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## INFLUENCE SHAPE IMPERFECTIONS ON STOCHASTIC STABILITY OF ELASTIC SHELL PARAMETRIC VIBRATIONS

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Mathematical modeling of stochastic parametric oscillations of an elastic cylindrical tank shell with real and modelled shape imperfections under the action of a random axial load was performed. Finite element models of the imperfect shell were generated in the NASTRAN software. A functional approach was applied to the formation of a reduced model of parametric shell oscillations in the form of a system of differential equations of the first Markov approximation with respect to moment functions of the second order, taking into account specific values of the constant components of the parametric load. The stochastic component of the parametric load on the dynamic behavior of the shell was investigated using the fourth-order Runge-Kutta method. Response realizations and phase trajectories of the shell with real and simulated imperfections at a given frequency of the hidden periodicity of the stochastic load, damping coefficient, and correlation parameter were obtained. The stochastic stability problem was reduced to determining the characteristic indices of a linear autonomous system. The influence of real and simulated imperfections of the shell shape on the stability of parametric oscillations at different values of the constant and stochastic components of the shell shape on the stability of parametric oscillations at different values of the constant and stochastic components of the axial load was estimated.

Keywords: stochastic stability, parametric vibrations, finite element method, cylindrical shell, shape imperfections.

The mathematical aspects of the theory of stochastic stability elastic systems were presented in [1-6]. The methods and approaches of the stochastic stability theory aim to answer the question: will the trajectory of the system's motion in some stochastic sense deviate sufficiently little from the unexcited trajectory for a given initial perturbation or a perturbation acting in time. The numeral methods allow to explore dynamic problems of the thin shells including parametric vibrations [7-14]. To form of the reduced models of parametric vibrations of shells was developed the numeral approach [11] that was based on the finite element method and computational procedures of the NASTRAN software and special program to modeling of the imperfections. As an example, the influence of real and modelled imperfections on natural frequencies and modes of reservoir shell parametric vibrations excited by axial load was investigated by the authors in the articles [14]. The importance of considering influence the constant component of parametric load on natural frequencies and modes of reservoir shell was shown. The continuation of these investigations is analysis of influence shape imperfections on the stochastic stability of reservoir shell parametric vibrations. Mathematical modeling of stochastic parametric oscillations of the elastic cylindrical shell with real and modelled shape imperfections under the action of an axial delta-correlated perturbation was performed. Using the functional approach and splitting of the correlation of the external influence, a reduced finite element model of parametric shell oscillations was formed in the form of a system of differential equations of the first Markov approximation for second-order moment functions. The influence of real and modelled shape imperfections on the dynamic behavior of an elastic shell under the action of constant and random components of a parametric load was investigated using the fourth-order Runge-Kutta method. The stochastic stability of parametric oscillations of a shell with imperfections was investigated using generalized Hill determinants.

# 1. Reduced mathematical model of parametric oscillations of elastic shell with shape imperfections

A cylindrical reservoir shell with a perfect surface, real and modelled shape imperfections in the form the first bifurcation buckling mode of perfect shell under uniform pressure was considered by the authors in the article [14]. The mathematical model of parametric oscillations of elastic shells was constructed according to the numerical approach [11]. The random parametric axial load was given as a sum of the constant and stochastic components. Reduced matrices were obtained under the condition of normalizing the forms of natural oscillations by the mass matrix. The members of the reduced matrix of stiffnesses were the squares of the frequencies of the shell natural oscillations. To obtain the members of the reduced matrix of shell geometric stiffnesses, a two-stage calculation was performed: the boundary value problem of statics under the action of a specific values of the constant components of the parametric load was solved using the Newton-Raphson method and the modal analysis of the prestressed shell by Lanczos method.

Then the reduced model was taked in the form of a system of uncoupled equations that describe the parametric oscillations of the shell taking into account a specific value of the constant component of the random parametric load

$$\ddot{y}_i(t) + 2\varepsilon_i \omega_i^2 \dot{y}_i(t) + \tilde{\omega}_{i(z_0)}^2 y_i(t) - \tilde{z}(t) \frac{g_{ii(z_0)}}{z_0} y_i(t) = 0, \ i = 1, 2, ..., m ,$$
(1)

where  $\omega_i$  and  $\tilde{\omega}_i$  – circular frequency of natural oscillations of the shell and the shell loaded with a specific value of constant component  $z_0$  of the parametric load (s<sup>-1</sup>) [14];  $g_{ii(z_0)}$  – member of the reduced geometric stiffness matrix taking into account the constant component  $z_0$ ;  $\tilde{z}(t)$  – stochastic component of the parametric load, which was given in the form of a delta-correlated random load with a correlation function

