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HYPERBOLIC MODELS IN THE ANALYSIS OF HEAT AND MOISTURE EXCHANGE IN INHOMOGENEOUS POROUS MATERIALS

Y.V. Chovnyuk[1]

P.P. Cherednichenko[1]

A.S. Moskvitina[1]

M.O. Shyshyna[1]

N.S. Shudra[1]

E.A. Ivanov[2]

[1]*Kyiv National University of Construction and Architecture*
avenue Povitrianykh syl 31, Kyiv, Ukraine, 03037

[2]*National Aviation University*
Lubomyr Huzar avenue,1, Kyiv, Ukraine, 03058

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The paper uses hyperbolic models for the analysis of heat and moisture exchange in inhomogeneous porous materials in which short heat pulses propagate. The heat transfer in sharply inhomogeneous media at room temperature is not described by Fourier and Cattaneo laws, but is modeled by Guyer-Krumhansl-type equations. The O.V. Lykov system of equations of interrelated heat and mass transfer taking into account the finiteness of heat and mass (moisture) transfer rates is solved using a one-dimensional formulation. However, the heat propagation velocity is of the order of the sound speed, so due to the short relaxation time, the solutions of the hyperbolic equation of thermal conductivity largely coincide with the solutions of the classical parabolic equation, although there are some significant differences. They depend on processes occurring on the surface (in thin layers) of porous bodies. The moisture diffusion rate in capillary-porous materials is approximately $10^6 \ldots 10^7$ and more times lower than the heat propagation rate, so, accordingly, the relaxation time of diffusion processes is much longer and should be considered in mass transfer equations. Exact analytical solutions of the one-dimensional Guyer-Krumhansl equation are obtained using the operator method. This equation is also used to study heat pulses of different shapes in the medium with respect to phonon/ballistic methods of heat transfer. The obtained results are used to model the heat and moisture propagation in thin films of capillary-porous bodies with account taken of molecular effects in systems of reduced dimension. The very short heat pulses propagation simulating isolated heat waves is modeled with reference to Knudsen number, as well as the solutions for the periodic initial function. The exact solutions of the above problems in the model of thin films of capillary-porous bodies are obtained.

Keywords: Guyer-Krumhansl equation, heat and mass transfer, thin films, capillary-porous bodies, inhomogenuity, Knudsen number, hyperbolic equation of heat and moisture transfer.

1. Introduction

The development of computer methods together with the computer technology's advance made in recent decades has substantially simplified the solution of many mathematical problems. Computer methods are widely used to analyze mathematical models of physical processes. However, it is often necessary to obtain and study analytical solutions for a deep understanding of physical phenomena and their proper explanation. All the tools of mathematical physics are used for this purpose. One of the most commonly used laws of physics in everyday life is the Fourier law of thermal conductivity relating temperature changes to the heat flow using a linear dependence [1]. This model well describes the phenomenon of thermal conductivity in homogeneous solid undeformed bodies under normal conditions. However, Fourier's law does not apply to all materials (especially at low temperatures < 15K) and, as noted by *L*. Onsager in [2], it can be considered only as an "approximate description of thermal conductivity which neglects the time required to accelerate the heat flow", because it supposes an instantaneous heat flow increase simultaneously at all points. The most significant physical phenomenon that goes beyond Fourier's law is the so-called second sound [3], first discovered during experimental studies of crystals in which heat pulses propagate [4-7]. To describe this phenomenon, a phonon mechanism of thermal conductivity and corresponding Cattaneo equation are proposed [8]:

$(\tau\cdot\partial_t^2+\partial_t)T=D_T\cdot\nabla^2 T$, where D_T – thermal conductivity of the material, τ – relaxation time, T – temperature, ∇^2 – Laplace operator, t – time. According to this theory, temperature fluctuations propagate as attenuated waves in a medium with the velocity $v_T=\sqrt{D_T/\tau}$. The Cattaneo model supposes a finite rate of heat flow increase that follows a change in temperature at the boundary of the area. This delay is characterized by time after the appearance of the temperature gradient, which in its turn reflects the properties of the medium and describes the temporal relation between the beginning of the temperature change and the reaction of the heat flow to this change. The time required to begin the heat transfer is associated with phonon interactions that transfer heat and are a measure of the medium's thermal inertia. However, the Cattaneo model has contradictions of both physical and mathematical nature [9-13]. In addition, the Cattaneo equation, although qualitatively describing the second sound, gave incorrect values for the velocity of heat wave propagation $\sqrt{D_T/\tau}$, which differ from the experimental data on the propagation of heat pulses in extremely pure nonmetallic Bi and NaF crystals at low temperatures.

Therefore, the modeling of heat transfer processes with finite velocity in materials requires further comprehensive research and improvement.

2. Literature review and problem statement

The most common Guyer-Krumhansl model [14] replaced the Cattaneo model and came down to the corresponding one-dimensional thermal conductivity equation in the one-dimensional case [15]:

$$\begin{cases}\left(\frac{\partial^2}{\partial t^2}+\varepsilon\frac{\partial}{\partial t}-\delta\frac{\partial^3}{\partial t\partial x^2}\right)\cdot F(x,t)=\left(\alpha\frac{\partial^2}{\partial x^2}+\beta^2\right)\cdot F(x,t),\\ (\alpha,\varepsilon,\delta,\beta)=const.\end{cases} \tag{1}$$

The linear component $k\neq 0$ means the presence of sources in the equation (1) and the model proposed by Guyer and Krumhansl assumes that $k=0$. The Guyer-Krumhansl model complements the Fourier heat diffusion and heat wave propagation with another heat transfer component acting on scales *L,* which are of the same order as the average free path length of phonons *l* [16, 17]. Indeed, the theories of Fourier [1] and Casimir [18] are insufficient to describe the heat transfer which depends not only on the collisions between phonons, but also on the interaction of phonons at the boundary of the medium. These so-called ballistic conditions (when the free path length of phonons is of the same order with the size of the whole system) are actually observed in structures of small dimension, i.e. in ultrathin films or fibers of capillary-porous bodies. Studies of ballistic thermal conductivity have recently been a central preoccupation of scientists and have conducted mainly using numerical methods [19-26]. Understanding whether heat transfer is determined by ballistic conditions in each case is important from a practical standpoint, because ballistic phonon heat transfer requires an assessment of conditions at the boundary of the medium and depends on them, not only on the properties of the medium itself [17, 25, 27].

At the same time, it was shown that the Guyer-Krumhansl equation also describes the thermal conductivity in macroscopic three-dimensional objects with significant internal inhomogeneity [28, 29]. Other studies [30-35] confirmed that the heat transfer in substantially inhomogeneous media at room temperature is not described by Fourier and Cattaneo laws, but rather modeled by Guyer-Krumhansl-type equations [19, 21, 28] despite the absence of actual ballistic conditions $l\approx L$. Similar results were obtained during studies of short heat pulses' propagation [36] at normal temperature in porous/capillary-porous materials. These results were considered afterward in a more general context in [37]. Thus, obtaining solutions and studying the Guyer-Krumhansl equation are an urgent task with many practical applications.

