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THERMALLY STRESSED STATE OF ASPHALT CONCRETE LAYER ON A METAL BASE

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The results of a finite element study of the thermally stressed and deformed states of a fragment of a two-layer bridge structure of a road bridge running deck, consisting of a load-bearing metal orthotropic slab with a layer of asphalt concrete applied to it, are presented. It is believed that the materials of the layers are characterized by different thermomechanical parameters, which determine the heterogeneity of the stress and strain fields. An analogue of these phenomena can be the effect of transforming into an electrothermal relay the thermal effect on a bimetallic plate with different coefficients of thermal linear expansion into its mechanical movements used to operate electrical switches and circuit breakers.

Using computer modeling, it has been established that these factors lead to the concentration of stresses and deformations and a change in the stress-strain state of the bridge structure of the running deck, which is not taken into account in modern practice in the design and operation of bridges, and is one of the reasons for premature destruction of the asphalt concrete pavement of a road bridge. To eliminate these shortcomings, based on finite element algorithms, a theoretical analysis of the thermally stressed state of a metal orthotropic slab with an asphalt concrete coating was carried out at different ratios of their thicknesses. It is shown that an increase in the thickness of the top layer can lead to an increase in the contacting and normal tensile stresses initiated in it. Therefore, when designing bridge structures, these features must be taken into account additionally.

Key words: bridge structure, asphalt concrete pavement, thermal effects, shear thermal stress, normal thermal stress.

1. Introduction. The factors that most influence the strength and durability of asphalt concrete road surfaces include temperature disturbances associated with daily and seasonal changes in ambient temperature [7, 9, 10, 17-21, 23]. Primarily, they are caused by the fact that asphalt concrete materials are characterized by a relatively low coefficient of thermal conductivity, and with typical dimensions of the structure, it does not have time to warm up or cool down to a greater depth during the day. As a result, noticeable high-gradient temperature changes occur predominantly only in the upper layer, and the temperature field takes on the form of an edge effect. At the same time, intense normal and tangential stresses are also concentrated in the upper layer, which contributes to its detachment from the metal orthotropic slab and cracks in the asphalt concrete pavement and further intensive destruction from the effects of transport and climatic influences. In the general theory of thermal conductivity, such effects have been known for a long time, and the equations they describe are called singularly perturbed [16]. Solving such problems (as confirmed in [2-4]) is associated with great difficulty.

An unexpected thermoelastic effect was also found to be due to the reinforcement of the top layer [4]. Here, the reinforcement has a reinforcing effect only at the same values of the coefficients of thermal expansion, when the deformations of asphalt concrete and reinforcement are compatible. If they are different, then to ensure compatibility of deformations, additional stresses are generated on the contact surface, contributing to premature local destruction of asphalt concrete.

The fields of thermal deformations and stresses are particularly specific if the asphalt concrete layer is arranged on a metal orthotropic slab, or reinforced rods with increased rigidity are included. At the same time, as our calculations showed, the difference in the values of their coefficients of thermal linear expansion has a great influence on the formation of stress fields. Since when the temperature changes due to this difference, different components of the array tend to lengthen or shorten by different amounts, to ensure the compatibility of their movements and deformations, intense shear stresses are formed on the surfaces of their contact, contributing to the destruction of the bonds between them. Moreover, it turned out that, for example, in a bridge structure, these shear stresses quickly increase with increasing thickness of the asphalt concrete layer. This leads to the paradoxical conclusion that in order to reduce the level of thermal stresses, it is necessary to reduce the thickness of the asphalt concrete pavement layer.

Since this effect is not obvious, we believe that it should be considered and commented on in more detail.

2. Statement of the problem of the thermally stressed state of an inhomogeneous structure. A typical example of an unexpected deterioration in the conditions of elastic operation of a layered structure under the influence of thermal disturbances is the case associated with an increase in the thickness of the asphalt concrete layer on a metal base. The study of this issue is the development of a solution to the problem set out in [1, 3-4, 22, 23]. As shown, an asphalt concrete layer is used as a coating on the top surface of a bridge structure of a road bridge. If the materials of the coating and the metal base have different values of coefficients of thermal linear expansion, then when the ambient temperature fluctuates, the elements of each of these materials elongate and contract differently, leading to their different deformations and movements on the plane of their communication. To connect these deformations and movements, significant tangential stresses must arise in the contact zone of these materials, excluding their mutual sliding and ensuring their joint deformation. In works [1, 2, 4], finite element modeling of the main features of these effects was performed. Using the theory of thermoelasticity, it is shown that the highest shear stresses between the layers of asphalt concrete and the metal base are concentrated in the edge zone of the system, and normal longitudinal stresses predominate in the central sections. Note that similar features occur in the mechanics of composite materials [13–15].