$$K(\tau) = \sigma_0^2 e^{-\alpha \tau} \left[ \cos \theta_\alpha \tau + \frac{\alpha}{\theta_\alpha} \sin \theta_\alpha \tau \right], \tag{2}$$

and a finite correlation radius

$$\tau_0 = \frac{1}{\sigma_0^2} \int K(\tau) d\tau = \frac{2\alpha}{\alpha^2 + \theta_\alpha^2},\tag{3}$$

where  $\sigma_0^2$  – the intensity of the stochastic influence;  $\alpha$  – the correlation parameter,  $\theta_{\alpha}$  – the frequency of the hidden periodicity.

To study the stochastic stability of the shell, we considered the question of transforming the system of equations (1) with respect to second-order moment functions in normal form. The system of equations in the form of a system of three deterministic differential equations of the first Markov approximation with respect to moment functions of the second order in the phase variables  $\langle \zeta_1(t) \rangle = y_i(t), \ \langle \zeta_2(t) \rangle = \dot{y}_i(t)$  for each frequency of shell natural oscillations of the shell with initial conditions  $\zeta_1(0) = y_{i0}, \ \zeta_2(0) = \dot{y}_{i0}$ 

$$\frac{d}{dt}\langle\zeta_{1}^{2}(t)\rangle = 2\langle\zeta_{1}(t)\zeta_{2}(t)\rangle,$$

$$\frac{d}{dt}\langle\zeta_{1}(t)\zeta_{2}(t)\rangle = \langle\zeta_{2}^{2}(t)\rangle - \tilde{\omega}_{i(z_{0})}^{2}\langle\zeta_{1}^{2}(t)\rangle - 2\varepsilon_{i}\omega_{i}\langle\zeta_{1}(t)\zeta_{2}(t)\rangle,$$

$$\frac{d}{dt}\langle\zeta_{2}^{2}(t)\rangle = -4\varepsilon_{i}\omega_{i}\langle\zeta_{2}^{2}(t)\rangle - 2\tilde{\omega}_{i(z_{0})}^{2}\langle\zeta_{1}(t)\zeta_{2}(t)\rangle + \sigma_{0}^{2}\tau_{0}\tilde{\omega}_{i(z_{0})}^{4}a_{ii(z_{0})}^{2}\langle\zeta_{1}^{2}(t)\rangle, \quad i = 1, 2, ..., m.$$
(4)

Here  $a_{ii(z_0)} = g_{ii(z_0)} / z_0$ .

# **2.** Investigation of stochastic stability of shell parametric oscillations with respect to moment functions of the second order in the phase variables

The influence of the stochastic component of the parametric load  $\tilde{z}(t)$  on the dynamic behavior of the solution  $\langle \zeta_1^2(t) \rangle$  (m<sup>2</sup>) of system (4) and phase trajectories at the frequency of the hidden periodicity

of the stochastic load  $\theta_{\alpha} = \omega_1$ , the damping coefficient  $\varepsilon_1 = 0,001$  and the correlation parameter  $\alpha = \varepsilon_1 \omega_1$ . In the case when the value  $\langle \zeta_1^2(t) \rangle$  decreased over time, the state of the parametric oscillations of the shell was considered stable, when it increased, an unstable oscillation mode was observed.

The oscillation stable mode of the shell with the imperfection amplitude  $\delta = [0(1), real(2), h(3)]$  and the constant component of the parametric load  $z_0 = [100(a), 200(b), 300(c)]$ kN are presented in Fig. 1.



 $z_0 = [100(a), 200(b), 300(c)]$ kN

As an example, in Fig. 2 we can see the phase trajectories of oscillation unstable mode of the shell with the imperfection amplitude  $\delta = h$  and the constant component of the parametric load  $z_0 = [100(a), 200(b), 300(c)]$ kN.

The oscillation unstable mode of the shell with the imperfection amplitude  $\delta = [0(1), real(2), h(3)]$  and the constant component of the parametric load  $z_0 = [100(a), 200(b), 300(c)]$ kN are presented in Fig. 3.



Fig. 2. Phase trajectories of oscillation stable mode of the shell with the imperfection amplitude  $\delta = h$  and  $z_0 = [100(a), 200(b), 300(c)]$ kN





Fig. 3. Oscillation unstable mode of the shell with the imperfection amplitude  $\delta = [0(1), real(2), h(3)]$  and  $z_0 = [100(a), 200(b), 300(c)]$ kN

As an example, the phase trajectories of oscillation unstable mode of the shell with the imperfection amplitude  $\delta = h$  and the constant component of the parametric load  $z_0 = [100(a), 200(b), 300(c)]$ kN are presented in Fig. 4.



