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BELT CONVEYOR STARTING MODE OPTIMIZATION

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In the presented article the variational problem of optimization starting mode of a belt conveyor is stated and analytically solved. In order to solve it a conveyor was modelled as dynamical system with three (connected in a chain manner with elastic elements) masses dynamic model. Based on the equations, which describe their movement, the optimization criterion was formed. It is the root mean square value of the driving force during starting mode. Finding the minimum of the optimization criterion (integral functional) with satisfying the boundary conditions of motion presents the sense of a variational problem. In order to solve it the Euler-Poisson equation was applied.The obtained optimal operation mode improved the productivity, reliability and energy efficiency of the belt conveyor.

Keywords: dynamic model, motion mode, belt conveyor, drive mechanism, dynamic loads.

Introduction

The work is aimed at increasing the efficiency of belt conveyors by choosing the drive mechanism movement mode.

During transient processes of belt conveyors (starting, braking, speed changing, emergency stop) there are significant dynamic loads in the elements of the drive mechanism and the belt. In the article proposed to reduce the dynamic loads by optimizing the movement mode of the drive mechanism. For this purpose, a conveyor is modelled as a dynamical system with three masses. The corresponding system of equations have been developed. Optimization of the conveyor movement mode has been carried out by minimizing of the integral criterion, which is the root mean square of the driving force (it calculated during starting process). As a result of the carried out optimization, the start mode of a belt conveyor, which minimizes the action of dynamic forces, has been defined. The starting mode allows to increase the reliability of the conveyor and reduce the energy losses of the drive.

Analysis of publications

In the articles [2, 3] a mathematical model of metal suspended structure of a belt conveyor section was developed. The problem of minimization of its weight was stated and solved. Researches involved various conveyor parameters: capacity, profile of the metal structure, length of conveying, etc. Obtained results may be used by designers for optimal development of conveyor supported structure.

In the scientific papers [4, 5] authors considered multiengine variable speed drives of belt conveyor to decrease overloads in the belt. This goal was achieved by development of control algorithm of the asynchronous conveyor drives with frequency inverters. Such approach is effective due to the fact, that inverters are commonly used in modern conveying machinery. In the work [10] the statement about exploitation of frequency inverters in belt conveyor drives is grounded in a similar manner.

In the article [6] the dynamical model of a belt conveyor was proposed. The corresponding mathematical model – is a system of partial differential equations. However, focusing on the dynamical processes in the belt authors has avoided the influence of the drive. Among other result, obtained in the work, it is necessary to stress the optimization problem. Researchers selected six the laws of belt conveyor accelerations and proved, that combination of polynomial and periodical (cos function) terms formed ideal acceleration law. Such a finding decreases dynamical forces in spite of quite limited domain of optimal control search.

Forcing by the need of improving belt conveyor performance (in terms of electric energy and bulk material losses), authors of the work [7] optimized the main parameters of conveyor and developed a fuzzy-control system, that varies belt speed, based on the flow of the material. The closed-loop system of control is fed with error of speed and its first time derivative. Thus, here fuzzy-PD-controller was obtained. The model of the plant (conveyor), which is presented of third order transient function with delay, was quite simple and it may be applied only as an initial approximation.

Article [8] proposes an estimation-calculation-optimization method to determine the minimum speed adjustment time to ensure healthy transient operations (the accelerating and the decelerating). With the suggested adjustment time, unexpected risks were avoided and the developed finite-element model of a belt conveyor shown it's an appropriate dynamic behaviour.

Analysis of these and other works revealed the need of deep substantiation of the optimal control law of a belt conveyor during its acceleration. It should be found on the as much as possible domain. Such an approach is presented in the current article.

Statement of the problem

During the movement of belt conveyors in the elements of the drive mechanism and the belt, there are significant dynamic loads, which significantly affect the reliability of their work and energy losses. Particularly dangerous are the dynamic loads during the transition processes (start, brake, change the speed, emergency stop). At this point, high-frequency oscillations of both the belt and the drive elements are generated. In the belt there are significant braking forces, which create additional stress and, as a consequence, contribute to premature failure.

