UDC 539.3

INVESTIGATION OF NONLINEAR DEFORMATION, BUCKLING AND NATURAL VIBRATIONS OF ELASTIC SHELLS UNDER THERMOMECHANICAL LOADS USING A UNIVERSAL THREE-DIMENSIONAL FINITE ELEMENT

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DOI: 10.32347/2410-2547.2025.114.35-43

The finite element method for solving problems of nonlinear deformation, stability, post-buckling behavior and natural vibrations of a wide class of thin inhomogeneous shells under the action of thermomechanical loads is considered. An effective computer algorithm for analyzing the behavior of shells is developed based on the geometrically nonlinear relations of the three-dimensional theory of thermoelasticity and the use of the finite element moment scheme.

Keywords: shell of inhomogeneous structure, universal 3D finite element, geometrically nonlinear deformation, buckling, natural vibrations, finite element moment scheme

Introduction

At present, the finite element method has become one of the most common numerical methods used to study the stress-strain state (SSS), stability, and dynamics of shell structures [1-7 and others]. On its basis, effective approaches to studying the behavior of shells of different classes have been developed. Various finite elements (FE) specially developed for these purposes are often used to calculate shells. The FEs are created mainly on the basis of shell theories or, less often, on the basis of three-dimensional equations of the theory of thermoelasticity. The elements used have their own range of application, as a rule. Modern and effective approaches are those in which the shell is considered as a three-dimensional body with a small thickness [2, 4, 8-12]. In this case, two-dimensional theories of plates and shells and one-dimensional theories of rods are not used to describe the behavior of the shell. However, certain hypotheses regarding the stress-strain state of a thin shell are usually accepted. Almost any real shell structure has a complex geometric shape. In addition to having non-canonical outlines, shell shapes can have different geometric features, such as ribs, channels, holes, variable thickness, etc. Under real operating conditions, thin-walled shell structures can usually be subject to mechanical and thermal loads [9, 13, 14]. This necessitates the use of refined approaches from the standpoint of three-dimensional thermoelasticity theory to analyze their behavior [2, 9, 15, 16]. A detailed review of approaches and methods for numerical modeling of nonlinear deformation processes, stability and post-buckling behavior of elastic shells of inhomogeneous structure can be found in the monograph [9] and in the overview article [8].

Problem Formulation and the Research Method

The method of finite element analysis of the stress-strain state, buckling, post-buckling behavior and natural vibrations of shells is based on geometrically nonlinear relations of the three-dimensional theory of thermoelasticity and the principles of the finite element moment scheme [9, 15, 16]. This approach made it possible to develop a universal three-dimensional finite element and a method for studying the behavior of thin elastic multilayer shells with various geometric features along the thickness under the action of a complex thermomechanical load. Thus, thin multilayer shells can have complex geometric shape, constant and step-variable thickness, ribs, holes, channels, sharp bends in the mid-surface, etc. We will call such shells of different classes: inhomogeneous shells, shells of inhomogeneous structure, shells of inhomogeneous stiffness, and shells with thickness-variable parameters [15, 16]. A distinctive feature of the developed universal 3D FE is the presence of its additional variable parameters. This approach allowed the use of a single universal FE in all sections when modeling shells with different inhomogeneities. The single computational finite-element shell model (FESM) created on this basis takes into account various geometric features of the structural elements and inhomogeneities of the thin shell material. This significantly expanded the range of problems under consideration and simplified the numerical implementation of the calculation method. The approximation of a thin multilayer shell of a non-uniform structure is realized by one FE in the thickness direction.

The shell is modeled by a nonlinear elastic continuum subject to large displacements and small strains. The displacements $u^{k'}$ of an arbitrary point are identified in a global Cartesian coordinate system $x^{k'}$. The components of the strain tensor ε_{ij} are set in the local coordinate system x^i by the tensor of finite Cauchy–Green strains:

$$\varepsilon_{ij} = \frac{1}{2}\left(C_j^{k'} \frac{\partial u^{k'}}{\partial x^i} + C_i^{k'} \frac{\partial u^{k'}}{\partial x^j} \right) + \frac{1}{2} \frac{\partial u^{k'}}{\partial x^i} \frac{\partial u^{k'}}{\partial x^j}, \qquad (1)$$

де $C_i^{k'} = \partial x^{k'} / \partial x^i$ are the components of the coordinate transformation tensor.

