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ENERGY-BASED ASSESSMENT OF THE ULTIMATE LIMIT STATE OF A PHYSICALLY NONLINEAR STRUCTURE

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The paper addresses issues of nonlinear analysis of load-bearing structural members. It is proposed to use the work done by external forces as a measure for determining the ultimate load. An incremental procedure is analyzed, through which the equilibrium state curve is constructed and this work is calculated. The paper highlights the issue of numerical instability in the computational process as it approaches the failure load. As a way to address this problem, it is suggested to consider a state of the structure as ultimate when it significantly loses its ability to resist the increasing load. The algorithm for searching for dangerous load combinations relies on a plausible hypothesis that the energy-based composition of load combinations leading the system to its ultimate state, due to the global nature of energy assessments, will be the same as in the case of linear analysis.

Keywords: nonlinear analysis, ultimate load, design load combination, strain energy.

Introduction

In order to achieve a more realistic description of the behavior (response) of structures under applied loads, computer simulation, including nonlinear finite element analysis, is increasingly being used in practice. Notably, among the new sections introduced in the second generation of Eurocodes, particularly in EN 1990 [1] and the draft prEN 1992 [2], there are new requirements related to the nonlinear finite element analysis of structures and structural systems.

The requirements related to nonlinear analysis of structures have been included in the design codes due to growing interest in this issue, driven by the transition to new design methods, as well as by new opportunities arising from advancements in computer technology and software of various levels, which enable such design [3-5]. As a rule, nonlinear analysis is based on an incremental procedure, i.e. simulation of the loading process.

As pointed out by G. Papazafeiropoulos et al. [6], traditional methods based on forces and displacements focus on extreme values, while the loading history is usually not taken into account, as it plays no role for elastic (conservative) systems. For physically nonlinear elastic-plastic systems, on the other hand, accounting for the loading history is important, and it is quite natural that relevant studies have emerged.

First of all, the behavior of structures under seismic loads began to be analyzed assuming the possibility of local damage and energy dissipation in plastic zones. Unlike forces or displacements, the values of which fluctuate during cyclic loading, energy dissipation continuously accumulates as the material yields. Thus, the analysis of energy changes in the system is a more suitable parameter for seismic design, and one of the first examples of the direct application of the energy-based approach to the analysis of structural behavior was the problem of earthquake resistance.

In 1956, G.W. Housner [6] was the first to propose comparing the energy input into a structure with the energy required for its failure. Since then, the energy-based approach to seismic resistance evaluation has significantly advanced [8-10], and found its place in practical design. Additionally, an energy-based method for assessing the degree of damage has also been developed [11, 12].

The damage index has found a certain application in studies of structural robustness [14-17], where critical damages of the structure, including collapse states, are considered. In this work, such states (or those close to them) are analyzed to establish the actual safety margins of structures operating beyond their elastic limits.

Ultimate load

The use of nonlinear analysis of system behavior highlights a number of inconsistencies with conventional methods, including those established in standards, regarding the application of the limit state design (the partial safety factor method [1]). The rules for checking strength and stability provided in the current standards are based on a two-step procedure where internal forces obtained from linear elastic analysis are used to verify individual critical sections with the help of nonlinear resistance models.

The limit state design method itself, based on the use of safety factors, is to some extent tied to the assumption of linearity of the design mathematical model. Its linearity, for example, allows the system to be analyzed under the characteristic load, and the transition from the characteristic value of the system's response (forces, displacements, etc.) to the design value is implemented by multiplying by the safety factor for load.

In nonlinear finite element analysis, the load is represented as a vector of nodal forces $\boldsymbol{f}$. The resistance of the structure is typically assessed through an incremental procedure, where the corresponding load is increased from its initial value to the design level by raising the load intensity $p_0 f$ to the design value $p_d \boldsymbol{f}$, where p is the load intensity parameter. To determine the safety margin, the incremental process should continue until the ultimate limit state of the structure is reached at $p_{lim} f$, and, hence, the safety factor is determined $k = p_{lim}/p_d$. Such an analysis is inherently a global assessment, where all structural elements of the system and their sections interact with each other.

There is no separation here between the stage of determining internal forces from design loads and the stage of section analysis. When such analysis is performed incrementally with monotonically increasing load intensity, the structure and all its design sections undergo various stress states, and their behavior is governed by the specified material stress-strain diagram. In this process, the resistance of all sections is evaluated, and if some sections reach their ultimate limit due to material yielding, their internal forces no longer increase. The structure's resistance to the increasing load is then ensured by other elements of the system that have not yet reached their load-bearing capacity limits. The entire process stops when the structure turns into a mechanism (i.e., when the system reaches its ultimate load-bearing capacity).