Using the methods of thermodynamics of irreversible processes authors [38, 39] created a theory of interrelated heat and mass transfer during phase transformations. The obtained system of differential equations in the one-dimensional case (taking into account the finiteness of heat propagation rates and mass transfer) is as follows:

$$\begin{cases} \dfrac{\partial T}{\partial t}+\tau_{pt}\dfrac{\partial^2 T}{\partial t^2}=\dfrac{\partial}{\partial x}\left(\dfrac{\lambda}{c\rho_0}\cdot\dfrac{\partial T}{\partial x}\right)+\psi\cdot\rho_0\cdot Q\cdot\dfrac{\partial u}{\partial t}, \\ \dfrac{\partial u}{\partial t}+\tau_{pm}\dfrac{\partial^2 u}{\partial t^2}=\dfrac{\partial}{\partial x}\left(a_m\cdot\dfrac{\partial u}{\partial x}\right)+\dfrac{\partial}{\partial x}\left(a_m\cdot\delta\cdot\dfrac{\partial T}{\partial x}\right), \end{cases} \tag{2}$$

where T – temperature, u – moisture content, τ_{pt}, τ_{pm} – temperature and mass (moisture) transfer relaxation time respectively, t – time, λ – heat-conductivity coefficient, ρ_0 – dry body density, c – specific heat capacity, $\lambda/(c\rho_0)=a$ – temperature conductivity coefficient, ψ – phase transition criterion which characterizes the ratio of changes in moisture content due to the evaporation and total change in moisture content, Q – evaporation heat. It is assumed in this study that a_m – moisture diffusivity, σ – thermal-gradient coefficient, as well as a – temperature conductivity coefficient does not depend on the spatial coordinate x.

In addition, the situation when $\psi \to 0$ further is being considered. As a result, the system of equations (2) takes a simplified form:

$$\begin{cases} \dfrac{\partial T}{\partial t}+\tau_{pt}\dfrac{\partial^2 T}{\partial t^2}=a\dfrac{\partial^2 T}{\partial x^2}, \\ \dfrac{\partial u}{\partial t}+\tau_{pm}\dfrac{\partial^2 u}{\partial t^2}=a_m\dfrac{\partial^2 u}{\partial x^2}+a_m\cdot\delta\cdot\dfrac{\partial^2 T}{\partial x^2}. \end{cases} \tag{3}$$

It is pointed out that the last term in the right part of the second equation of system (3) describes the thermal diffusion of moisture process. The system of equations (3), as well as (2), plays an important role in the study of the drying process of wet materials.

For example, use of the O.V. Lykov's system of equations (2) or (3) allows to solve the two-dimensional problem of non-isothermal moisture transfer in the wood material together with the equation of moisture elasticity. As a result, the subsurface stresses during drying and limit strength values under the assumption of orthotropic structure of wood material are determined. Studies showed that the effect of thermal conductivity negatively affected the duration of the wood drying process.

Another example is the modeling of non-stationary interrelated processes of heat and moisture transfer in plant materials under combined power supply in parallel to a constant and impulse ultra-high frequency (UHF) exposure (as well as when irradiated with electromagnetic waves of the millimeter range with a carrier frequency f = 50…60 GHz). It is shown that the creation of pulse and pulse-step UHF/EHF (extremely high frequency range corresponding to mm range of electromagnetic waves) modes allows to reduce the temperature effect on the processed material. Kinetic dependences with respect to the finite rate of moisture transfer are established.

The relaxation time of thermal stress during the heat propagation in metals is ~ 10^{-9} s. The heat distribution rate is of the same order as the sound velocity.

Due to the short relaxation time, the solution of the hyperbolic equation of thermal conductivity almost coincides with the solution of the classical parabolic equation (but it is not true for short and ultrashort thermal pulses). The moisture diffusion rate in capillary-porous materials is approximately 10^6…10^7 and more times lower, accordingly, the relaxation time of diffusion processes is much longer, so it should be taken into account in the equations of mass transfer [38-42]. The relaxation time of the mass is related to the moisture conductivity coefficient by the ratio $\tau_{pm}=a_m/v_m^2$, where v_m – mass propagation velocity.

3. The aim and objectives of the study

The objective of this paper is to establish hyperbolic models for the analysis of heat and moisture exchange in inhomogeneous porous materials during the propagation of short heat pulses.

To achieve this aim, it is necessary to solve the following tasks:

– to solve the O.V. Lykov`s system of equations of interrelated heat and mass transfer taking into account the finiteness of heat and mass (moisture) transfer rates;

– to obtain analytical solutions of the one-dimensional Guyer-Krumhansl equation;

– to model the heat and moisture propagation in thin films of capillary-porous bodies with account taken of molecular effects in systems of reduced dimension.

4. Investigation of the solutions to a system of differential equations describing an interconnected heat and mass transfer processes in capillary-porous materials

Methods of mathematical physics together with the operator approach to the solution of differential equations [43-45] and Laplace transforms allow to obtain an accurate analytical description of the system's thermal behavior taking into account similar effects at different values of its parameters [46-48]. For this purpose, the equation (1) should be written as shown below:

$$\left(\tau\cdot\frac{\partial^2}{\partial t^2}+\frac{\partial}{\partial t}\right)\cdot F(x,t)=\left(k_b\cdot\frac{\partial^3}{\partial t\partial x^2}+k_T\cdot\frac{\partial^2}{\partial x^2}+\mu\right)\cdot F(x,t), \tag{4}$$

where $\tau=1/\varepsilon$, $\mu=\beta^2/\varepsilon$, $k_T=\alpha/\varepsilon$ – Fourier-type thermal conductivity, and $k_b=\sigma/\varepsilon$ – thermal conductivity of ballistic type. If compared (4) to the first equation of system (3), then:

$$F(x,t)=T(x,t),\quad \tau=\tau_{pt},\quad k_T=a,\quad k_b\equiv 0,\quad \mu\equiv 0. \tag{5}$$

Therefore, the natural generalization of the first equation of system (3) is the following (taking into account ballistic effects):

$$\frac{\partial T}{\partial t}+\tau_{pt}\frac{\partial^2 T}{\partial t^2}=\left(a\cdot\frac{\partial^2}{\partial x^2}+k_b\cdot\frac{\partial^3}{\partial t\partial x^2}+\mu\right)\cdot T, \tag{6}$$

where $\mu\neq 0$ means that heat sources are available in the system (3).

It should first be focused on the exact solutions of equation (6).

5. Exact solutions of Guyer-Krumhansl-type equation

It should be noted that the equation (6) is a particular case of a more general equation of the following type:

$$\left(\frac{\partial^2}{\partial t^2}+\hat{\varepsilon}(x)\cdot\frac{\partial}{\partial t}\right)\cdot F(x,t)=\hat{D}(x)\cdot F(x,t)\,, \tag{7}$$

where $\hat{D}(x)$ – differential operator affecting the x coordinate.