Since the effect of thermomechanical loads is considered, the strains (1), which are called total [17-19], consist of two different types of strains: elastic $\overset{e}{\varepsilon}_{kl}$ and temperature $\overset{T}{\varepsilon}_{kl}$:

$$\varepsilon_{kl} = \overset{e}{\varepsilon}_{kl} + \overset{T}{\varepsilon}_{kl}. \qquad (2)$$

Consider a steady-state thermal process in which the temperature field in the shell $T = T(x^i)$ is a known function of coordinates, independent of the SSS. Thus, we neglect the coupling of the strain and thermal fields of the shell. Since the shell is thin, the temperature may be considered linearly distributed throughout the thickness of the layer. The effect of the mechanical Q and thermal T fields on the shell is represented as a single process of loading. It is characterized by a common parameter $P = P(Q,T)$. In the algorithm, this relationship is specified as a function describing the effect of the applied thermomechanical load. This approach allows us to analyze the behavior of elastic shells under the influence of various modes of complex thermomechanical loading, including combined ones [9, 15].

The shell layers are assumed to be linearly elastic and are described by the generalized Duhamel–Neumann law [18]:

$$\sigma^{ij} = C^{ijkl} \overset{e}{\varepsilon}_{kl} = C^{ijkl}\left(\varepsilon_{kl} - \overset{T}{\varepsilon}_{kl} \right) = C^{ijkl}\left(\varepsilon_{kl} - \alpha_{kl} T \right) = \overline{\sigma}^{ij} - \overset{T}{\sigma}{}^{ij}, \qquad (3)$$

where

$$\overline{\sigma}^{ij} = C^{ijkl} \varepsilon_{kl} \qquad (4)$$

– are stresses dependent on total strains;

$$\overset{T}{\sigma}{}^{ij} = C^{ijkl} \overset{T}{\varepsilon}_{kl} = C^{ijkl} \alpha_{kl} T \qquad (5)$$

– are stresses dependent on thermal strains; C^{ijkl} are the components of the stiffness tensor; α_{kl} are the components of the tensor of thermal-expansion coefficients.

Anisotropic inhomogeneous material of shell layers is modeled by traditional materials (isotropic, transversely isotropic, and orthotropic [9]) and composite (unidirectional fibrous [16]). To describe curvilinear anisotropy, we introduce an orthotropy basis to set the orientation of the principal axes of the material relative to the mid-surface of the FE layer. The thermoelastic properties are assumed constant during a step of loading.

Two hypotheses are used to describe the features of the SSS of a thin inhomogeneous shell [9, 15, 16]. The static hypothesis compressive assumes that the stresses $\underset{n}{\sigma}{}^{11}$ in the fibers of the n th layer are constant throughout its thickness (along the x^1-axis):

$$\partial \underset{n}{\sigma}^{11} \Big/ \partial x^{1} = 0\,, \tag{6}$$

where $n = \overline{1,m}$; m is a number of layers in the FE.

To satisfy the static hypothesis (6), it is necessary to correct law (3). The second hypothesis is the non-classical kinematic hypothesis of a deformed straight line. The straight segment along the thickness of the shell remains straight during deformation, although it is stretched or shortened. This segment is not necessarily normal to the mid-surface of the shell. The displacements are assumed distributed linearly along the thickness, which is conventional in the theory of thin shells. The hypothesis allows us to combine three-dimensional FEs keeping the compatibility of coordinates and displacements, and to naturally model sharp bends of the middle surface of the shell, which are typical, for example, for faceted, folded and articulated shells.

The universal FE (Fig. 1) is based on the "standard" 3D 8-node isoparametric FE with polylinear shape functions for coordinates and displacements [1, 2, 9] (Fig. 1, b) which is a classic three-dimensional FE of the shell. It is named the casing finite element (CFE) (Fig. 1, a). The casing is understood as a shell body without geometric features in thickness. We have developed a unified model based on this universal FE that describes the multilayer structure of a material and geometrical features of structural elements of an inhomogeneous shell. The geometric features of the shell are: variable-thickness casing, ribs, overlays, channels, holes, sharp bends of the middle surface, etc.

The transformation of the CFE (Fig. 1, a) into a modified finite element (MFE) is achieved by introducing additional variable geometric parameters of the "standard" FE. Using these parameters, the necessary changes in the FE thickness and its shift along the x^1-axis are implemented for modeling sections of shells with a step-variable thickness. The MFE$^+$ (Fig. 1, c) is used to approximate sections of shells with a step-increased thickness (e.g. with ribs). The MFE$^-$ (Fig. 1, d) is used on sections of a step-decreased thickness (e.g. with channels).