Fig. 4. Phase trajectories of oscillation unstable mode of the shell with the imperfection amplitude  $\delta = h$  and  $z_0 = [100(a), 200(b), 300(c)]$ kN

## **3.** Investigation of stochastic stability of shell parametric oscillations using the generalized Hill determinants

Using the method of generalized Hill determinants, the influence of the shape imperfections on shell stochastic stability can be investigated in the first approximation. The system of deterministic differential equations with respect to moment functions of the second order (4) is rewritten in the form of a linear autonomous system

$$\frac{d}{dt} \begin{cases} \langle \zeta_1^2(t) \rangle \\ \langle \zeta_1(t) \zeta_2(t) \rangle \\ \langle \zeta_2^2(t) \rangle \end{cases} = G(t) \begin{cases} \langle \zeta_1^2(t) \rangle \\ \langle \zeta_1(t) \zeta_2(t) \rangle \\ \langle \zeta_2^2(t) \rangle \end{cases},$$
(5)

where G(t) – matrix whose coefficients are  $2\pi/\omega_i$  – periodic functions

$$G(t) = \begin{vmatrix} 0 & 2 & 0 \\ -\tilde{\omega}_{i(z_0)}^2 & -2\varepsilon_i\omega_i & 1 \\ \sigma_0^2 \tau_0 \tilde{\omega}_i^4 a_{ii(z_0)}^2 & -2\tilde{\omega}_{i(z_0)}^2 & -4\varepsilon_i\omega_i \end{vmatrix}.$$
 (6)

Analysis of the shell stability is reduced to the problem of the stability of trivial solutions of system (5). Solving an algebraic problem for eigenvalues, the characteristic indicators are determined.

In Fig. 5 we can see the dependence of Hill's characteristic indicators on intensity of the random load for the shell with the imperfection amplitude  $\delta = [0(a), real(b), h(c)]$  when the constant component of the parametric load  $z_0 = 100 \text{ kN}$  and  $\theta_{\alpha} = [\omega_1(1); \omega_3(2); \omega_5(3); \omega_7(4); \omega_9(5)]$ . The positive real parts of the characteristic indicators, which correspond to the unstable mode of oscillations, lie in the upper half-plane. The points of intersection of the solid curve of the coordinate axis correspond to the critical values of the stochastic component of parametric fluctuations.

The dependence of Hill's characteristic indicators on intensity of the random load for the shell with the imperfection amplitude  $\delta = [0(a), real(b), h(c)]$  when the constant component of the parametric load  $z_0 = 200 \text{ kN}$  and  $\theta_{\alpha} = [\omega_1(1); \omega_3(2); \omega_5(3); \omega_7(4); \omega_9(5)]$  is presented in Fig. 6.

In Fig. 7 we can see the dependence of Hill's characteristic indicators on intensity of the random load for the shell with the imperfection amplitude  $\delta = [0(a), real(b), h(c)]$  when the constant component of the parametric load  $z_0 = 300$  kN and  $\theta_{\alpha} = [\omega_1(1); \omega_3(2); \omega_5(3); \omega_7(4); \omega_9(5)]$ .

The dependence of Hill's characteristic indicators on intensity of the stochastic load for the shell with  $\delta = [0(1), real(2), h(3)]$  when  $\theta_{\alpha} = \omega_1$ ,  $\varepsilon_1 = 0,001$  and the constant component of the parametric load  $z_0 = [100, 200, 300]$ kN we can see in Fig. 8.



(a)



(b)



Fig. 5. The dependence of Hill's characteristic indicators on intensity of the stochastic load for the shell with  $\delta = [0(a), real(b), h(c)]$  when  $z_0 = 100$  kN and  $\theta_{\alpha} = [\omega_1(1); \omega_3(2); \omega_5(3); \omega_7(4); \omega_9(5)]$ 











Fig. 6. The dependence of Hill's characteristic indicators on intensity of the stochastic load when  $z_0 = 200$  kN for the shell with  $\delta = [0(a), real(b), h(c)]$  and  $\theta_{\alpha} = [\omega_1(1); \omega_3(2); \omega_5(3); \omega_7(4); \omega_9(5)]$ 



(a)



(b)