If the behavior of structures under load is illustrated by an equilibrium curve (Fig. 1), that relates the load intensity parameter p to the displacement λ, then the limit state is characterized by the value $g = dp/d\lambda = 0$. In this state, the system experiences infinitely large displacements with any infinitesimally small increase in load, meaning it behaves like a mechanism.

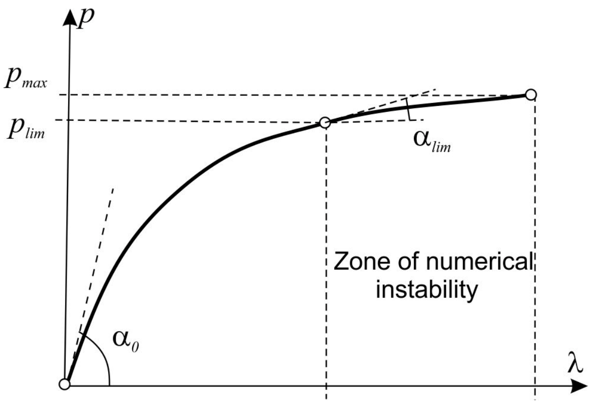

Fig. 1

It should be noted that it is often impossible to carry out a numerical analysis up to the maximum load-bearing capacity p_{max}, due to numerical instability of the process as the system's stiffness matrix approaches a degenerate state, which is exactly what characterizes the transformation of the structure into a mechanism. The practical solution to this problem is to consider the structure to have reached its limit state when it loses a significant part of its initial ability to resist increasing loads. In fact, this approach is almost always used in experimental studies of structural behavior, where the experiment is stopped when, for example, a rapid increase in deflections begins.

This concept was proposed in [18] and is based on the characteristic of the system's behavior, determined by the rate of change of the system's response to external action. It is proposed to use the decrease in the slope of the equilibrium state curve as a measure

$$\alpha = arctg(dp/d\lambda), \tag{1}$$

which characterizes the ability to resist increasing loads (system's resistance) and can indicate the approach of the structure to a state of complete failure, occurring at zero resistance.

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The limit resistance value

$$\alpha_{lim} = \beta\alpha_0 \tag{2}$$

is determined by the reduction factor β, the value of which should apparently be standardized. The results of control calculations show that the load value at which resistance decreases by two orders of magnitude compared to the initial value (β≈0,01) can be used as a criterion. This approach provides a reasonable margin compared to the failure load.

It should be noted that when a nonlinear analysis approaches the failure state of the system, it is almost always accompanied by significant displacements. If the limit resistance of the structure p_{lim}, determined using formula (2), is achieved at sufficiently large displacements, there is a need to adjust the value of p_{lim}.

Note that a system is considered resistant if, under any deformation, the change in potential energy is positive (the energy increases, which means that work is required to deform the system, and therefore, the system resists deformation) [19].

If the so-called characteristic displacement, which is energetically related to the load [19], is used as the abscissa axis λ of the equilibrium state curve, then the area under this curve indicates the work of external forces. The value of this work

$$U = \int_0^{\lambda_{lim}} p \cdot d\lambda \tag{3}$$

is an energy characteristic of the ultimate load-bearing capacity of the system. When the finite element method is used and nonlinear analysis is performed using the incremental method

$$U = \sum_{s=1}^{K} U_s = \sum_{s=1}^{K} \sum_{i=1}^{n} \Delta f_{si} \cdot \Delta z_{si}. \tag{4}$$

Here s is the step number, Δf_{si} is the fraction of the load at node i introduced into the calculation at step s, Δz_{si} is the increment of the nodal displacement calculated at step s. The values of U_s (s=1,2,…,K) are determined by **SCAD** during the implementation of the incremental procedure.

Design Load Combinations

The transition to global reliability analysis presents a very important and relatively underexplored problem. The issue is that the absence of the superposition principle in nonlinear analysis makes it impossible to use the approach [20], based on finding the most unfavorable combination of results from analyses of individual load cases. This necessitates performing nonlinear analysis to assess the effect of some postulated load combinations. The selection of such combinations creates a very difficult problem, for which a theoretical solution has not yet been found.