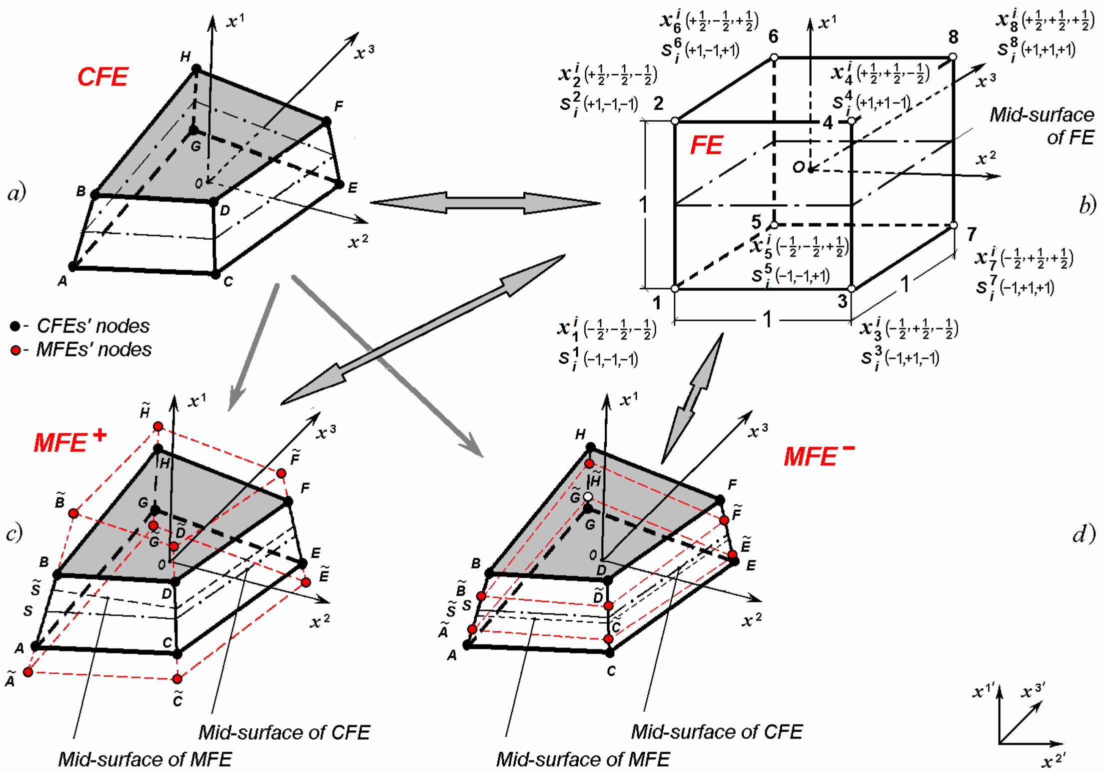
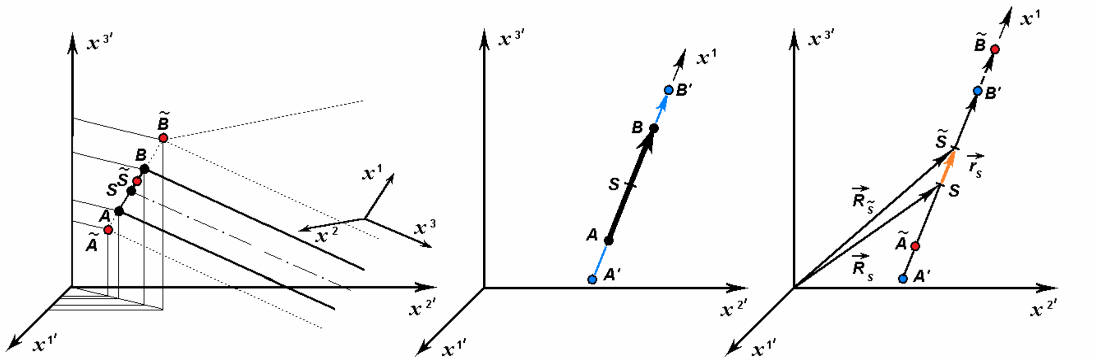


Fig. 1. Transformation of the universal three-dimensional finite element

For this purpose, a method of linear transformation of the coordinates of the nodes of the three-dimensional CFE into the corresponding nodal coordinates of the MFE in the thickness direction, along the x^1-axis (Fig. 2, a) is used [9, 16]. This approach is a consequence of the adopted polylinear law of coordinate change within the FE and the formulated non-classical kinematic hypothesis of the deformable straight line. Figure 2 shows the transformation of the edge AB belonging to the CFE into the edge $\widetilde{A}\widetilde{B}$ of the MFE. At first, the edge AB increases (or decreases) to the value $A'B'$ (Fig. 2, b), and then shifts by the value $\vec{r}_s$ from point S to point $\widetilde{S}$ (Fig. 2 c).

