= 0,3(0≤z/l≤1,0), γ_b =-0,5 (0≤z/l≤0,5), γ_b = 0,25(0,5<z/l≤1,0)	0,1703	90,97
4	γ_h=0 (0≤z/l≤1,0), γ_b= -0,5(0≤z/l≤0,225), γ_b =0,3 (0,225<z/l≤1,0)	0,1875	100%

Conclusions. A methodological approach was developed to determine the rational topology of steel I-beams with variable stiffness under uniformly distributed loading. It was shown that for such beams, where both web height and flange width vary, the maximum normal stresses do not occur in the cross-section with the maximum bending moment.

The problem of determining the optimal cross-sectional height was solved using the Lagrange multipliers method along with the Kuhn–Tucker conditions. For steel I-beams with variable geometry, the sufficient condition for structural optimality that the web area equals the total area of the flanges was confirmed.

However, under linear reduction of web height and flange width along the direction of decreasing bending moments, there are cross-sections where the normal stresses in the flanges exceed those in the section where the maximum moment acts. This indicates the presence of multiple governing (critical) sections in beams with variable stiffness.

Accordingly, the physical–mathematical model of the stress–strain state in bending was expanded to include the possibility of finding a rational structure by accounting for reverse variation of the flange width parameter— i.e., in certain segments, the beam height decreases or remains constant, while the flange width and area increase relative to the section with the maximum bending moment.

Such a design solution helps reduce stresses under linear variation of the web height and flange width along the beam length and provides justification for a rational topology of a steel I-beam with stepped, smooth stiffness variation and different rates of change in web and flange geometry.

The numerical studies demonstrated the possibility of reducing steel consumption and increasing the efficiency of variable cross-section I-beams based on the obtained results. In addition, the existence of an admissible set of rational solutions for the stated optimization problem was confirmed.

Thus, the task of determining a rational cross-section of a steel I-beam with linearly variable flange width and web height constitutes an optimization problem with well-defined and appropriate conditions.

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РАЦІОНАЛЬНА ТОПОЛОГІЯ СТАЛЕВИХ ДВОТАВРОВИХ БАЛОК ІЗ РІЗНИМИ ГРАДІЄНТАМИ ЗМІНИ ВИСОТИ СТІНКИ І ШИРИНИ ПОЛИЦЬ НА ВИЗНАЧЕНИХ СЕКЦІЯХ ПО ДОВЖИНІ КОНСТРУКЦІЇ

Розроблено методичний підхід до пошуку раціональної топології сталевих двотаврових балок змінної жорсткості під час дії рівномірно розподіленого навантаження по довжині конструкції. Показано, що для таких балок зі змінною висотою стінки і полиць максимальна напруга не виникає в перерізі, де діє максимальний згинальний момент. Задача пошуку оптимальної висоти перерізу вирішується з використанням методу множників Лагранжа та з використанням умов Куна-Такера. Для сталевих двотаврових балок зі змінною висотою стінки і шириною полиць підтверджено достатні умови оптимальності всієї конструкції: рівність площі стінки дорівнює площі двох полиць. Але при лінійному зменшенні висоти стінки і ширини полиць у бік зменшення згинальних моментів по довжині конструкції маються нові розрахункові перерізи, в яких нормальні напруження в полицях перевищують нормальні напруження в перерізі, де діє максимальний згинальний момент. Це означає, що в балці змінної жорсткості є кілька розрахункових перерізів. Запропонована вдосконалена фізико-математична модель напружено-деформованого стану двотаврової балки при згині. Сталева балка двотаврового перерізу з новою топологією має можливість адаптуватися до напружено-деформованого стану з урахуванням зворотного зміни параметра ширини полів (висота балки у визначеному січенні зменшується або залишається постійною, а ширина і, відповідно, площа поперечного січення полиць збільшується відносно січення, де діє максимальний згинальний момент). Таке удосконалене конструктивне рішення дозволяє вирішити задачу досягнення напруження в поточних перерізах, які не перевищують міцність сталі за границею текучості, по всій довжині сталевої двотаврової балки. Проведені числові дослідження показали можливість знайти нові конструктивні рішення раціональних конструкцій сталевих двотаврових балок змінного перерізу. Також показана допустима множина раціональних рішень за результатами виконаних досліджень. Таким чином, задача пошуку раціональної топології сталевих двотаврових балок з лінійно-змінною шириною полки і висотою стінки є задачею з адекватними умовами проектування.