One of the ways to reduce the dynamic loads in the elements of the drive and the belt is to choose a movement mode of the drive mechanism during the transition process. The greatest effect may be obtained by exploitation of an optimization approach. Therefore, it is advisable to optimize a start mode in a belt conveyor according to some criteria. Since the belt reliability (it is well-known, that belt is the most expensive element of the conveyor), is affected by driving forced, it is desirable to form such a criterion on that point. Minimizing such a criterion provides favourable (in terms of forces in the belt and torques in the different parts of transmission of the machine) mode of movement of a belt conveyor.

Thus, the purpose of the work is connected with improving of the belt conveyor efficiency by optimizing the drive mechanism□s movement mode.

To achieve this goal, it is necessary to solve the following tasks: to develop a mathematical model of the belt conveyor dynamics; to ground the criterion to minimize, which reflect the undesirable dynamical forces in the belt; to optimize the belt conveyor starting mode; to analyse the obtained results.

Research results

To optimize the belt conveyor movement mode, we have presented a conveying machine in the form of a three-mass dynamic model (Fig. 1). Developing a dynamic model involves some assumptions: all elements of a belt conveyor are solids except the belt, which has stiffness properties.

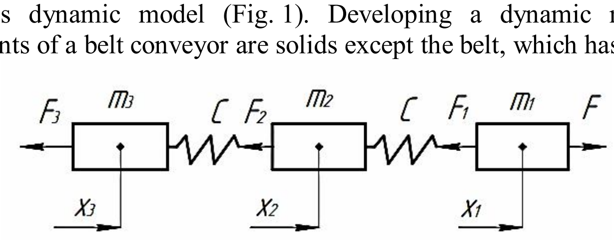

Fig. 1. A dynamic model of the belt conveyor

In this model all the dynamic values and characteristics are reduced to the drive drum. In fig. 1 the following notations are used: m_1 – reduced mass of the drive and drive drum; m_2 – reduced mass of the conveyed load and the working branch of the conveyor; C – reduced stiffness coefficient of the half of belt on the conveyor working branch; F – reduced the driving force of the conveyor electric motor; F_1, F_2, F_3 – reduced forces of resistance of the first, second and third masses, respectively; x_1, x_2, x_3 – are coordinates of the centers of masses the first, second and third masses, respectively.

Based on the accepted dynamic model, we may write a mathematical model of a belt conveyor. In order to do this, we have used d'Alemberts method. We should consider the equilibrium each of the massed under the action of active forces, resistance forces, inertia forces, and stiffness connections forces between the masses. As a result, we obtain a system of three second-order differential equations, which has the following form:

$$\begin{cases} m_1 \cdot \ddot{x}_1 = F - C \cdot (x_1 - x_2) - F_1; \\ m_2 \cdot \ddot{x}_2 = C \cdot (x_1 - x_2) - C \cdot (x_2 - x_3) - F_2; \\ m_3 \cdot \ddot{x}_3 = C \cdot (x_2 - x_3) - F_3. \end{cases} \quad (1)$$

We may reduce the system (1) to one sixth-order differential equation. In order to do this, from the last equation (1) we express the coordinate of the second mass x_2 and its time derivatives via the coordinate of the third mass and its time derivatives. In the following the designation has been done $x_3 = x$. In addition, we make the assumption that values F_1, F_2 and F_3 are constant. It allows to carry out some calculations:

$$x_2 = x + \frac{m_3}{C} \cdot \ddot{x} + \frac{F_3}{C}; \quad (2)$$

$$\dot{x}_2 = \cdot \dot{x} + \frac{m_3}{C} \cdot \dddot{x}; \quad (3)$$

$$\ddot{x}_2 = \ddot{x} + \frac{m_3}{C} \cdot \overset{IV}{x}. \quad (4)$$

Taking into account the dependencies (2)-(4), from the second equation of system (1) we express the coordinate of the first mass and its time derivatives via the coordinate of the third mass. As a result, we obtain:

$$x_1 = x + \frac{m_2 + 2 \cdot m_3}{C} \cdot \ddot{x} + \frac{m_2 \cdot m_3}{C^2} \cdot \overset{IV}{x} + \frac{F_2 + 2 \cdot F_3}{C}; \quad (5)$$

$$\dot{x}_1 = \dot{x} + \frac{m_2 + 2 \cdot m_3}{C} \cdot \dddot{x} + \frac{m_2 \cdot m_3}{C^2} \cdot \overset{V}{x}; \quad (6)$$

$$\ddot{x}_1 = \ddot{x} + \frac{m_2 + 2 \cdot m_3}{C} \cdot \overset{IV}{x} + \frac{m_2 \cdot m_3}{C^2} \cdot \overset{VI}{x}. \quad (7)$$

After substituting the dependences (2)-(7) in the first equation of system (1) we obtain a differential equation of the sixth order. It expresses the dependence of the drive mechanism driving force on time derivatives of the third mass coordinate and dynamic model parameters:

$$F = a_0 + a_1 \cdot \ddot{x} + a_2 \cdot \overset{IV}{x} + a_3 \cdot \overset{VI}{x}; \quad (8)$$

$$\begin{cases} a_0 = F_1 + F_2 + F_3; \\ a_1 = m_1 + m_2 + m_3; \\ a_2 = (m_1 \cdot m_2 + 2 \cdot m_1 \cdot m_3 + m_2 \cdot m_3) / C; \\ a_3 = m_1 \cdot m_2 \cdot m_3 / C^2; \\ a_{0,1,2,3} = const. \end{cases} \quad (9)$$

From the previously conducted dynamic analysis of the belt conveyor [12], it was found that during the starting process in the elements of the drive and the belt there are significant dynamical and energy overloads, which depend on the driving force of the drive. In addition, high-frequency oscillations of the belt are observed. To reduce the negative factors acting on the belt conveyor, we should optimize its movement during starting mode. Since the undesirable properties of the belt conveyor depend on the magnitude of the driving force, it should be the basis of the optimization criterion. In addition, the driving force must be reflected in the optimization criteria throughout the starting process, i.e., the optimization criterion must be presented in the integral form. To avoid the possible compensation actions of the driving force negative and positive values, the integrand must be written in the quadratic form. Therefore, the criterion for optimizing the belt conveyor movement mode is the RMS value of the driving force during starting:

$$F_{ск} = \left[\frac{1}{t_1} \cdot \int_0^{t_1} F^2 dt \right]^{1/2}, \tag{10}$$

where t – time; t_1 – the duration of the conveyor starting process.

Since the criterion (10) reflects the undesirable properties of the belt conveyor (the action of the dynamic forces) during the starting, it must be minimized.

In this regard, to determine the motion mode, we state a variational problem: to find the motion law $x = x(t)$, $0 \le t \le t_1$, that minimizes the criterion (10) and satisfies the boundary conditions of motion:

$$\left[\begin{array}{l} t=0:\ x_1=0,\ \dot{x}_1=0,\ x_2=0,\ \dot{x}_2=0,\ x_3=x=0,\ \dot{x}_3=\dot{x}=0; \\ t=t_1:\ x_1=x_2=x_3=x_к,\ \dot{x}_1=v,\ \dot{x}_2=v,\ \dot{x}_3=\dot{x}=v. \end{array} \right. \tag{11}$$

Obtained expressions (2)-(7) provide the basis for the boundary conditions (11) rewriting:

$$\left[\begin{array}{l} t=0:\ x_1=0,\ \dot{x}=0,\ \ddot{x}=-\dfrac{F_3}{m_3},\ \dddot{x}=0,\ \overset{IV}{x}=\dfrac{C}{m_2 \cdot m_3}\left(\dfrac{m_2}{m_3}\cdot F_3 - F_2\right),\ \overset{V}{x}=0; \\ t=t_1:\ x_1=x_к,\ \dot{x}=v,\ \ddot{x}=-\dfrac{F_3}{m_3},\ \dddot{x}=0,\ \overset{IV}{x}=\dfrac{C}{m_2 \cdot m_3}\left(\dfrac{m_2}{m_3}\cdot F_3 - F_2\right),\ \overset{V}{x}=0. \end{array} \right. \tag{12}$$

In conditions (11) and (12) x_k – the position of the third mass of the dynamic model at the end of the starting process, which is an unknown value and should be determined in further studies.