A partial solution of equation (7) is obtained after applying Laplace transforms, provided that the integral coincides:

$$F(x,t)=C\cdot\exp\left[-\frac{t}{2}\cdot\hat{\varepsilon}(x)\right]\cdot\frac{t}{4\pi}\int_0^\infty\frac{d\xi}{\xi\sqrt{\xi}}\cdot\exp\left(-\frac{t^2}{16\xi}\right)\times\exp\left[-\xi\cdot\hat{\varepsilon}^2(x)\right]\cdot\exp\left[-4\xi\cdot\hat{D}(x)\right]\cdot f(x), \tag{8}$$

where $F(x,0)=f(x)$ – initial condition, and C – a constant determined from another initial or boundary condition.

In particular for the operator $\hat{D}(x)=\left(\alpha\cdot\partial_x^2+\beta\right)\equiv\left(\alpha\cdot\partial_x^2+\mu\right)$, a partial solution of the telegraph equation (TE) is obtained:

$$\begin{cases}\left(\dfrac{\partial^2}{\partial t^2}+\varepsilon\dfrac{\partial}{\partial t}\right)\cdot F(x,t)=\left(\alpha\cdot\dfrac{\partial^2}{\partial x^2}+\beta\right)\cdot F(x,t),\\ (\varepsilon,\alpha,k)=const,\end{cases} \tag{9}$$

or:

$$\begin{cases}\left(\dfrac{\partial^2}{\partial t^2}+\dfrac{1}{\tau_{pT}}\cdot\dfrac{\partial}{\partial t}\right)\cdot T(x,t)=\left(\dfrac{\alpha}{\tau_{pT}}\cdot\dfrac{\partial^2}{\partial x^2}+\dfrac{\mu}{\tau_{pT}}\right)\cdot T(x,t),\\ \varepsilon\equiv\dfrac{1}{\tau_{pT}},\quad \alpha\equiv\dfrac{\alpha}{\tau_{pT}}\,,\quad \beta\equiv\dfrac{\mu}{\tau_{pT}}.\end{cases} \tag{10}$$

The equation of (9) or (10) type is also called a hyperbolic equation of thermal conductivity with an absolute term. The solution (8) of equation (9) or (10) attenuating on a semi-infinite interval of time at

$F(x,0)=f(x)$, $F(x,\infty)=0$ (as a reminder $F(x,t)\equiv T(x,t)$) is one of the branches of the general solution:

$$F(x,t)=\exp\left(-\frac{t\cdot\varepsilon}{2}\right)\left\{C_1(x)\cdot\exp\left[-\frac{t}{2}\cdot\sqrt{\varepsilon^2+4\widehat{D}(x)}\right]+C_2(x)\cdot\exp\left[+\frac{t}{2}\cdot\sqrt{\varepsilon^2+4\widehat{D}(x)}\right]\right\}, \quad (11)$$

where $\widehat{D}(x)=\alpha\cdot\partial_x^2+\beta$ and $C_{1,2}(x)$ are determined by boundary conditions.

Bounded solution of equation (1) or (4) or (6) at $t\to\infty$ with the initial condition $F(x,0)=f(x)$ can be obtained by the method of operators using the technique developed in [41, 42,49]. It can be written as shown below:

$$F(x,t)=\frac{\exp(-(t/2)\cdot\varepsilon)\cdot t}{4\pi}\cdot\int_0^\infty\frac{d\xi}{\xi\sqrt{\xi}}\cdot\exp\left\{-\frac{t^2}{16\xi}-\xi\cdot\left(\varepsilon^2+4\beta^2\right)\right\}\times\int_0^\infty\exp(-\zeta^2)\cdot\widehat{S}f(x)d\zeta,$$
$$\widehat{S}=\exp\left\{\left(\frac{t\cdot\delta}{2}-4\xi\cdot a+2\xi\cdot\varepsilon+i\cdot2\sqrt{\xi}\cdot\delta\cdot\zeta\right)\partial_x^2\right\},\quad i^2=-1. \quad (12)$$

Thermal conductivity operator $\widehat{S}$:

$$\widehat{S}=\exp\left(\alpha\cdot t\cdot\partial_x^2\right) \quad (13)$$

gives a solution of the Fourier thermal conductivity equation:

$$\partial_t F(x,t)=\alpha\cdot\partial_x^2F(x,t), \quad (14)$$

using the Gauss transforms of initial condition $F(x,0)=f(x)$ [40, 43, 49]:

$$F(x,t)=\widehat{S}f(x)=\frac{1}{2\sqrt{\pi t\alpha}}\cdot\int_{-\infty}^{\infty}\exp\left\{-\frac{(x-\xi)^2}{4t\alpha}\right\}f(\xi)d\xi. \quad (15)$$

Thus, for the initial distribution $f(x)=\exp(-x^2)$:

$$\widehat{S}\exp(-x^2)\equiv\exp\left(\alpha\cdot t\cdot\partial_x^2\right)\cdot\exp(-x^2)=\frac{1}{\sqrt{1+4\alpha t}}\cdot\exp\left(-x^2/(1+4\alpha t)\right) \quad (16)$$

and for Dirac σ-function:

$$\widehat{S}\delta(x)\equiv\exp\left(\alpha\cdot t\cdot\partial_x^2\right)=\frac{1}{2\sqrt{\pi t\alpha}}\cdot\exp\left(-x^2/4\alpha t\right). \quad (17)$$

For the equation of thermal conductivity in the form (10) while neglecting the finiteness of heat propagation (its propagation velocity):

$$\alpha\equiv a,\ \ \overline{f}(x)=\exp\left(-K\cdot x^2\right)\cdot q_{2m}, \quad (18)$$

where $\overline{f}(x)$ – distribution of specific heat flow in the direction *OX*-axis, W/m^2; *K* – coefficient of heat flow concentration of the source, 1/m^2; q_{2m} – the highest heat flow in the center of the hot spot, W/m^2. For this case:

$$T(x,t)=q_{2m}\cdot\frac{1}{\sqrt{1+4a\cdot K\cdot t}}\cdot\exp\left(-\frac{K\cdot x^2}{1+4\alpha\cdot K\cdot t}\right). \quad (19)$$

If a heat source enters into the material (e.g. plate) *q* J of heat every second, then it can be written as shown below:

$$q=\int_0^\infty q_{2m}\cdot e^{(-K\cdot x^2)}dx=q_{2m}\int_0^\infty e^{(-K\cdot x^2)}dx=\frac{\sqrt{\pi}\cdot q_{2m}}{2\sqrt{K}}\cdot\Phi, \quad (20)$$

where $\Phi(z)=\left(2/\sqrt{\pi}\right)\int_0^z\exp(-z^2)dz$ – probability integral.