Thus, by varying the additional parameters, the “standard” 3D FE is endowed with the properties of a universal 3D FE, which allows for a unified modeling of a wide class of inhomogeneous shell structures.



(a) (b) (c)

Fig. 2. The procedure for converting the CFE into the MFE

To derive the governing finite-element equations for displacements, use is made of the moment finite-element scheme (MFES) the principles of which have been developed by Sakharov [2]. We have applied the MFES to the problem of geometrically nonlinear deformation of thin multilayer shells of step-variable thickness under the action of thermomechanical loads [9, 16]. The MFES approximations of displacements and strains guarantee a correct description of the rigid-body displacements of FEs, which enhances the convergence and accuracy of solutions on coarse meshes.

The MFES represents the total strains (1) in the local coordinate system x^i as truncated Maclaurin series about the FE center, the point O (Fig. 1, d):

$$\varepsilon_{ij} = \varepsilon_{ij}^{0} + \varepsilon_{ij}^{0f}\,\omega_{(ij)}^{(ff)}x^f + \varepsilon_{ij}^{0fh}\,\delta_{(i)}^{(j)}\,\omega_{(ffh)}^{(hii)}x^f x^h\,, \tag{7}$$

where

$$\varepsilon_{ij}^{0} = \left(\varepsilon_{ij}\right)\Big|_{(0)},\ \varepsilon_{ij}^{0f} = \left(\frac{\partial\varepsilon_{ij}}{\partial x^f}\right)\Bigg|_{(0)},\ \varepsilon_{ij}^{0fh} = \frac{1}{2}\left(\frac{\partial^2\varepsilon_{ij}}{\partial x^f\partial x^h}\right)\Bigg|_{(0)},\ \ \omega_{ff\ldots}^{hi\ldots} = \omega_f^h\omega_f^i\ldots,\ \omega_f^h = \begin{cases}0, & f = h\\ 1, & f \neq h\end{cases}.$$

The thermal strains $\overset{T}{\varepsilon}_{kl} = \alpha_{kl}T$ are given within the nth layer as truncated Taylor series about its center, point O_n:

$$\underset{n}{\overset{T}{\varepsilon}}_{ij} = \underset{n}{\overset{T}{\varepsilon}}{}_{ij}^{0} + \underset{n}{\overset{T}{\varepsilon}}{}_{ij}^{0p}(x^p - x_{O_n}^p)\,\omega_{(ij)}^{(pp)} + \underset{n}{\overset{T}{\varepsilon}}{}_{ij}^{0ph}(x^p - x_{O_n}^p)(x^h - x_{O_n}^h)\,\delta_{(i)}^{(j)}\,\omega_{(pph)}^{(hii)}\,, \tag{8}$$

where

$$\underset{n}{\overset{T}{\varepsilon}}{}_{ij}^{0} = \left(\overset{T}{\varepsilon}_{ij}\right)\Big|_{(O_n)},\ \underset{n}{\overset{T}{\varepsilon}}{}_{ij}^{0p} = \left(\frac{\partial\overset{T}{\varepsilon}_{ij}}{\partial x^p}\right)\Bigg|_{(O_n)},\ \underset{n}{\overset{T}{\varepsilon}}{}_{ij}^{0ph} = \frac{1}{2}\left(\frac{\partial^2\overset{T}{\varepsilon}_{ij}}{\partial x^p\partial x^h}\right)\Bigg|_{(O_n)}.$$

With (7) and (8), stresses (2) are presented in the form of linear segments of Taylor series in powers of local coordinates x^i about the center of the n th layer of the FE:

$$\underset{n}{\bar{\sigma}}{}^{ij} = \underset{n}{N}{}^{ij} + \underset{n}{M}{}^{ij}_{1}(x^1 - x^1_{O_n}) + \underset{n}{M}{}^{ij}_{\alpha} x^{\alpha} + 2\underset{n}{M}{}^{ij}_{1\alpha}(x^1 - x^1_{O_n})x^{\alpha} + \underset{n}{M}{}^{ij}_{\alpha\beta}\omega^{(\beta)}_{(\alpha)}x^{\alpha}x^{\beta}, \tag{9}$$

$$\overset{T}{\underset{n}{\sigma}}{}^{ij} = \overset{T}{\underset{n}{N}}{}^{ij} + \overset{T}{\underset{n}{M}}{}^{ij}_{1}(x^1 - x^1_{O_n}) + \overset{T}{\underset{n}{M}}{}^{ij}_{\alpha} x^{\alpha} + 2\overset{T}{\underset{n}{M}}{}^{ij}_{1\alpha}(x^1 - x^1_{O_n})x^{\alpha} + \overset{T}{\underset{n}{M}}{}^{ij}_{\alpha\beta}\omega^{(\beta)}_{(\alpha)}x^{\alpha}x^{\beta}. \tag{10}$$