Variation problem (10), (11) can be rewritten in an equivalent form:

$$\int_0^{t_1} F^2 dt \to \min. \tag{13}$$

Optimal law should meet the conditions (12).

In order to solve the stated problem, we introduce following notation:

$$\ddot{y}(t) = \ddot{x}(t) + \frac{a_0}{a_1},\ \ 0 \le t \le t_1 \Leftrightarrow \ddot{x}(t) = \ddot{y}(t) - \frac{a_0}{a_1},\ \ 0 \le t \le t_1. \tag{14}$$

Taking into account notation (14) and the following mathematical features $\overset{IV}{x}(t) = \overset{IV}{y}(t)$, $\overset{VI}{x}(t) = \overset{VI}{y}$, $0 \le t \le t_1$, we may write:

$$F = a_0 + a_1 \cdot \ddot{x} + a_2 \cdot \overset{IV}{x} + a_3 \cdot \overset{VI}{x} = a_1 \cdot ((a_0/a_1) + \ddot{x}) + a_2 \cdot \overset{IV}{x} + a_3 \cdot \overset{VI}{x} = a_1 \cdot \ddot{y} + a_2 \cdot \overset{IV}{y} + a_3 \cdot \overset{VI}{y} = \left[a_1 \cdot \frac{d^2}{dt^2} + a_2 \cdot \frac{d^4}{dt^4} + a_3 \cdot \frac{d^6}{dt^6} \right] \cdot y. \tag{15}$$

Variotainal problem (12), (13) may be solved via the necessary condition of the functional (13) minimum, which is the Euler-Poisson equation [12]. For considered case it has the following form:

$$\frac{d^2}{dt^2} \frac{\partial F^2}{\partial \ddot{y}} + \frac{d^4}{dt^4} \frac{\partial F^4}{\partial \overset{IV}{y}} + \frac{d^6}{dt^6} \frac{\partial F^6}{\partial \overset{VI}{y}} = 0. \tag{16}$$

The result of substituting expression (15) into equation (16) and using the rule of differentiation of a complex function is presented below:

$$\begin{array}{c} \dfrac{d^2}{dt^2}(2 \cdot F \cdot a_1) + \dfrac{d^4}{dt^4}(2 \cdot F \cdot a_2) + \dfrac{d^6}{dt^6}(2 \cdot F \cdot a_3) = 0 \Leftrightarrow \\ \Leftrightarrow a_1 \cdot \dfrac{d^2 F}{dt^2} + a_2 \cdot \dfrac{d^4 F}{dt^4} + a_3 \cdot \dfrac{d^6 F}{dt^6} = 0 \Leftrightarrow \left[a_1 \cdot \dfrac{d^2}{dt^2} + a_2 \cdot \dfrac{d^4}{dt^4} + a_3 \cdot \dfrac{d^6}{dt^6} \right]^2 \cdot y = 0. \end{array} \tag{17}$$

The obtained equation (17) is a linear, homogeneous differential equation of the 12-th order concerning the unknown function $y(t)$, $0 \le t \le t_1$. In order to solve it, we have find the roots of a characteristic polynomial:

$$Q(\lambda) = \left[a_1 \cdot \lambda^2 + a_2 \cdot \lambda^4 + a_3 \cdot \lambda^6 \right]^2. \tag{18}$$