For Dirac σ-function:

$$\widehat{S}\delta(x)=\exp\left(\alpha\cdot t\cdot\partial_x^2\right)\delta(x)=\frac{1}{2\sqrt{\pi\alpha t}}\cdot\exp\left(-\frac{x^2}{4\alpha t}\right). \quad (21)$$

In the context of thermal conductivity, the initial condition $F(x,0)=\sigma(x)$ simulates the propagation of an instantaneous laser pulse in a material. This experimental technique is well-established for determining the thermal conductivity of the substance [43, 50] and allows to make "sensitive" measurements, that is why obtaining accurate solutions if $F(x,0)=\sigma(x)$ has a significant practical value.

Exact bounded by $t\to\infty$ solution of Guyer-Krumhansl-type equation (1), (4) or (6) ($\mu\not\equiv 0$) with the initial condition $f(x)=\sigma(x)$ is written as follows:

$$F(x,t)\Big|_{f(x)=\delta(x)}=\frac{\exp\left(-\frac{t}{2}\cdot\varepsilon\right)\cdot t}{8\pi^{\frac{3}{2}}}\cdot\int_0^{\infty}\frac{d\xi}{\xi\sqrt{\xi}}\cdot\exp\left\{-\frac{t^2}{16\xi}-\xi\cdot\left(\varepsilon^2+4\beta^2\right)\right\}\times\int_{-\infty}^{\infty}\frac{\exp\left[-\zeta^2-\frac{x^2}{4\left(\bar{a}+ib\zeta\right)}\right]d\zeta}{\sqrt{\left|\bar{a}+ib\zeta\right|}},\quad(22)$$

where

$$\hat{S}\delta(x)=\exp\left(\alpha\cdot t\cdot\partial_x^2\right)\delta(x)=\frac{1}{2\sqrt{\pi\alpha t}}\cdot\exp\left(-\frac{x^2}{4\alpha t}\right).\quad(23)$$

It can be written for equation (6) in a similar way:

$$T(x,t)\Big|_{f(x)=\delta(x)}=\frac{\exp\left(-\frac{t}{2}\cdot\frac{1}{\tau_{pT}}\right)\cdot t}{8\pi^{\frac{3}{2}}}\times\int_0^{\infty}\frac{d\xi}{\xi\sqrt{\xi}}\cdot\exp\left\{-\frac{t^2}{16\xi}-\xi\cdot\left(\frac{1}{\tau_{pT}^2}+\frac{4\mu^2}{\tau_{pT}^2}\right)\right\}\cdot\int_{-\infty}^{\infty}\frac{\exp\left[-\zeta^2-\frac{x^2}{4\left(\tilde{a}+i\tilde{b}\zeta\right)}\right]d\zeta}{\sqrt{\left|\tilde{a}+i\tilde{b}\zeta\right|}},\quad(24)$$

where

$$\tilde{a}=t\cdot k_b\cdot\frac{1}{\tau_{pT}}\cdot\frac{1}{2}-4\xi\cdot\frac{a}{\tau_{pT}}+2\xi\cdot\frac{1}{\tau_{pT}^2}\cdot k_b,\quad\tilde{b}=2\sqrt{\xi}\cdot k_b\cdot\frac{1}{\tau_{pT}}.\quad(25)$$

If the function of the initial condition can be approximated by the polynomial $f(x)=\sum_n x^n$, the solution will be as follows:

$$F(x,t)\Big|_{f(x)=\sum_n x^n}=\sum_n\frac{\exp(-\frac{t}{2}\cdot\varepsilon)\cdot t}{4\pi}\cdot\int_0^{\infty}\frac{d\xi}{\xi\sqrt{\xi}}\cdot\exp\left\{\frac{t^2}{16\xi}-\xi\cdot(\varepsilon^2+4\beta^2)\right\}\times\int_{-\infty}^{\infty}\exp(-\zeta^2)\cdot H_n\left(x,\bar{a}+2\cdot i\cdot\delta\cdot\zeta\cdot\sqrt{\xi}\right)d\zeta,\quad(26)$$

where $H_n(x,y)$ – Hermite polynomials of two variables [44 - 46]:

$$\begin{cases}H_n(x,y)=\exp\left(y\cdot\frac{\partial^2}{\partial x^2}\right)\cdot x^n=n!\cdot\frac{x^{n-2r}\cdot y^r}{(n-2r)!r!},\\ \sum_{n=0}^{\infty}\frac{t^n}{n!}H_n(x,y)=\exp\left(xt+yt^2\right),\end{cases}\quad(27)$$

where $[n/2]$ – means the quotient of $n/2$.

Non-periodic solutions of equation (10) with $\mu\equiv 0$ can be further found using the paper's approach [47], i.e.:

$$\frac{a}{\tau_{pT}}\cdot\frac{\partial^2}{\partial x^2}\{T(x,t)\}-\frac{\partial^2}{\partial t^2}\{T(x,t)\}=\frac{1}{\tau_{pT}}\cdot\frac{\partial}{\partial t}\{T(x,t)\}.\quad(28)$$

The equation (28) is given as follows:

$$\frac{\partial^2}{\partial x^2}\{T(x,t)\}-\frac{1}{C_1^2}\cdot\frac{\partial^2}{\partial t^2}\{T(x,t)\}=\frac{1}{a}\cdot\frac{\partial}{\partial t}\cdot\{T(x,t)\},\quad(29)$$

where $C_1^2=a/\tau_{pT}$, C_1 – propagation velocity of thermal excitation (pulse) in the medium.

Non-periodic ("integral" according to Fourier method) solutions of the equation (29) can be found.

It is necessary to enter the characteristic relaxation time of thermal field τ_{char}:

wave number $\tilde{k}$:

$$\tilde{k}=\frac{\omega}{C_1}\cdot\sqrt{1+2i(\omega\cdot\tau_{char})^{-1}}, \tag{31}$$

where ω –characteristic circular frequency of the short heat pulse. Along with the solution (31) and the solution represented as path integrals, the equation (29) also describes non-sinusoidal thermal fields in the time-space.

Using normalized variables:

$$T=T_0\cdot\bar{f},\quad \bar{\tau}=t\cdot\tau_{char}^{-1},\quad \eta=x\cdot(C_1\cdot\tau_{char})^{-1}, \tag{32}$$

the equation (29) can be rewritten in dimensionless form:

$$\frac{\partial^2\bar{f}}{\partial\eta^2}-\frac{\partial^2\bar{f}}{\partial\bar{\tau}^2}=2\cdot\frac{\partial\bar{f}}{\partial\tau}, \tag{33}$$

Exact analytical solutions of the dimensionless telegraph equation (33) for describing variable non-periodic thermal fields are presented as follows [47, 48]:

$$\bar{f}=\sum_{\tilde{q}}a_{\tilde{q}}\cdot\overline{\overline{f_{\tilde{q}}}}, \tag{34}$$

$$\overline{\overline{f_{\tilde{q}}}}=\frac{1}{2}\left(\Theta_{\tilde{q}-1}+\Theta_{\tilde{q}+1}-2\Theta_{\tilde{q}}\right)=\frac{\partial\Theta_{\tilde{q}}}{\partial\bar{\tau}}, \tag{35}$$

$$\Theta_{\tilde{q}}=\exp(-\bar{\tau})\cdot\left(\frac{\bar{\tau}-\eta}{\bar{\tau}+\eta}\right)^{\frac{q}{2}}\cdot I_{\tilde{q}}\left(\sqrt{\bar{\tau}^2-\eta^2}\right),\quad \bar{\tau}=\eta. \tag{36}$$

$I_{\tilde{q}}$ is modified Bessel function in the equation (36); the index $\tilde{q}$ is determined by the boundary conditions on the surface of the deformed material $\eta=0$.