Thus, we have similar series (8) and (9). Their coefficients are obtained using the static hypothesis (6) [9]. Using this hypothesis corrects the components of the stiffness tensor $\underset{n}{C}{}^{ijkl}$ for the terms of higher degrees of these series: $\underset{n}{B}{}^{ijkl} = \underset{n}{C}{}^{ijkl} - \left(\underset{n}{C}{}^{ij11}\underset{n}{C}{}^{11kl}\right)\Big/\underset{n}{C}{}^{1111}$.

The study of the processes of geometrically nonlinear shell deformation is based on the general Lagrangian formulation of the variational problem in increments, when the deformation process is given as a sequence of equilibrium states at sufficiently small steps of thermomechanical loading. At the current step of loading, the new shell geometry, the SSS and its prehistory are known. The Lagrange variational equation in the finite element approximation has the form:

$$\delta\Pi = \sum_{FE}\left(\delta W_{FE} - \delta A_{FE}\right) = 0, \tag{11}$$

where Π is the strain energy of the FESM; δW_{FE} and δA_{FE} are the works done by internal and external forces of the FE, respectively; $\sum_{FE}$ is the sum over finite elements of the FESM.

With the Duhamel–Neumann law (2), the virtual work of internal forces of the FE is given by

$$\delta W_{FE} = \int_{V_{FE}} \sigma^{ij}\delta\overset{e}{\varepsilon}_{ij}dV == \int_{V_{FE}} \bar{\sigma}^{ij}\delta\varepsilon_{ij}dV - \int_{V_{FE}} \overset{T}{\sigma}{}^{ij}\delta\varepsilon_{ij}dV = \delta\overline{W}_{FE} - \delta\overset{T}{W}_{FE}. \tag{12}$$

After substituting the strain and stress functions (7-10) into the dependencies (11) and (12), all the defining relations of the SE are obtained, and in explicit form [9].

It is common practice to use the Cartesian displacements $u^{k'}_{s_1s_2s_3}$ of FE nodes as unknowns for a 3D FE (Fig. 1, b). For thin shells, it is appropriate to use the set of displacements of nodal points on the mid-surface and the differences of nodal displacements on the bounding surfaces of the FE, as unknown functions [9].

The equations obtained for three-dimensional finite element are universal, since they are derived in a local coordinate system for a general FE (Fig. 1, d). When obtaining the system of governing nonlinear equations for the FEMS, a method is used that takes into account the eccentric placement of the FE on different sections of a shell with stepwise-varying thickness.

The static problem of geometrically nonlinear deformation of the shell is solved by a step-by-step method. The algorithm for solving the stability problem employs the parameter continuation method, a modified Newton–Kantorovich method, and a procedure for automatic correction of algorithm parameters [9]. At each step s, characterized by a parameter $\lambda_s = \lambda_s(P,U)$, the stress-strain state of the shell is determined: its new coordinates (deformed shape) and increments of the strain and stress fields. Each step corresponds to an increase (or decrease) of the external load parameter P, which is associated with the parameters of mechanical (Q) and temperature (T) effects. The solution of the nonlinear problem is the relationship between the parameter P and the displacement field U of the FESM. This relationship is determined at each step of loading increment $\Delta_s P$ and is usually represented by *a load P-deflection U* ($P-U$) curve at the characteristic point of the shell (Fig. 3). This curve reflects the behavior of the shell in the processes of nonlinear deformation, buckling, and post-buckling behavior. The value P corresponding to the first maximum of the $P-U$ curve (it is a point "a" in Fig. 3) is taken as the upper critical load P^{up}_{cr}. There is a loss of shell stability "in large". This process is accompanied by a sharp transition from one stable equilibrium state of the shell to another. If there is a branching point in the pre-buckling domain of the $P-U$ curve this is (it is a point

" g " in Fig. 3), then it determines the loss of stability of the shell "in the small". In this case, there is a smooth change in the SSS, and the critical load can be taken as the load value P^* corresponding to the first bifurcation point [9, 15, 16].