Since the polynomial (18) is a square of a polynomial of the 6-th order, it has 6 roots of the second order. To determine these roots, we write the equation: $a_1 \cdot \lambda^2 + a_2 \cdot \lambda^4 + a_3 \cdot \lambda^6 = 0$, in which to reduce the degree we introduce the notation $\mu = \lambda^2$. As a result we have:

$$\mu \cdot (a_1 + a_2 \cdot \mu + a_3 \cdot \mu^2) = 0 . \tag{19}$$

The roots of the equation have been defined as follows:

$$\mu_{1,2} = \frac{-a_2 \pm \sqrt{a_2^2 - 4 \cdot a_1 \cdot a_3}}{2 \cdot a_3}; \tag{20}$$

$$\mu_3 = 0 . \tag{21}$$

For the belt conveyor with parameters: $m_1 = 490\ kg$, $m_2 = 425\ kg$, $m_3 = 95\ kg$, $F_1 = 900\,\mathrm{N}$, $F_2 = F_3 = 0\,\mathrm{N}$, $C = 18000\,\mathrm{N/m}$, $v = 1.6\,\mathrm{m/s}$, $t_1 = 3\,\mathrm{s}$. Numerical values of constants are: $a_0 = 900\,\mathrm{N}$, $a_1 = 1010\,\mathrm{kg}$, $a_2 = 18{,}98\ \mathrm{kg \cdot s^2}$, $a_3 = 0{,}06106\ \mathrm{kg \cdot s^4}$. The roots of equation (19) are determined by these constants according to (20): $\mu_1 = -68{,}22$; $\mu_2 = -242{,}67$; $\mu_3 = 0$. The roots of a characteristic polynomial may be found:

$$\lambda_{1,2} = \pm\sqrt{\mu_1} = \pm 8.26 \cdot i = \pm \alpha_1 \cdot i; \quad \lambda_{3,4} = \pm\sqrt{\mu_2} = \pm 15.58 \cdot i = \pm \alpha_2 \cdot i; \quad \lambda_{5,6} = \pm 0 ,$$

where $i = \sqrt{-1}$ is an imaginary unit.

All roots λ_1, λ_2, λ_3, …, λ_6 of characteristic polynomial $Q(\lambda)$ are second-order roots. Then the general solution of the linear, homogeneous differential equation (17) has the following form:

$$\begin{aligned} y(t) = & (C_1 + C_2 \cdot t) \cdot \cos\alpha_1 \cdot t + (C_3 + C_4 \cdot t) \cdot \sin\alpha_1 \cdot t + (C_5 + C_6 \cdot t) \cdot \cos\alpha_2 \cdot t + \\ & + (C_7 + C_8 \cdot t) \cdot \sin\alpha_2 \cdot t + C_9 \cdot t^3 + C_{10} \cdot t^2 + C_{11} \cdot t + C_{12}; \end{aligned} \tag{22}$$

$$\begin{aligned} \dot{y}(t) = & (C_2 + C_3 \cdot \alpha_1 + C_4 \cdot \alpha_1 \cdot t) \cdot \cos\alpha_1 \cdot t + (C_4 - C_1 \cdot \alpha_1 - C_2 \cdot \alpha_1 \cdot t) \cdot \sin\alpha_1 \cdot t + \\ & + (C_6 + C_7 \cdot \alpha_2 + C_8 \cdot \alpha_2 \cdot t) \cdot \cos\alpha_2 \cdot t + (C_8 - C_5 \cdot \alpha_2 - C_6 \cdot \alpha_2 \cdot t) \cdot \sin\alpha_2 \cdot t + \\ & + 3 \cdot C_9 \cdot t^2 + 2 \cdot C_{10} \cdot t + C_{11}; \end{aligned} \tag{23}$$