The characteristic properties of integral functions (34) describing thermal fields in deformed media are confined largely to the following:

1) $\Theta_{\tilde{q}}(\bar{\tau},\eta)\big|_{\bar{\tau}=\eta}=0,\ \tilde{q}>0$; (37)

2) using the known asymptotics of functions:

$$I_{\tilde{q}}(u)-I_{\tilde{q}}(u)\Big|_{u=1}=\frac{\exp(-u)}{\sqrt{2\pi u}}\cdot\sum_{n=0}^{\infty}\frac{(-1)^n}{(2\pi)^n}\pi\cdot\frac{\Gamma(\tilde{q}+(1/2)+n)}{\Gamma(\tilde{q}+(1/2)-n)}, \tag{38}$$

where Γ is gamma function, law of decrease of the thermal field $\overline{\overline{f_{\tilde{q}}}}$ (35) can be found in any section at $\bar{\tau}=\eta$:

$$\overline{\overline{f_{\tilde{q}}}}\Big|_{\bar{\tau}=\eta}=-\frac{1}{2\sqrt{2\pi}}\cdot(\bar{\tau})^{-3}. \tag{39}$$

Values of function $\overline{\overline{f_{\tilde{q}}}}(\bar{\tau})$ (35), which characterizes the law of decrease of the thermal field in any section of the material / body during its processing by short wave pulses $(\bar{\tau}=\eta)$ are given in Table 1.

Table 1

Dependence $\overline{\overline{f}}_{\tilde{q}}$ on $\bar{\tau}$ $(\bar{\tau}=\eta)$

τ	$-\overline{\overline{f}}_{\tilde{q}}$	τ	$-\overline{\overline{f}}_{\tilde{q}}$	τ	$-\overline{\overline{f}}_{\tilde{q}}$
0,1	200	1	0,2	50	$1{,}6\cdot10^{-6}$
0,2	25	10	$2\cdot10^{-4}$	100	$2{,}0\cdot10^{-7}$
0,3	7,4	20	$2{,}5\cdot10^{-4}$	-	-
0,5	1,6	30	$7{,}4\cdot10^{-6}$	-	-
0,8	0,4	40	$3{,}1\cdot10^{-6}$	-	-

6. Analysis of solutions of Guyer-Krumhansl equation for the plate with the finite thickness $(\bar{\delta})$. Using the Fourier method

The solution of equation (4) at $\mu \equiv 0$ after simple transformations takes the following form:

$$\left(\frac{\partial^2}{\partial t^2}+\frac{1}{\tau_{pT}}+\frac{\partial}{\partial t}\right)\cdot T=\left(\frac{a}{\tau_{pT}}\cdot\frac{\partial^2 T}{\partial x^2}\right)+\left(\frac{k_b}{\tau_{pT}}\right)\cdot\frac{\partial^3 T}{\partial t \partial x^2}\,, \tag{40}$$

as the known method of variables dividing (Fourier method) is used.

The required solution (40) is presented in the following form:

$$T(x,t)=T(t)\cdot X(x), \tag{41}$$

and as object for which the temperature field $T(x,t)$ is found the plate of finite thickness $\bar{\delta}$ is chosen (fig. 1).

Boundary conditions however are as follows:

$$\left(\frac{\partial^2}{\partial t^2}+\frac{1}{\tau_{pT}}+\frac{\partial}{\partial t}\right)\cdot T=\left(\frac{a}{\tau_{pT}}\cdot\frac{\partial^2 T}{\partial x^2}\right)+\left(\frac{k_b}{\tau_{pT}}\right)\cdot\frac{\partial^3 T}{\partial t \partial x^2}\,, \tag{42}$$

$$-\lambda_0\cdot\left(\frac{\partial T}{\partial t}\right)=-\alpha_1\cdot T,\ \text{at}\ x=0;\qquad \lambda_0\cdot\left(\frac{\partial T}{\partial t}\right)=-\alpha_2\cdot T,\ \text{at}\ x=\bar{\delta}\,; \tag{43}$$

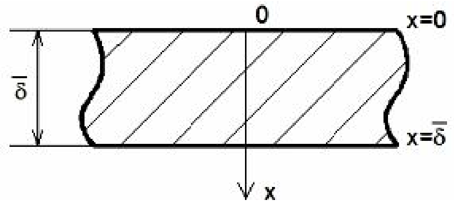

Fig. 1. The plate
for which the temperature field is determined

and the following is chosen for initial conditions of the problem:

$$T\big|_{t=0}=T_0\,,\ x=0\,;\ T\big|_{t=0}=T_{\bar{\delta}}\,,\ x=\bar{\delta}\,,\ \dot{T}\big|_{t=0}=\dot{T}_0\,,$$
$$x=0\,,\ \dot{T}\big|_{t=0}=\dot{T}_{\bar{\delta}}\,,\ x=\bar{\delta}\,. \tag{44}$$

The following notations are used in relations (43): λ_0 – thermal conductivity of the material (at $T=0$), $\alpha_{1,2}$ – heat-exchange/heat-removal coefficients of surfaces $x=0$ and $x=\bar{\delta}$, respectively. In relations (43) T_0, $T_{\bar{\delta}}$, $\dot{T}_0$, $\dot{T}_{\bar{\delta}}$ – constants (a point above the function means the time differentiation t).

Functions $X(x)$ in (41) are found using the following relations:

$$X_n(x)=C_{1n}\cdot\sin(\lambda_n\cdot x)+C_{2n}\cdot\cos(\lambda_n\cdot x)\ ,\ (C_{1n},C_{2n})=const\ , \tag{45}$$

and eigenvalues λ_n are found using transcendental equation:

$$\text{ctg}\left\{\lambda_n\cdot\bar{\delta}\right\}=\frac{\lambda_n^2-\alpha_1\cdot\alpha_2/\lambda_0^2}{\lambda_n\cdot(\alpha_1+\alpha_2)/\lambda_0}\,,\ n=1,2,3\ldots. \tag{46}$$