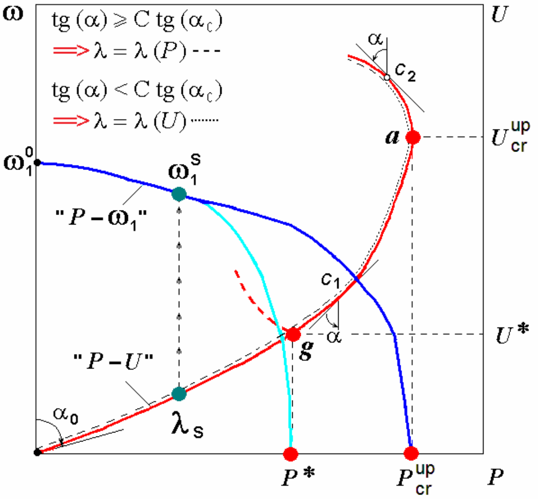


Fig. 3. Algorithm for the complex solution of problems of buckling and natural vibrations of flexible shells

The adopted incremental approach allows not only to analyze the geometrically nonlinear deformation of the shell at each step of the thermomechanical load, but also to determine the frequencies and shapes of natural vibrations of the structure taking into account the prestressed deformed state.

The modal analysis procedure is implemented at each moment s of load increment corresponding to the parameter λ_s. The result of the calculation is *a load* P – *lowest frequency* ω_1" curve ($P-\omega_1$) (Fig. 3), the specified spectrum of natural frequencies (ω_i^s , $i=J$), and the corresponding shapes of natural vibrations. If there is a branching point " g " in the pre-buckling domain of the $P-U$ curve then the load $P=P^*$ for which $\omega_1=0$ can be taken as the upper critical according to the dynamic stability criterion. If there are no branch points, then the maximum point " a " of the $P-U$ curve corresponds to the upper critical load P_{cr}^{up} according to both the static criterion and the dynamic one ($\omega_1=0$).

The algorithm for solving the buckling problem obtains branching points on the $P-U$ curve. Of course, the branching points are fixed with the accuracy of the load increment step. Qualitative theory is used to accurately determine the branching point. The presence of at least one negative eigenvalue of the linearized stiffness matrix for the FESM corresponds to a new equilibrium shell configuration. We use the method to more accurately determine the branching point on the $P-U$ curve and to draw the adjacent deformation mode in its neighborhood. An asymmetric small imperfection defined by the parameter η is introduced into the perfect initial shell shape. If η is small, it affects the neighborhood of the branching point. This allows the branching point to become a critical one and we may to obtain an adjacent deformation mode.

In addition, the developed complex algorithm (Fig. 3) for solving problems of stability and modal analysis of shells under the action of thermomechanical loads also allows, using a dynamic criterion, to determine the presence of the first branching point in the pre-buckling domain, if it exists.

Numerical calculation and analysis of results

We will illustrate the capabilities of the approach using the classical problem of determining the critical load for a axisymmetric spherical panel subject to uniform pressure q. The panel is clamped at the edges. The possibility of the appearance of axisymmetric and non-axisymmetric shapes of buckling for panels of different curvatures $k=H/h$ is investigated. The change in curvature of the panel is achieved by changing the thickness h keeping the rise constant ($H=5$).

The deformed state of the shell of revolution of ideal shape at the beginning of the load is axisymmetric. This corresponds to the axisymmetric formulation of the problem, in which the characteristic shape of buckling is axisymmetric. However, at high load levels, significant compressive forces may arise in the mid-surface of the shell. This, in turn, can lead to the transition of an axisymmetric form of equilibrium to a non-axisymmetric one. This moment on the $P-U$ curve is defined as the appearance of a branching point [20-22]. Moreover, this can occur in the pre-buckling domain. After branching, two types of solution branches can be formed, on which the shell will either take on additional load or not. In the example under consideration, the branching point is identified using the approaches described above – by the number of negative eigenvalues, by introducing a small perturbation into the initial shape of the shell, and by conducting a modal analysis (MA).

The perturbation of the perfect shape of the mid-surface of the shell is introduced as an initial non-axisymmetric imperfection by the formula $\eta \sin(\pi r/a)\cos(\varphi)$, where $\eta = 0.001$ is the accepted value of the disturbance parameter, r and φ are the polar coordinates, a is the radius of the support boundary. Modal analysis is carried out for shells with ideal shape ($\eta = 0$).