In practice, the selection of a design load combination (one or several competing combinations) relies on engineering intuition, especially in the case of a more or less familiar set of loads acting on a well-studied type of structure. Below is a proposed method for solving this problem, which has the advantage of being sufficiently general.

It is evident that, in the case of nonlinear (global) analysis, the selection of a design load combination should be based not on a local criterion but on a global one, which determines the set of loads and actions creating the design combination. A general indicator like deformation energy could be used as such a criterion, since it is clear that, for the system to reach its ultimate state, the external load must perform some work. The combination where the work of external forces is maximal and the system reaches its ultimate bearing capacity sooner can be decisive here.

To develop an algorithm for finding a dangerous load combination, we will use a plausible hypothesis that the energy-based composition of the load combination leading the system to its ultimate state will be similar to that in linear analysis, due to the global nature of energy evaluations. This hypothesis is supported by the fact that the relationship between load intensity and the level of accumulated deformation energy is monotonically increasing [25], similarly to a linear analysis.

Since energy is a positive quantity, all loads can be considered for the design load combination, except for those that cannot physically occur simultaneously. However, among independent loads, there may be pairs that counteract each other. To avoid their simultaneous action, additional verification is required. This verification can be based on calculating the work of one load on the displacements caused by another load. If this work is determined to be negative, the load pair can be classified as energetically alternative, where one load partially counteracts the other, and thus they should not be included together in the same combination. This information is obtained from linear analysis but is also used in nonlinear calculations.

The idea of the method is that, starting with the permanent load as a baseline, all other loads are sequentially checked to see whether the total potential energy increases or decreases when they are included in the design load combination.

Based on the finite element method, we will assume that the loading is characterized by a pair of vectors **f** – nodal forces, **z** – displacements of the system nodes, and their scalar product

$$U = \mathbf{f}^T \mathbf{z}, \tag{5}$$

which for a linear system is the doubled value of the potential energy of the system.

If the search begins with a permanent load, which is always taken into account (or the sum of permanent loads if there is more than one), and the corresponding potential energy is determined

$$U_1 = \mathbf{f}_1^T \mathbf{z}_1. \tag{6}$$

A candidate for inclusion in the design load combination is evaluated based on twice the value of the potential energy of the system subjected to the sum of the loads

$$U_{1,i} = \left(\mathbf{f}_1 + \mathbf{f}_i\right)^T \left(\mathbf{z}_1 + \mathbf{z}_i\right) = \mathbf{f}_1^T \mathbf{z}_1 + 2\mathbf{f}_1^T \mathbf{z}_i + \mathbf{f}_i^T \mathbf{z}_i. \tag{7}$$

The validity of this formula follows from the linearity of the problem, taking into account that based on the Betti's theorem of reciprocal works $\mathbf{f}_1^T \mathbf{z}_i = \mathbf{f}_i^T \mathbf{z}_1$. The first and last terms on the right side of (7) are positive, and it is easy to see that the sign of the scalar product $\mathbf{f}_1^T \mathbf{z}_i$ determines whether the *i*-th load increases the energy $(U_{1,i} \geq U_1)$ or decreases it $(U_{1,i} < U_1)$.

Such checks are performed for all loads, allowing the set of temporary loads to be divided into two non-overlapping subsets $I_{incr} = \left\{i_{i,1}, i_{i,2}, ..., i_{i,p}\right\}$ - those that increase the work of external forces and $I_{decr} = \left\{i_{d,1}, i_{d,2}, ..., i_{d,p}\right\}$ - those that decrease this work.

The energetically unfavorable design load combination is defined as the sum of permanent loads and temporary loads with indices I_{incr} or I_{decr}.

Several examples were analyzed to test the proposed method for determining the design load combination. All of them led to load combinations that were found to be design combinations according to the verification, in which the value of deformation energy was calculated using a nonlinear analysis by exhaustively evaluating all possible combinations. Naturally, such confirmation cannot be considered proof, and it is possible that in some cases the assumptions underlying the hypothesis may not hold, but the authors were unable to find such a case.

Illustrative example

A frame structure is considered (Fig. 2). All the bar elements are made of I-beam 20H1. Nodal loads are applied to the frame, grouped into four independent load cases (Fig. 3), where the first one is permanent.

The linear analysis yielded the nodal displacements **z** (mm), corresponding to the components of the load vector **f**.