Ключові слова: сталеві конструкції балок, моделювання, сталеві двотаврові балки змінного перерізу, оптимальна топологія, цільова функція, умови Куна-Такера, метод множників Лагранжа, сталеві балки із різними параметрами швидкості зміни висоти стінки і ширини полиць на окремих ділянках, раціональна топології сталевої двотаврової балки з адекватними умовами проектування.

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RATIONAL TOPOLOGY OF STEEL I-BEAMS WITH VARIOUS GRADIENTS OF CHANGING WALL HEIGHT AND SHELF WIDTH AT SPECIFIED SECTIONS ALONG THE LENGTH OF THE BEAM

A methodological approach has been developed for determining the rational topology of steel I-beams with variable stiffness under uniformly distributed loading along the beam length. It has been shown that for such beams, with varying web height and flange width, the maximum stress does not occur in the section where the maximum bending moment acts. The problem of finding the optimal cross-sectional height is solved using the Lagrange multipliers method in conjunction with the Kuhn–Tucker conditions. For steel I-beams with variable web height and flange width, the sufficient condition for structural optimality is confirmed: the area of the web is equal to the total area of the two flanges. However, under linear reduction of web height and flange width in the direction of decreasing bending moments, new critical cross-sections arise along the beam length in which the normal stresses in the flanges exceed those in the section with the maximum bending moment. This indicates that beams with variable stiffness may have multiple governing sections. An improved physical–mathematical model of the stress–strain state of I-beams in bending is proposed. A steel I-beam with the proposed new topology has the ability to adapt to its stress–strain state by introducing reverse variation of flange width: in selected sections, the beam height decreases or remains constant, while the flange width and accordingly the flange cross-sectional area increases relative to the section where the maximum bending moment acts. This improved design approach allows for achieving stress levels in all current cross-sections that do not exceed the yield strength of steel along the entire length of the I-beam. The numerical studies conducted demonstrate the possibility of finding new rational design solutions for variable cross-section steel I-beams. The existence of an admissible set of rational solutions based on the obtained results has also been confirmed. Thus, the problem of determining the rational topology of steel I-beams with linearly varying flange width and web height represents a design task with appropriately formulated and adequate design condition.

Keywords: steel beam structures, modeling, variable cross-section steel I-beams, optimal topology, objective function, Kuhn–Tucker conditions, Lagrange multipliers method, steel beams with different rates of web height and flange width variation in specific segments, rational topology of steel I-beams with adequately formulated design conditions.

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У статті розроблено методологічний підхід раціонального проектування сталевих двотаврових балок при дії рівномірно розподіленого навантаження з урахуванням різної лінійної зміни висоти стінки і ширини полиць у визначених секціях по довжині конструкції.
Іл. 2. Табл. 1. Бібліогр. 42 назв.

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Dzhanov L.V., Bilyk S.I., Bilyk A.S. **Rational topology of steel i-beams with various gradients of changing wall height and shelf width at specified sections along the length of the beam //** Strength of Materials and Theory of Structures. Scientific-and-technical collected articles. – K.: KNUBA, 2025. – Issue 114. – P. 155-164.
The article presents a generalized methodology for determining the dynamic coefficients for deflections and bending moments of the dynamic operation of a steel roof truss structure under the action of a concentrated impulse and after the load has ceased.
Figs. 2. Tabl 1. Refs. 42.

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