The dependence of the dimensionless critical load $\hat{q}_{cr}^{up} = \dfrac{q_{cr}^{up}}{E}\dfrac{\sqrt{12(1-\nu^2)}}{4}\left(\dfrac{R}{h}\right)^2$ versus the dimensionless shallowness parameter $b = \sqrt{6.6k}$ for an axisymmetric spherical panel is determined, where R is the radius of the spherical shell; E, ν are the elastic modulus, Poisson's ratio. The obtained curves $\hat{q}_{cr}^{up} - b$ are compared with the data in [20] (Fig. 4).

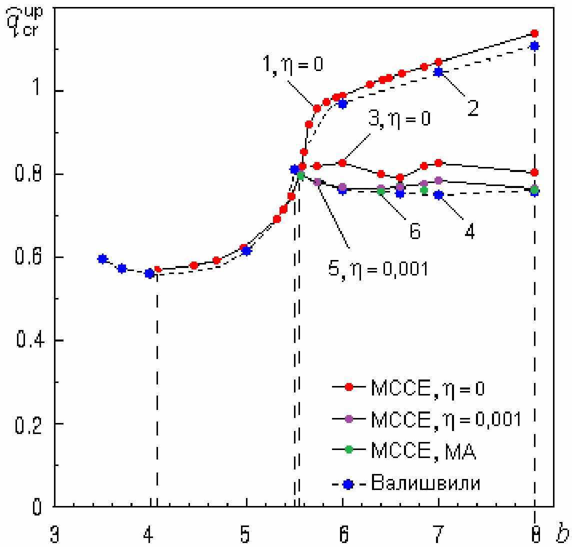


Fig. 4. Dependence of the critical load and buckling shape versus on the curvature of spherical shells of revolution

According to the data of [20], the axisymmetric buckling mode (curve 2) changes into an adjacent non-axisymmetric mode when $b \approx 5.5$ (the origin of curve 4). According to the MFES for the perfect panel ($\eta = 0$) the branching point, determined by qualitative theory, is located near $b = 5.57$ (the origin of curve 3). For a panel with a perturbation of the perfect shape ($\eta = 0.001$), the non-axisymmetric buckling mode is also at a nearby point $b = 5.56$ (the origin of curve 5). Modal analysis also gives a close value $b = 5.56$ (the origin of curve 6). So, good agreement is observed between the $\hat{q}_{cr}^{up} - b$ curves obtained in [22] and the MFES-based calculations.

Calculations using the MFES show that the values of critical loads obtained with $b < 5.56$ for panels of perfect initial shape ($\eta = 0$) and panels with a perturbation shape ($\eta = 0.001$) coincide, having an axisymmetric buckling shape. With the increase of the shallowness parameter b, the difference between the values of critical loads gradually grows to more and thanreaches almost 50% at $b = 8$. This occurs due to the transformation of the bifurcation point at $b = 5.56$ into a critical one and the change of the pre-critical axisymmetric deformation shape into a non-axisymmetric buckling mode.

Studies of the effect of parameters of structural elements, thermal and mechanical loads, boundary conditions, and other factors on the buckling and natural vibrations of flexible shells of a inhomogeneous structure can be found in a number of scientific works by the authors, for example [9, 15, 16].

Conclusions

The proposed method, constructed from the unified positions of the three-dimensional geometrically nonlinear theory of thermoelasticity and the use of the moment finite element scheme, allows solving static problems of nonlinear deformation, buckling, post-buckling behavior and natural vibrations of a wide class of thin elastic inhomogeneous shells under the action of thermomechanical loads.

The method is applied to identify the branching point of the solution on the load-deflection curve. The analysis is performed using three approaches: by the number of negative eigenvalues, by introducing a small perturbation into the perfect initial shape of the shell, and by conducting a modal analysis. It can be concluded that the developed method is an effective tool for numerical study of buckling and modal analysis of shells with detection of branching points and transition to adjacent branches of the solutions.

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Стаття надійшла 27.03.2025

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ДОСЛІДЖЕННЯ НЕЛІНІЙНОГО ДЕФОРМУВАННЯ, СТІЙКОСТІ ТА ВЛАСНИХ КОЛИВАНЬ ПРУЖНИХ ОБОЛОНОК ПРИ ДІЇ ТЕРМОМЕХАНІЧНИХ НАВАНТАЖЕНЬ З ВИКОРИСТАННЯМ УНІВЕРСАЛЬНОГО ТРИВИМІРНОГО СКІНЧЕННОГО ЕЛЕМЕНТА