For function $\tilde{T}(t)$:

$$T_n(t)=\left\{A_{1n}\cdot e^{(-\gamma_n\cdot t)}\cdot\sin(\Omega_n\cdot t)+A_{2n}\cdot e^{(-\gamma_n\cdot t)}\cdot\cos(\Omega_n\cdot t)\right\}\,, \tag{47}$$

where A_{1n}, A_{2n} – constants, and γ_n and Ω_n are found using the following relations:

$$\begin{cases}\gamma_n=\dfrac{1}{2}\cdot\left\{\dfrac{1}{\tau_{pT}}+\lambda_n^2\cdot\left(\dfrac{k_b}{\tau_{pT}}\right)\right\}, \quad n=1,2,3,\ldots; \\[2ex] \Omega_n=\left\{\lambda_n^2\cdot\dfrac{a}{\tau_{pT}}-\dfrac{1}{4}\cdot\left[\dfrac{1}{\tau_{pT}}+\lambda_n^2\cdot\left(\dfrac{k_b}{\tau_{pT}}\right)\right]^2\right\}^{1/2}, \quad \Omega_n>0, \quad n=1,2,3,\ldots.\end{cases} \tag{48}$$

The general solution (40) can be presented as follows:

$$T(x,t)=\sum_{n=1}^{\infty}\tilde{T}(t)\cdot X_n(x)=\sum_{n=1}^{\infty}\left\{S_{1n}\cdot e^{(-\gamma_n\cdot t)}\cdot\sin(\Omega_n\cdot t)\cdot\sin(\lambda_n\cdot x)+S_{2n}\cdot e^{(-\gamma_n\cdot t)}\cdot\sin(\Omega_n\cdot t)\cdot\cos(\lambda_n\cdot x)+\right.$$

$$+S_{3n}\cdot e^{(-\gamma_n\cdot t)}\cdot\cos(\Omega_n\cdot t)\cdot\sin(\lambda_n\cdot x)+S_{4n}\cdot e^{(-\gamma_n\cdot t)}\cdot\cos(\Omega_n\cdot t)\cdot\cos(\lambda_n\cdot x)\Big\}, \quad (49)$$

and constants $S_{1n},S_{2n},S_{3n},S_{4n}$ are found using initial conditions (43) and orthogonality of functions $\{\sin(\lambda_n\cdot x),\cos(\lambda_n\cdot x)\}$ realized on the interval $x\in[0,\bar{\delta}]$.

7. Harmonic solutions of telegraph equation

Considering the evolution of the harmonic function $\exp(i\cdot n\cdot x)$, $i^2=-1$ in the equation (10) at $\mu\equiv 0$. This result is of interest for any function which can be expanded in a Fourier series. The operator $\exp\{t\cdot\partial_x^2\}$ action examined in [40-42], the exponential differential operator $\exp\{\hat{D}(x)\}$ does not add the new harmonics to the existing ones at $t=0$. There are the following initial conditions:

$$T(x,t)\big|_{t=0}=Ge^{inx},\quad \left.\frac{\partial T(x,t)}{\partial t}\right|_{t=0}=Be^{inx}. \quad (50)$$

In a similar way to (41), the following solution of the telegraph equation is obtained using (10) at $\mu\equiv 0$:

$$\begin{cases} T(x,t)\big|_{T(x,t)\sim\exp(inx)}=B_1\exp\left\{inx-\frac{t}{2}\cdot\left(\frac{1}{\tau_{pT}}+\sqrt{V}\right)\right\}+B_2\exp\left\{inx-\frac{t}{2}\cdot\left(\frac{1}{\tau_{pT}}-\sqrt{V}\right)\right\},\\ V=\frac{1}{\tau_{pT}^2}+4\left(-\frac{a}{\tau_{pT}}\cdot n^2\right), \end{cases} \quad (51)$$

where the coefficients B_1,B_2 are expressed in terms of initial conditions at $t=0$:

$$B_1+B_2=G;\quad B_1\cdot\left(\frac{1}{\tau_{pT}}+\sqrt{V}\right)+B_2\cdot\left(\frac{1}{\tau_{pT}}-\sqrt{V}\right)=-2B\ , \quad (52)$$

Therefore:

$$B_1=\frac{-2B+G\cdot\left(\frac{1}{\tau_{pT}}+\sqrt{V}\right)}{2\sqrt{V}},\quad B_2=\frac{2B+G\cdot\left(\frac{1}{\tau_{pT}}+\sqrt{V}\right)}{2\sqrt{V}}. \quad (53)$$

8. Harmonic solutions of interrelated heat and mass exchange equations in thin capillary-porous films

The O.V. Lykov system of equations describes the processes of heat and mass exchange in thin capillary-porous films. In this respect, the finiteness of the of heat propagation velocity and the ballistic effects of thermal conductivity are taken into account in the equation for $T(x,t)$. In view of the above, this system of equations has the following form:

$$\begin{cases} \frac{\partial T}{\partial t}+\tau_{pt}\frac{\partial^2 T}{\partial t^2}=a\cdot\frac{\partial^2 T}{\partial x^2}+k_b\cdot\frac{\partial^2 T}{\partial t\partial x^2},\\ \frac{\partial u}{\partial t}+\tau_{pm}\frac{\partial^2 u}{\partial t^2}=a_m\cdot\frac{\partial^2 u}{\partial x^2}+a_m\cdot\delta\cdot\frac{\partial^2 T}{\partial x^2}. \end{cases} \quad (54)$$

The solution of the first equation of system (54) is found under the following periodic harmonic conditions:

$$T(x,t)\big|_{t=0}=Ge^{inx},\quad \left.\frac{\partial T(x,t)}{\partial t}\right|_{t=0}=Be^{inx},\quad i^2=-1. \quad (55)$$

$$\begin{cases} T(x,t)=C_1\exp\left\{inx-\frac{t}{2}\cdot\left(\frac{1}{\tau_{pT}}+n^2\cdot k_b\cdot\frac{1}{\tau_{pT}}\sqrt{U}\right)\right\}+C_2\exp\left\{inx-\frac{t}{2}\cdot\left(\frac{1}{\tau_{pT}}+n^2\cdot k_b\cdot\frac{1}{\tau_{pT}}-\sqrt{U}\right)\right\}, \\ U=\left(\frac{1}{\tau_{pT}}+n^2\cdot k_b\cdot\frac{1}{\tau_{pT}}\right)^2-4\frac{a}{\tau_{pT}}. \end{cases} \quad (56)$$

The solution (56) completely defines the dependence $T(x,t)$ under the conditions (55). In this respect, the constants C_1,C_2 are found using the following relations:

$$\begin{cases} C_1=\dfrac{-2B+G\cdot\left(n^2\cdot k_b\cdot\frac{1}{\tau_{pT}}+\frac{1}{\tau_{pT}}-\sqrt{U}\right)}{2\sqrt{U}}; \\ C_2=\dfrac{2B+G\cdot\left(n^2\cdot k_b\cdot\frac{1}{\tau_{pT}}+\frac{1}{\tau_{pT}}+\sqrt{U}\right)}{2\sqrt{U}}. \end{cases} \quad (57)$$

The solution of the second equation of system (54) is given as follows:

$$u(x,t)=u_{on}(x,t)+u_{part}(x,t)\ , \quad (58)$$

the boundary conditions for $u(x,t)$ however have the following form:

$$u(x,t)\big|_{t=0}=G_1\cdot e^{inx};\quad \left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0}=B_1\cdot e^{inx}\ . \quad (59)$$