Fig. 7. The dependence of Hill's characteristic indicators on intensity of the stochastic load for the shell with  $\delta = [0(a), real(b), h(c)]$  when  $z_0 = 300$  kN and  $\theta_{\alpha} = [\omega_1(1); \omega_3(2); \omega_5(3); \omega_7(4); \omega_9(5)]$ 



(a)







Fig. 8. The dependence of Hill's characteristic indicators on intensity of the stochastic load for the shell with  $\delta = [0(1), real(2), h(3)]$  when  $\theta_{\alpha} = \omega_1$  and  $z_0 = [100(a), 200(b), 300(c)]$ kN

In Tab. 1 the dependence of critical values of the stochastic component intensity of parametric load on the frequency of the hidden stochastic load periodicity for shell models without and with imperfections were showed.

Table 1

| Shell<br>model  | z <sub>0</sub> ,kN | The frequency of the hidden stochastic load periodicity |                              |                              |                              |                              |
|-----------------|--------------------|---|------------------------------|------------------------------|------------------------------|------------------------------|
|                 |                    | $\theta_{\alpha} = \omega_1$                            | $\theta_{\alpha} = \omega_3$ | $\theta_{\alpha} = \omega_5$ | $\theta_{\alpha} = \omega_7$ | $\theta_{\alpha} = \omega_9$ |
| $\delta = 0$    | 100                | 354,181   | 391,157                      | 542,064                      | 735,221                      | 975,145                      |
|                 | 200                | 198,003   | 225,567                      | 308,029                      | 554,854                      | 596,342                      |
|                 | 300                | 5,906   | 11,888                       | 11,906                       | 296,996                      | 285,723                      |
| $\delta = real$ | 100                | 354,452   | 354,375                      | 571,678                      | 716,457                      | 634,165                      |
|                 | 200                | 197,940   | 225,043                      | 318,735                      | 578,084                      | 698,105                      |
|                 | 300                | 5,905   | 11,887                       | 11,891                       | 297,005                      | 307,953                      |
| $\delta = h$    | 100                | 308,272   | 342,446                      | 403,078                      | 512,391                      | 934,967                      |
|                 | 200                | 189,913   | 207,021                      | 254,993                      | 525,885                      | 682,130                      |
|                 | 300                | 5,963   | 5,905                        | 11,867                       | 180,606                      | 265,239                      |

Critical values of the stochastic component intensity of parametric load, kN

**Conclusion.** Decrease of the critical value of the stochastic component intensity of the parametric load was observed with an increase in the constant component. Influence the modelled imperfections on the critical values of the stochastic load intensity was bigger than influence real ones.

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Лук'янченко О.О., Геращенко О.В., Костіна О.В., Палій О.М.

#### ВПЛИВ НЕДОСКОНАЛОСТЕЙ ФОРМИ НА СТОХАСТИЧНУ СТІЙКІСТЬ ПАРАМЕТРИЧНИХ КОЛИВАНЬ ПРУЖНОЇ ОБОЛОНКИ

Виконано математичне моделювання стохастичних параметричних коливань пружної циліндричної оболонки резервуару з реальними і змодельованими недосконалостями форми при дії випадкового осьового навантаження. Скінченноелементні моделі недосконалої оболонки сформовані у програмному комплексі NASTRAN. Застосовано функціональний підхід до формування редукованої моделі параметричних коливань оболонки у вигляді системи диференціальних рівнянь першого Марківського наближення відносно моментних функцій другого порядку з урахуванням конкретних значень сталих складових параметричного навантаження. Стохастична складова параметричного навантаження задана у вигляді дельта-корельованого випадкового навантаження. Членами матриці жорсткості редукованої математичної моделі є квадрати частот власних коливань досконалої оболонки, що отримані методом Ланцоша. Члени редукованої матриці геометричної жорсткості оболонки без і з недосконалостями форми отримано за допомогою двоетапного розрахунку. На першому етапі розв'язана нелінійна задача статики від дії сталої складової параметричного навантаження методом Ньютона-Рафсона. На другому етапі виконано модальний аналіз методом Ланцоша з урахуванням попередньо напруженого стану оболонки. Досліджено вплив стохастичної складової параметричного навантаження на динамічну поведінку оболонки за допомогою методу Рунге-Кутти четвертого порядку. Отримано реалізації відгуку та фазові траєкторії оболонки з реальними і модельованими недосконалостями на заданій частоті схованої періодичності стохастичного навантаження, коефіцієнті затухання і параметрі кореляції. Режим параметричних коливань оболонки вважався стійким, коли амплітуда параметричних коливань з часом зменшувалась, нестійким - збільшувалась. Досліджена стохастична стійкість параметричних коливань недосконалої оболонки за допомогою узагальнених визначників Хілла. Задача стохастичної стійкості зводилась до визначення характеристичних показників лінійної автономної системи. Тривіальний розв'язок системи вважався стійким, якщо у всіх характеристичних показників дійсна частина була менша за нуль. У випадку, коли хоча б у одного розв'язку дійсна частина характеристичного показника була більша за нуль, то тривіальний розв'язок системи вважався нестійким. Якщо максимальна дійсна частина характеристичних показників дорівнювала нулю, то такий стан відповідав границі області стохастичної нестійкості оболонки. Оцінено вплив реальних і змодельованих недосконалостей форми оболонки на стійкість параметричних коливань при різних значеннях сталої і стохастичної складової осьового навантаження.