This effect is one of the factors explaining the intense detachment of the asphalt concrete layer from the metal base in the winter-spring period. It can be argued that the intensity of the indicated interlayer tangential stresses is determined primarily by the difference in the values of the coefficients of thermal linear expansion and the thickness of the asphalt concrete layer, which affects the value of incompatible deformations and movements of the contacting materials to be combined. In this case, the thickness of the metal layer of the bridge structure obviously plays a lesser role due to the low elastic deformability of steel.

To verify the above reasoning, finite element calculations of the thermally stressed state of a fragment of the bridge structure, the cross section of which is shown in Fig. 1. In this case, the thickness of the asphalt concrete layer was h = 0,07 m, the thickness of the bridge deck slab was -0,014 m. The value of thermomechanical characteristics for the asphalt concrete pavement material was $E = 5 \cdot 10^9$ Pa, v = 0,2, $\alpha_T = 2,46 \cdot 10^{-5}$ K⁻¹; for steel $E = 2,1 \cdot 10^{11}$ Pa, v = 0,3, $\alpha_T = 1,3 \cdot 10^{-5}$ K⁻¹. It was assumed that the initial temperature was and it was $T_0 = 0$ assumed that at the end of cooling the coating and the metal plate had the same temperature -25°C.

The problem of thermoelastic deformation of a road surface is solved in a linear formulation. This allows you to analyze only changes in strain and stress that are caused by temperature changes.



Fig. 1. Dimensions of structural elements

In the zones of each layer, the evolution of the temperature field is determined by the equation of non-stationary thermal conductivity [8]

$$\nabla^2 T - \frac{1}{a} \frac{\partial T}{\partial t} = 0.$$
 (1)

Here a – is the thermal diffusivity coefficient, the term $\nabla^2 T$ is equivalent to the expression $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$.

We believe that the process is quasi-static. Then the field of elastic displacements is described by the vector equation [8, 11]

$$u\nabla^2 \mathbf{u} + (\lambda + \mu) grad \ div \, \mathbf{u} - (3\lambda + 2\mu)\alpha_T grad(T) = 0, \qquad (2)$$

where λ and μ – are the isothermal Lame parameters.

At the conventional ends of the selected area, it is assumed that there are no heat flows in the normal directions, therefore the derivative of T in the normal direction **n** is zero,

$$\frac{\partial T}{\partial \mathbf{n}} = 0. \tag{3}$$

When formulating the boundary conditions for the function $\mathbf{u}(x, y, z)$, it was assumed that on the upper surface the medium is free from normal and tangential stresses, and on the side and lower surfaces of the conditional boundaries, normal displacements and tangential stresses are equal to zero. On the contact surfaces of the coating layers with each other and with the soil mass, the conditions of continuity of the values of the functions T, displacement functions and deformation components were accepted.

The accepted formulation of the problem of thermoelastic deformation of a selected multilayer array made it possible to use an algorithm for solving it, in which the problem of unsteady thermal conductivity for equation (1) is initially solved over the entire time t range of 12 hours. Then, at the moments of time necessary for the analysis, the fields of displacements, deformations and stresses were determined using the constructed temperature fields $T(x, y, z, t_i)$ using equations (2).

The solution of these equations is carried out by moving to a finite element model [12]

$$[K_T] \{T\} - [A] \{T\} = \{T_f(t)\},$$

$$[K_u] \{u\} = [L] \{T(t_i)\}.$$

$$(4)$$

Here $[K_T]$ – is the matrix of coefficients of the finite element model of the thermal conductivity equation, [A] – is the matrix of model coefficients with the derivative \dot{T} , $\{T_f(t)\}$ – is the vector of specified temperature values on the surface of the coating, $[K_u]$ – is the stiffness matrix for the finite element model of an elastic massif, [L] – is a matrix reflecting the influence of temperature on the movements of the massif elements.

After calculating the values of the displacement vector components $\{u\}$ in the nodes of the finite element model, the components of the strain ε_{jk} and σ_{jk} stress tensors are calculated. They are determined using equalities [9]

$$\varepsilon_{jk} = (1/2)(u_{j,k} + u_{k,j}), \ \sigma_{jk} = 2\mu\varepsilon_{jk} + [\lambda\varepsilon_{ll} - (3\lambda + 2\mu)\alpha_T \cdot T]\delta_{jk},$$
(5)

discredited at each node of the model.