For $u_{on}(x,t)$, the following relations can be written as follows:

$$\begin{cases} u(x,t)\big|_{t=0}=\bar{C}_1\exp\left\{inx-\frac{t}{2}\cdot\left(\frac{1}{\tau_{pm}}+\sqrt{U_1}\right)\right\}+\bar{C}_2\exp\left\{inx-\frac{t}{2}\cdot\left(\frac{1}{\tau_{pm}}-\sqrt{U_1}\right)\right\}, \\ U_1=\frac{1}{\tau_{pm}^2}-4\frac{a_m}{\tau_{pm}}. \end{cases} \quad (60)$$

The constants $\bar{C}_1$ and $\bar{C}_2$ can be found in the boundary conditions (59) using the following formulae:

$$\bar{C}_1=\frac{2B_1+G_1\cdot\left(\frac{1}{\tau_{\mathrm{pm}}}-\sqrt{U_1}\right)}{2\sqrt{U_1}},\quad \bar{C}_2=\frac{2B_1+G_1\cdot\left(\frac{1}{\tau_{pm}}+\sqrt{U_1}\right)}{2\sqrt{U_1}}. \quad (61)$$

the solution $u_{part}(x,t)$ can be found in the following form:

$$u_{part}(x,t)=A_1\exp\left\{inx-\frac{t}{2}\cdot\left(\frac{1}{\tau_{pT}}+n^2\cdot k_b\cdot\frac{1}{\tau_{pT}}+\sqrt{U}\right)\right\}+A_2\exp\left\{inx-\frac{t}{2}\cdot\left(\frac{1}{\tau_{pT}}+n^2\cdot k_b\cdot\frac{1}{\tau_{pT}}-\sqrt{U}\right)\right\}. \quad (62)$$

The constants A_1 and A_2 are found using the following relations:

$$A_1\cdot\left\{\left(-\frac{1}{2}\right)\cdot\left(\frac{1}{\tau_{pT}}+n^2\cdot k_b\cdot\frac{1}{\tau_{pT}}+\sqrt{U}\right)+\tau_{pm}\cdot\left(\frac{1}{4}\right)\cdot\left(\frac{1}{\tau_{pT}}+n^2\cdot k_b\cdot\frac{1}{\tau_{pT}}+\sqrt{U}\right)^2+a_m\cdot n^2\right\}=a_m\cdot\delta\cdot\left(-n^2\right)\cdot C_1; \quad (63)$$

$$A_2\cdot\left\{\left(-\frac{1}{2}\right)\cdot\left(\frac{1}{\tau_{pT}}+n^2\cdot k_b\cdot\frac{1}{\tau_{pT}}+\sqrt{U}\right)+\tau_{pm}\cdot\left(\frac{1}{4}\right)\cdot\left(\frac{1}{\tau_{pT}}+n^2\cdot k_b\cdot\frac{1}{\tau_{pT}}+\sqrt{U}\right)^2+a_m\cdot n^2\right\}=a_m\cdot\delta\cdot\left(-n^2\right)\cdot C_2; \quad (64)$$

9. Discussion of the results of hyperbolic models establishing for the analysis of heat and moisture exchange in inhomogeneous porous materials

Using the method of operators, integral transformations, extended forms of orthogonal polynomials, special functions and methods of non-Fourier analysis, exact analytical solutions for thermal conductivity equation in Guyer-Krumhansl model are established, as well as for hyperbolic equation of thermal conductivity. The propagation of heat pulses of different shapes simulating real experimental conditions (such as isolated heat wave, laser pulse, space-periodic heating) is investigated on the basis of obtained exact solutions. The solutions are studied for a wide range of conditions, including ballistic conditions together with the phonon heat transfer mechanism and heat diffusion. The validity of the obtained analytical solutions is checked by direct substitution into the equation.

If the term corresponding to the Fourier diffusion prevails in the equation, then the solution is expected to attenuate rapidly; however the phonon way of heat transfer is suppressed. Conditions at the boundary of the medium corresponding to the ballistic method of heat transfer also do not play a role in this case, and the linear component makes a small contribution. It should be noted that even when the contributions of all components of heat transfer (phonon, diffusion, ballistic) are of the same order of magnitude, the minimum of solution can be achieved within the area of interaction of the material and the thermal field.

10. Conclusions

1. The Guyer-Krumhansl-type equation proves that the maximum principle may not be applied for it (the maximum value of the thermal field amplitude in the material is reached either at the initial moment $t = 0$, or at one of the material's boundaries), e.g. under strong ballistic conditions. The effective thermal conductivity in the Guyer-Krumhansl-type equation significantly depends on the shape of the initial pulse. Short point pulses propagate much faster than smooth heat waves. This fact is very important for experimental measurements of thermal conductivity where sensitive experiments with thermal pulses are well-established practice. The sources of the latter are usually pulse lasers. It is shown that the shape of the initial pulse is of paramount importance for determining the presence of a ballistic type of heat transfer in each case. This fact should be taken into account during experimental measurements.

2. When modeling the processes of heat propagation and heat and mass transfer in thin films, the impact of the Knudsen number K_n, which is often used in the analysis of flow dynamics, was revealed. In this case, the Knudsen number characterizes the conditions for ballistic heat transfer. Thus, if $K_n = 1$ the contribution of all heat transfer mechanisms in the Guyer-Krumhansl equation is approximately the same, and if $K_n = 0{,}1$ the thermal conductivity a and ballistic component k_b are two orders of magnitude smaller than Cattaneo $\tau_{pT} \cdot \partial^2 T / \partial t^2$ and Fourier $\partial T / \partial t$ terms. The propagation of a heat wave in a thin film is weakly dependent on the Knudsen number. If the initial point pulse (such as Dirac δ-function) interacts with a thin film of material then the solution significantly depends on the Knudsen number. If $K_n = 1$ the value of the pulse is one order of magnitude greater than if $K_n = 0.1$ at the same spatio-temporal point. Moreover, the corresponding relaxation time of this solution at $K_n = 0{,}1$ exceeds the relaxation time by 2 orders of magnitude if $K_n = 1$: $\tau_{K_n = 0{,}1} / \tau_{K_n = 1} \approx 100$. Thus, the propagation of the instantaneous point pulse significantly depends on the Knudsen number. This is confirmed by studies of the propagation of the periodic function's harmonics $f(x) = exp(i \cdot n \cdot x)$ in thin films having heat and mass transfer processes inside. The high value of the Knudsen number improves the thermal conductivity, especially for higher harmonics.

3. The effective thermal conductivity determined in the context of the second law of thermodynamics without local maxima of solutions in the Cattaneo equation (hyperbolic equation of thermal conductivity) is practically constant and the effective thermal conductivity in the Guyer-Krumhansl equation increases with the number of the spatial harmonic $\sim exp(inx)$.