The doubled values of potential energy, calculated using formula (7), turned out to be equal to

$\mathbf{f}_1^T \mathbf{z}_2$ =-4,933 tm; $\mathbf{f}_1^T \mathbf{z}_3$ = -0,137 tm; $\mathbf{f}_1^T \mathbf{z}_4$ =6,667 tm,

indicating that the combinations of the first with the fourth load or the first with the second and third loads were design combinations.

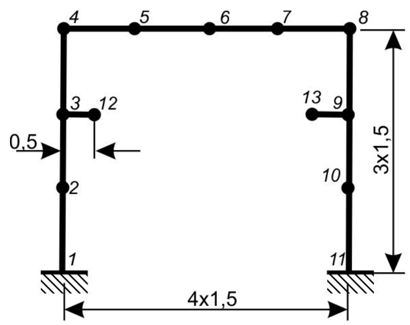

Fig. 2

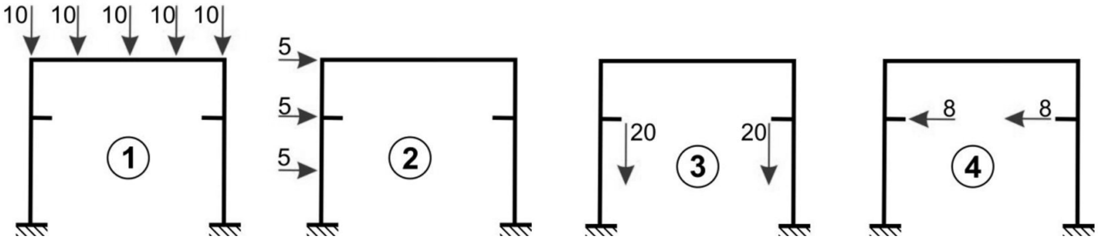

Fig. 3

To verify this, a physically nonlinear analysis of the system was performed for all 15 possible load combinations. It was assumed that the deformation theory of plasticity was used, and the material behavior was characterized by a bilinear σ-ε diagram with a yield strength of 240 MPa and a strain hardening coefficient of 0.01. The results shown in Table 2 indicated that the design combination predicted in the linear analysis, combining the first and fourth loads, was correctly identified.

Table 1

Node	$\mathbf{f}_1$ (t)	$\mathbf{z}_1$	$\mathbf{z}_2$	$\mathbf{z}_3$	$\mathbf{z}_4$
2	0,00	-0,46	28,17	0,04	-34,21
3	0,00	-0,92	72,40	0,09	-95,42
4	-10,00	-1,38	98,82	0,13	-133,33
5	-10,00	-56,82	98,74	-3,00	-133,33
6	-10,00	-85,37	98,66	5,50	-133,33
7	-10,00	-56,82	98,58	11,25	-133,33
8	-10,00	-1,38	98,50	-0,13	-133,33
9	0,00	-0,92	61,77	-0,09	-95,42
10	0,00	-0,46	20,09	-0,04	-34,21
12	0,00	-1,14	72,40	-12,63	-95,48
13	0,00	-1,14	61,77	14,31	-95,48

Table 2

Combination	Loads				Work of external forces
1	1	...	...	...	102,432
2	...	2	...	...	8,006
3	...	...	3	...	0,697
4	...	...	...	4	99,662
5	1	2	...	...	196,143
6	1	...	3	...	78,412
7	1	...	...	4	**419,924**
8	...	2	3	...	20,551
9	...	2	...	4	0,717
10	...	...	3	4	116,122
11	1	2	3	...	192,943
12	1	2	...	4	94,352
13	1	...	3	4	417,535
14	...	2	3	4	1,176
15	1	2	3	4	91,352

Conclusion

The proposed method for evaluating the work of external forces necessary to subject a structure to conditions near failure allows this energy measure to be used as a universal tool for analyzing the nonlinear behavior of structures.

The paper suggests a possible approach to solving the fundamentally important problem of unfavorable loading of a nonlinear deformable system. Using the energy measure of the ultimate load and the hypothesis of the similarity of unfavorable load combinations in linear and nonlinear analyses makes it possible to solve the problem of finding such a combination.

The mentioned hypothesis is empirical and has been confirmed by many individual tests, but the pressing issue is the theoretical search for the conditions under which it is valid.