In these equalities, the indices j,k,l run through the values 1, 2, 3; in this case the directions

 x_1, x_2, x_3 correspond to the directions x, y, z; $u_{j,k} = \frac{\partial u_j}{\partial x_k}$ and $\varepsilon_{ll} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$; δ_{jk} – Kronecker symbol, equal to 0 at $j \neq k$ and equal to 1 at j = k.

In our case, the structure under consideration has the property associated with the fact that it is in free contact with the air environment. Therefore, for example, at night (in the absence of solar thermal radiation), the temperature of all its elements manages to level out and, instead of the initial value $T_0 = 0$, takes the same value $T_0 = -25^{\circ}$ C.

As in [4], we assume that the metal base slab has a thickness of h = 0,014 m. When studying the influence of the thickness of the asphalt concrete layer on the thermally deformed state of the system, its thickness was considered equal to $h_1 = 0,035$ m, $h_2 = 0,07$ m, $h_3 = 0,105$ m.

For this case, the graphs of the shear stress function $\tau_{xy}(x)$ at the left end of the cross section of the bridge deck are shown in Fig. 2. This feature appears to spike at the edge and then drop off quickly as you move away from the edge. At the locations of the vertical reinforcing ribs it has small splashes. This type of these functions promotes local stratification of the system at this edge.





Note that the resultant of these forces, test integral

$$\int_{0}^{A} \tau_{xy} dx = P(X) \tag{6}$$

equal to the tensile force *P* in the transverse vertical section x = X. Analyzing the force graph $\tau_{xy}(x)$ in Fig. 2, we can conclude that tangential stresses $\tau_{xy}(x)$ predominate only in the contour zone on the segment $0 \le x \le X$ and throughout the rest of the section they are close to zero. At the same time, the opposite effect occurs for stress σ_{xx} and force P(x). These functions increase along a segment $0 \le x \le X$ and then remain almost constant throughout the entire section.

Taking into account the above considerations, a study was carried out to determine the dependence of these functions on the thickness of the asphalt concrete layer. Three cases were considered, when $h_1 = 0,035$ m, $h_2 = 0,07$ m and $h_3 = 0,105$ m. The function graphs $\tau_{xy}(x)$ for these cases are shown in Fig. 2(a), (b), (c), respectively. They appear to have concentrations at the edge x = 0 and then quickly decrease to zero. Note that additional bursts of these functions occur at the locations of the lower ribs of the metal structure of the bridge (Fig. 1).

Calculations confirmed the assumption that with increasing thickness h of the asphalt concrete layer, the maximum values of tangential stresses practically do not change, however, with increasing thickness they spread over a larger area of the contact surface. This can be verified by analyzing the value of integrals (6) from stresses $\tau_{xy}(x)$ in areas $0 \le x \le X$ equal to the colored areas in Fig. 2. Apparently, they are minimum at h_1 and maximum at h_3 . The values of integrals (6) for the selected cases at X = 0,103 m were $P_1 = 21263$ N, $P_2 = 25391$ N, $P_3 = 34160$ N.

The question of the distribution of normal stresses $\sigma_{xx}(y)$ in the vertical central section of a x = const two-layer structure is of significant interest. The graphs of this function for three cases of asphalt concrete layer thickness are shown in Fig. 3. It can be seen that with an increase in the thickness of the asphalt concrete layer, not only quantitative, but also qualitative changes in these fields occur. Thus, at $h_1 = 0,035$ m (Fig. 3(a)), the stresses $\sigma_{xx}(y)$ remain constant in both layers, and the asphalt concrete layer is stretched, and the metal slab is compressed in a given direction. In this case, naturally, the function $\sigma_{xx}(y)$ is discontinuous on the contact surface of the layers. As one would expect, the resultant stress in these layers is zero, that is

$$\int_{b}^{b} \sigma_{xx}(y)dy + \int_{b}^{b} \sigma_{xx}(y)dy = 0.$$
(7)



Fig. 3. Function distribution graphs σ_{xx} in the central vertical section of the structure: (a) $h_1=0.035$ m, (b) $h_2=0.07$ m, (c) $h_3=0.105$ m

With an increase in the thickness of the top layer to $h_2 = 0,07$ m (Fig. 3(b)), the function $\sigma_{xx}(y)$ in it became alternating, its tensile (the most dangerous) stress increased, and the compressive stress in the slab decreased. Thus, a bending of the upper layer is observed here, although condition (7) is preserved. With an increase in thickness h_i to $h_3 = 0,105$ m (Fig. 3(c)), these effects became even more noticeable.