У статті надані головні особливості методу розв’язування статичних задач нелінійного деформування, стійкості, закритичної поведінки та власних коливань широкого класу тонких пружних неоднорідних оболонок різної форми і структури при дії термомеханічних навантажень. Метод побудований з єдиних позицій тривимірної геометрично нелінійної теорії термопружності на основі методу скінченних елементів. Використовується універсальний тривимірний скінченний елемент, відмінною рисою якого є наявність його додаткових змінних параметрів. Такий прийом дав змогу при моделюванні оболонок з різними неоднорідностями застосовувати на усіх ділянках єдиний універсальний скінченний елемент. На цій основі створено розрахункову модель, що враховує геометричні особливості конструктивних елементів і неоднорідності матеріалу тонкої оболонки (змінність товщини, злами та гранованість обшивки, ребра, накладки, виїмки, отвори, вставки, багатошарову структуру матеріалу). В алгоритмі розв'язання задачі стійкості оболонки визначаються точки розгалуження з можливістю побудови в околицях суміжних форм деформування. Розроблено метод комплексного розв’язування задач стійкість і власних коливань оболонок при дії термомеханічних навантажень. На базі такого підходу втрата стійкості визначається за статичним і динамічним критеріями. На чисельному прикладі продемонстрована ефективність методу. Метод застосовано для виявлення точки розгалуження розв'язку на кривій «навантаження-прогин» для панелей різної кривизни.

Ключові слова: оболонка неоднорідної структури, універсальний тривимірний скінченний елемент, геометрично нелінійне деформування, стійкість, власні коливання, моментна схема скінченних елементів.

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INVESTIGATION OF NONLINEAR DEFORMATION, BUCKLING AND NATURAL VIBRATIONS OF ELASTIC SHELLS UNDER THERMOMECHANICAL LOADS USING A UNIVERSAL THREE-DIMENSIONAL FINITE ELEMENT

The article presents the fundamentals and features of the method for solving static problems of nonlinear deformation, buckling, post-buckling behavior and natural vibrations of a wide class of thin elastic inhomogeneous shells of various shapes and structures under the action of thermomechanical loads. The method is developed from the unified positions of the three-dimensional geometrically nonlinear theory of thermoelasticity based on the finite element method. A universal 3D finite element is used. The distinctive feature of the finite element is the presence of its additional variable parameters. This approach allowed for the use of a single universal finite element in all sections when modeling shells with different inhomogeneities. On this basis, a unified model has been developed that takes into account the geometric features of the structural elements and the multilayer structure of a material of the thin shells (constant or piecewise variable thickness, ribs, cover plates, channels, holes, sharp bends in the middle surface, layers, etc.). The algorithm for solving the shell buckling problem finds the branching points and allows obtaining adjacent deformation modes in their neighborhood. A method for the integrated solution of problems of stability and natural vibrations of shells under the action of thermomechanical loads has been developed. Based on this approach, the loss of stability is determined by static and dynamic criteria. The efficiency of the method is demonstrated by a numerical example. The method is used to identify the branching point of the solution on the load-deflection curve for panels of different curvatures.

Keywords: shell of inhomogeneous structure, universal 3D finite element, geometrically nonlinear deformation, buckling, natural vibrations, finite element moment scheme.

УДК 539.3
Кривенко О.П., Лізунов П.П. **Дослідження нелінійного деформування, стійкості та власних коливань пружних оболонок при дії термомеханічних навантажень з використанням універсального тривимірного скінченного елемента** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – Київ: КНУБА, 2025. – Вип. 114. – С. 35-43.
Розглянуто метод скінченних елементів для розв'язання задач нелінійного деформування, стійкості, закритичної поведінки та власних коливань широкого класу тонких неоднорідних оболонок під дією термомеханічних навантажень. На основі геометрично нелінійних співвідношень тривимірної теорії термопружності та використання моментної схеми скінченних елементів побудовано ефективний комп'ютерний алгоритм аналізу поведінки оболонок. Числовий приклад демонструє ефективність розробленого підходу.
Іл. 4. Бібліогр. 22 назв.

UDC 539.3
Krivenko O.P., Lizunov P.P. **Investigation of nonlinear deformation, buckling and natural vibrations of elastic shells under thermomechanical loads using a universal three-dimensional finite element** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – Kyiv: KNUBA, 2025. – Issue 114. – P. 35-43.
The finite element method for solving problems of nonlinear deformation, stability, post-buckling behavior and natural vibrations of a wide class of thin inhomogeneous shells under the action of thermomechanical loads is considered. An effective computer algorithm for analyzing the behavior of shells is developed based on the geometrically nonlinear relations of the three-dimensional theory of thermoelasticity and the use of the finite element moment scheme. A numerical example demonstrates the effectiveness of the developed approach.
Fig. 4. Ref. 22.

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