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ЕНЕРГЕТИЧНА ОЦІНКА ГРАНИЧНОГО СТАНУ ФІЗИЧНО НЕЛІНІЙНОЇ КОНСТРУКЦІЇ

Стаття присвячена питанням нелінійного аналізу роботи несучих будівельних конструкцій. Вказується на його основну відмінність від традиційного підходу, що передбачає двоетапну процедуру, в якій статичний та динамічний розрахунок відокремлений від локальної перевірки надійності розрахункових перерізів, тоді як у нелінійному аналізі використовується одноетапний аналіз та глобальна оцінка поведінки конструкції за одночасної перевірки працездатності всіх перерізів.

Як міру, на підставі якої реалізується пошук граничного навантаження, пропонується використовувати роботу зовнішніх сил, що витрачається. Аналізується крокова процедура, з допомогою якої будується крива станів рівноваги така робота обчислюється. Вказується на проблему чисельної нестійкості обчислювального процесу при наближенні до руйнівного навантаження. Як спосіб вирішення цієї проблеми запропоновано вважати граничним такий стан конструкції, коли вона значною мірою втрачає здатність чинити опір зростанню навантаження (втрата відпірності).

Пропонується деякий прийом пошуку розрахункових поєднань незалежних навантажень, заснований на використанні енергетичного підходу, перевагою якого є достатня спільність. Вказується, що у разі нелінійного (глобального) аналізу вибір розрахункової комбінації навантажень має грунтуватися не на локальному, але у глобальному критерії, з якого визначається склад навантажень і впливів, створюють розрахункову комбінацію. Як такий критерій запропоновано використовувати енергію деформування. В основі алгоритму пошуку небезпечної комбінації навантажень використовується цілком правдоподібна гіпотеза, за якою енергетично обумовлений склад комбінації навантажень, що призводить до граничного стану, в силу глобального характеру енергетичних оцінок буде таким, як і у разі лінійного аналізу. Такий алгоритм дозволяє вирішити завдання, не звертаючись до перебору всіх можливих комбінацій навантажень.

Ключові слова: нелінійний аналіз, граничне навантаження, проектна комбінація навантажень, енергія деформування.

Perelmuter A.V., Perelmuter M.A.

ENERGY-BASED ASSESSMENT OF THE ULTIMATE LIMIT STATE OF A PHYSICALLY NONLINEAR STRUCTURE

The paper addresses issues of nonlinear analysis of load-bearing structural members. It highlights the main distinction from the traditional approach, which involves a two-step procedure where static and dynamic analyses are separated from the local reliability check of design sections. In contrast, the nonlinear analysis employs a single-step analysis and a global assessment of the structural behavior while simultaneously checking the performance of all sections.

It is proposed to use the work done by external forces as a measure for determining the ultimate load. An incremental procedure is analyzed, through which the equilibrium state curve is constructed and this work is calculated. The paper highlights the issue of numerical instability in the computational process as it approaches the failure load. As a way to address this problem, it is suggested to consider a state of the structure as ultimate when it significantly loses its ability to resist the increasing load (loss of resistance).

The paper proposes a method for finding design combinations of independent load cases, based on the energy approach, which offers sufficient applicability. It is noted that in the case of nonlinear (global) analysis, the selection of a design load combination should be based not on a local criterion but on a global one, which defines the composition of loads and actions constituting the design combination. The energy of deformation is suggested as such a criterion. The algorithm for searching for dangerous load combinations relies on a plausible hypothesis that the energy-based composition of load combinations leading the system to its ultimate state, due to the global nature of energy assessments, will be the same as in the case of linear analysis. This algorithm enables to solve the problem without resorting to an exhaustive evaluation of all possible load combinations.

Keywords: nonlinear analysis, ultimate load, design load combination, strain energy.

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Стаття присвячена питанням нелінійного аналізу роботи несучих будівельних конструкцій. Вказується, що у разі нелінійного (глобального) аналізу вибір розрахункової комбінації навантажень має грунтуватися не так на локальному, але у глобальному критерії, з якого визначається склад навантажень і впливів, створюють розрахункову комбінацію. Як такий критерій запропоновано використовувати енергію деформування.

Табл. 2. Іл. 3. Бібліогр. 20 назв.

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The paper addresses issues of nonlinear analysis of load-bearing structural members. It is noted that in the case of nonlinear (global) analysis, the selection of a design load combination should be based not on a local criterion but on a global one, which defines the composition of loads and actions constituting the design combination. The energy of deformation is suggested as such a criterion.

Tabl. 2. Figs. 3. Refs. 20.

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