Therefore, we can conclude that with increasing thickness of the asphalt concrete layer, the thermally stressed state of the system becomes more dangerous.

The general picture of the thermally deformed state of the system is shown in Fig. 4, it is typical for all three cases considered. It differs only in the deflection arrow, which is equal to H the difference between the vertical displacements of the edges of the system and its central point. These values were H_1 =0,0102 m, H_2 =0,0075 m, H_3 =0,0053 m, that is, with an increase in the size of the upper layer, the system deflection arrow decreases.



Fig. 4. Sectional diagram of the bridge structure in a thermally deformed state

Although the functions of deflection and longitudinal displacements are smooth and have relatively small values, the stress and strain fields caused by them are significantly inhomogeneous and in places of concentration their values are significant.

Thus, it can be concluded that if the thermal stresses in the top layer structure under consideration are dominant compared to the stresses caused by traffic loads, then an attempt to reinforce the system by increasing the thickness of the asphalt concrete layer leads to a negative result. In this case, the integral characteristics of thermal stresses in the upper layer only increase. It can be expected that in non-uniform temperature fields they are even more noticeable.

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ТЕРМОНАПРУЖЕНИЙ СТАН АСФАЛЬТОБЕТОННОГО ШАРУ НА МЕТАЛЕВІЙ ОСНОВІ

Наведено результати скінчено-елементного дослідження термонапруженого і деформованого станів фрагмента двошарової мостової конструкції їздового полотна автодорожнього мосту, що складається з несучої металевої ортотропної плити з нанесеним на неї шаром асфальтобетону. Вважається, що матеріали шарів характеризуються різними термомеханічними параметрами, які зумовлюють неоднорідність полів напружень і деформацій. Аналогом цих явищ може служити ефект перетворення в електротеплових реле теплового впливу на біметалічну пластину з різними коефіцієнтами теплового лінійного розширення в її механічні переміщення, які використовуються для приведення в дію електровимикачів і вмикачів.

Методом комп'ютерного моделювання встановлено, що ці фактори призводять до концентрації напружень та деформацій і зміни напружено-деформованого стану мостової конструкції їздового полотна, яка не враховується в сучасній практиці проектування і експлуатації мостів, і є однією з причин передчасних руйнувань асфальтобетонного покриття автодорожнього мосту. Для виключення цих недоліків на базі алгоритмів скінченних елементів виконано теоретичний аналізтермонапруженого стану металевої ортотропної плити з асфальтобетонним покриттям при різних відношеннях їх товщин. Показано, що збільшення товщини верхнього шару може приводити до зростання ініційованих в ньому дотичних і нормальних розтягуючих напружень. Тому при проектуванні конструкцій мостів ці особливості повинні бути враховані додатково.

Ключові слова: мостова конструкція, асфальтобетонне покриття, термічні впливи, дотичне термонапруження, нормальне термонапруження.

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THERMALLY STRESSED STATE OF ASPHALT CONCRETE LAYER ON A METAL BASE

The results of a finite element study of the thermally stressed and deformed states of a fragment of a two-layer bridge structure, consisting of a bearing metal orthotropic slab with a layer of asphalt concrete applied on it, are presented. It is believed that the materials of the layers are characterized by different thermomechanical parameters, which determine the inhomogeneity of the stress and strain fields. An analogue of these phenomena can be the effect of transformation in electric thermal relays of thermal action on a bimetallic plate with different coefficients of thermal linear expansion into its mechanical displacements, which are used to actuate the switch and switches.

Using the method of computer modeling, it was found that these factors lead to the concentration of stresses and strains and changes in the stress-strain state in all elements of the bridge structure end are not taken into account in the modern practice of designing and operating bridges, as well as are one of the reasons for the premature destruction of asphalt concrete pavements. To eliminate these shortcomings, on the basis of finite element algorithms, a theoretical analysis of the thermally stressed state of a metal bridge slab with an asphalt concrete pavement at various ratios of their thicknesses is carried out. It is shown that an increase in the thickness of the upper layer can lead to an increase in shear and normal tensile stresses initiated in it. Therefore, when designing bridge structures, these features should be additionally taken into account.

Keywords: bridge structure, asphalt concrete pavement, thermal effects, shear thermal stress, normal thermal stress.

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В статті наведені результати комп'ютерного аналізу напружено-деформованого стану асфальтобетонного дорожнього покриття на металевій основі під дією транспортних навантажень.

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