Ключові слова: стохастична стійкість, параметричні коливання, метод скінченних елементів, циліндрична оболонка, недосконалості форми.

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#### INFLUENCE SHAPE IMPERFECTIONS ON STOCHASTIC STABILITY OF ELASTIC SHELL PARAMETRIC VIBRATIONS

Mathematical modeling of stochastic parametric oscillations of an elastic cylindrical tank shell with real and modelled shape imperfections under the action of a random axial load was performed. Finite element models of the imperfect shell were generated in the NASTRAN software. A functional approach was applied to the formation of a reduced model of parametric shell oscillations in the form of a system of differential equations of the first Markov approximation with respect to moment functions of the second order, taking into account specific values of the constant components of the parametric load. The stochastic component of the parametric load was given in the form of a delta-correlated random load. The members of the stiffness matrix of the reduced mathematical model are the squares of the frequencies of the natural oscillations of the perfect shell, obtained by the Lanczos method. The members of the reduced matrix of the geometric stiffness of the shell without and with shape imperfections were obtained using a two-stage calculation. At the first stage, the nonlinear statics problem under the action of the constant component of the parametric load was solved by the Newton-Raphson method. At the second stage, a modal analysis was performed using the Lanczos method taking into account the pre-stressed state of the shell. The influence of the stochastic component of the parametric load on the dynamic behavior of the shell was investigated using the fourth-order Runge-Kutta method. Response realizations and phase trajectories of the shell with real and simulated imperfections at a given frequency of the hidden periodicity of the stochastic load, damping coefficient, and correlation parameter were obtained. The stochastic stability of parametric oscillations of an imperfect shell was investigated using generalized Hill determinants. The stochastic stability problem was reduced to determining the characteristic indices of a linear autonomous system. The influence of real and simulated imperfections of the shell shape on the stability of parametric oscillations at different values of the constant and stochastic components of the axial load was estimated.

Keywords: stochastic stability, parametric vibrations, finite element method, cylindrical shell, shape imperfections.

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Лук'янченко О.О., Геращенко О.В., Костіна О.В., Палій О.М. Вплив недосконалостей форми на стохастичну стійкість параметричних коливань пружної оболонки // Опір матеріалів і теорія споруд: наук.-тех. збірн. - К.: КНУБА, 2025. – Вип. 114. – С. 23-34.

Виконано математичне моделювання стохастичних параметричних коливань пружної циліндричної оболонки з реальними і змодельованими недосконалостями форми при дії випадкового навантаження. Застосовано функціональний підхід до формування редукованої скінченноелементної моделі параметричних коливань оболонки у вигляді системи диференціальних рівнянь першого Марківського наближення відносно моментних функцій другого порядку. Досліджено вплив недосконалостей форми на стохастичну стійкість параметричних коливань оболонки методом Рунге-Кутти четвертого порядку та за узагальнених визначників Хілла.

Табл. 1. Іл. 8. Бібліогр. 17 назв.

### UDC 539.3

Lukianchenko O.O., Geraschenko O.V., Kostina O.V., Paliy O.M. Influence shape imperfections on stochastic stability of elastic shell parametric vibrations // Strength of Materials and Theory of Structures. – 2025. – Issue. 114. – P. 23-34. Mathematical modeling of stochastic parametric vibrations of an elastic cylindrical shell with real and modelled shape imperfections under the action of an random axial load was performed. Using the functional approach, the reduced finite element model of shell parametric oscillations as a system of differential equations of the first Markov approximation for second-order moment functions was formed. Influence shape imperfections on the stochastic stability of parametric oscillations of elastic shell was investigated by the fourth-order Runge-Kutta method and generalized Hill determinants. Tab. 1. Fig. 8. References 17 items.

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