4. For the Cattaneo-type hyperbolic equation, the maximum principle is not followed. However, it is possible mathematically, but such heat transfer is impossible in the context of the second law of thermodynamics. The Guyer-Krumhansl equation realized in the model for thin films (in particular, in capillary-porous bodies) also violates the maximum principle. Local maxima come out from the addition of waves, however, they can be suppressed by an additional damping term $\sim \sigma = k_b \cdot 1/\tau_{pT}$ in the equation of Guyer-Krumhansl-type at physically reasonable values of parameters. It dampens the Cattaneo heat waves and temperature maxima, while restoring the relevant behavior of the system according to the second law of thermodynamics. The effective heat transfer increases with the harmonic number.

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Човнюк Ю.В., Чередніченко П.П., Москвітіна А.С., Шишина М.О., Шудра Н.С., Іванов Є.О.

ЗАСТОСУВАННЯ ГІПЕРБОЛІЧНИХ МОДЕЛЕЙ В АНАЛІЗІ ТЕПЛОВОЛОГООБМІНУ У НЕОДНОРІДНИХ ПОРИСТИХ МАТЕРІАЛАХ ПРИ РОЗПОВСЮДЖЕННІ КОРОТКИХ ІМПУЛЬСІВ ТЕПЛОТИ

У роботі використані гіперболічні моделі для аналізу тепловологообміну у неоднорідних пористих матеріалах, у яких розповсюджуються короткі імпульси теплоти. Теплопередача у різко неоднорідних середовищах при кімнатній температурі не описується законами Фур'є та Каттанео, а моделюється рівняннями типу Гюера-Крумхансля. У одновимірній постановці розв'язана система рівнянь взаємозв'язаного тепло- та масообміну, отримана О.В. Ликовим, яка враховує скінченність швидкостей переносу теплоти та маси (вологи). При цьому швидкість розповсюдження теплоти порядку швидкості звуку, тому внаслідок малості часу релаксації розв'язки гіперболічного рівняння теплопровідності багато у чому співпадають з розв'язками класичного параболічного рівняння, хоча і існують певні суттєві відмінності. Вони залежать від процесів, що відбуваються на поверхні (у тонких прошарках) пористих тіл. Швидкість дифузії вологи в капілярно-пористих матеріалах приблизно у 106…107 і більше разів менша за швидкість розповсюдження теплоти, тому, відповідно, час релаксації дифузійних процесів значно більший і у рівняннях масопереносу його треба враховувати. Отримані точні аналітичні розв'язки одновимірного рівняння Гюера-Крумхансля за допомогою операторного методу. Вказане рівняння також використане для вивчення імпульсів теплоти різної форми у середовищі з урахуванням фононного/балістичного способів теплопередачі. Отримані результати застосовані для моделювання розповсюдження теплоти і вологи у тонких плівках капілярно-пористих тіл із урахуванням молекулярних ефектів у системах зниженої розмірності. Моделюється розповсюдження дуже коротких теплових імпульсів, що моделюють ізольовані теплові хвилі із урахуванням числа Кнудсена, а також розв'язки для періодичної початкової функції. Досліджені точні розв'язки вищевказаних задач у моделі тонких плівок капілярно-пористих тіл.

Ключові слова: рівняння Гюера-Крумхансля, тепломасообмін, тонкі плівки, капілярно-пористі тіла, неоднорідність, число Кнудсена, гіперболічні рівняння тепло- і вологопереносу.

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Човнюк Ю. В., Чередніченко П.П., Москвітіна А.С., Шишина М.О., Шудра Н.С., Іванов Є.О. **Гіперболічні моделі в аналізі тепло- та вологообміну у неоднорідних пористих матеріалах** // Опір матеріалів і теорія споруд: наук.-тех. збірник – К.:КНУБА, 2024. . – Вип. 113. – С. 227-240.
У роботі проведений аналіз тепло- та вологообміну у неоднорідних пористих матеріалах, котрі використовуються при будівництві споруд різноманітного призначення.
Табл. 1. Іл. 1. Бібліогр. 50 назв.

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Chovniuk Yu.V., Cherednichenko P.P., Moskvitina A.S., Shyshyna M.O., Shudra N.S., Ivanov E.O. **Hyperbolic models in the analysis of heat and moisture exchange in inhomogeneous porous materials** // Strength of Materials and Theory of Structures: Scientificand-technical collected articles- K.: KNUBA, 2024. – Issue 113. – P. 227-240.
The paper analyzes the hyperbolic models in the analysis of heat and moisture exchange in inhomogeneous porous materials used for the constructions of various purposes.
Tab. 1. Fig. 1. Refs. 50.

Автор (науковий ступінь, вчене звання, посада): кандидат технічних наук, доцент, доцент кафедри фізичного виховання і спорту Човнюк Юрій Васильович
Адреса робоча: 03680 Україна, м. Київ, проспект Повітряних сил 31, Київський національний університет будівництва і архітектури.
Роб. тел. + +38(044) 245-48-29
E-mail: ychovnyuk@ukr.net
ORCID ID: http://orcid.org/0000-0002-0608-0203

Автор (науковий ступінь, вчене звання, посада): доцент, доцент кафедри міського будівництва Чередніченко Петро Петрович
Адреса робоча: 03680 Україна, м. Київ, проспект Повітряних сил 31, Київський національний університет будівництва і архітектури.
Роб. тел. + +38(044) 245-48-29
E-mail: petro_che@ukr.net
ORCID ID: http://orcid.org/0000-0001-7161-661x

Автор (науковий ступінь, вчене звання, посада): кандидат технічних наук, доцент, доцент кафедри теплогазопостачання і вентиляції Москвітіна Анна Сергіївна
Адреса робоча: 03680 Україна, м. Київ, проспект Повітряних сил 31, Київський національний університет будівництва і архітектури.
Роб. тел. +38(044) 245-48-33
E-mail: moskvitina.as@knuba.edu.ua
ORCID ID: http://orcid.org/0000-0003-3352-0646

Автор (науковий ступінь, вчене звання, посада): асистент кафедри теплогазопостачання і вентиляції Шишина Марія Олексіївна
Адреса робоча: 03680 Україна, м. Київ, проспект Повітряних сил 31, Київський національний університет будівництва і архітектури.
Роб. тел. +38(044) 245-48-33
E-mail: shyshyna.mo@knuba.edu.ua
ORCID ID: http://orcid.org/0000-0001-9384-7662

Автор (науковий ступінь, вчене звання, посада): ст. викладач кафедри інженерної геодезії Шудра Наталія Сергіївна
Адреса робоча: 03680 Україна, м. Київ, проспект Повітряних сил 31, Київський національний університет будівництва і архітектури
Робочий тел.: +38 (044) 249-72-51
E-mail: shudra.ns@knuba.edu.ua
ORCID ID: http://orcid.org/0000-0001-5416-7680

Автор (науковий ступінь, вчене звання, посада): ст. викладач кафедри іноземної філології та перекладу Іванов Євген Олександрович
Адреса робоча: 03058, Україна, м. Київ, Любомира Гузара проспект, 1, Національний авіаційний університет
Робочий тел.: +38 (044)406-74-22
E-mail: shifango@ukr.net
ORCID ID: http://orcid.org/0000-0002-1318-0472