$$\begin{aligned} \ddot{y}(t) = & (2 \cdot C_4 - C_1 \cdot \alpha_1 - C_2 \cdot \alpha_1 \cdot t) \cdot \alpha_1 \cdot \cos\alpha_1 \cdot t - (2 \cdot C_2 + C_3 \cdot \alpha_1 + C_4 \cdot \alpha_1 \cdot t) \cdot \alpha_1 \cdot \sin\alpha_1 \cdot t + \\ & + (2 \cdot C_8 - C_5 \cdot \alpha_2 + C_6 \cdot \alpha_2 \cdot t) \cdot \alpha_2 \cdot \cos\alpha_2 \cdot t - (2 \cdot C_6 + C_7 \cdot \alpha_2 + C_8 \cdot \alpha_2 \cdot t) \cdot \alpha_2 \cdot \sin\alpha_2 \cdot t + \\ & + 6 \cdot C_9 \cdot t + 2 \cdot C_{10} \cdot t; \qquad 0 \le t \le t_1 . \end{aligned} \tag{24}$$

Substituting the obtained explicit form of the function $\ddot{y}(t)$ from (24) into the dependence (14), we find the image of the optimal acceleration:

$$\begin{aligned} \ddot{x}(t) &= \ddot{y}(t) - \frac{a_0}{a_1}; \\ \dot{x}(t) &= \dot{y}(t) - \frac{a_0}{a_1} \cdot t + C_{13}; \\ x(t) &= y(t) - \frac{a_0}{2 \cdot a_1} \cdot t^2 + C_{13} \cdot t + C_{14} . \end{aligned} \tag{25}$$

From expressions (22), (23), and (25) we find the constants C_{13} and C_{14} using the initial conditions of motion, when $t = 0$, $x = 0$ and $\dot{x} = 0$. As a result, we have obtained:

$$\begin{cases} C_1 + C_5 + C_{12} + C_{14} = 0; \\ C_2 + C_3 \cdot \alpha_1 + C_6 + C_7 \cdot \alpha_2 + C_{11} + C_{13} = 0. \end{cases}$$

From the presented above system we may find:

$$C_{13} = -C_2 - C_3 \cdot \alpha_1 - C_6 - C_7 \cdot \alpha_2 - C_{11}; \tag{26}$$

$$C_{14} = -C_1 - C_5 - C_{12} . \tag{27}$$

Substituting expressions (22)-(24) into (25) and taking into account (26) and (27) we have:

$$x(t)=(C_1+C_2\cdot t)\cdot\cos\alpha_1\cdot t+(C_3+C_4\cdot t)\cdot\sin\alpha_1\cdot t+(C_5+C_6\cdot t)\cdot\cos\alpha_2\cdot t+$$
$$+(C_7+C_8\cdot t)\cdot\sin\alpha_2\cdot t+C_9\cdot t^3+\left(C_{10}-\frac{a_0}{2a_1}\right)\cdot t^2-(C_2+C_3\alpha_1+C_6+C_7\cdot\alpha_2)\cdot t-C_1-C_5\,; \tag{28}$$

$$\dot{x}(t)=(C_2+C_3\cdot\alpha_1+C_4\cdot\alpha_1\cdot t)\cdot\cos\alpha_1\cdot t+(C_4-C_1\cdot\alpha_1-C_2\cdot\alpha_1\cdot t)\cdot\sin\alpha_1\cdot t+$$
$$+(C_6+C_7\cdot\alpha_2+C_8\cdot\alpha_2\cdot t)\cdot\cos\alpha_2\cdot t+(C_8-C_5\cdot\alpha_2-C_6\cdot\alpha_2\cdot t)\cdot\sin\alpha_2\cdot t+ \tag{29}$$
$$+3\cdot C_9\cdot t^2+\left(2\cdot C_{10}-\frac{a_0}{a_1}\right)\cdot t-C_2-C_3\cdot\alpha_1-C_6-C_7\cdot\alpha_2\,;$$

$$\ddot{x}(t)=(2\cdot C_4-C_1\cdot\alpha_1-C_2\cdot\alpha_1\cdot t)\cdot\alpha_1\cdot\cos\alpha_1\cdot t-(2\cdot C_2+C_3\cdot\alpha_1+C_4\cdot\alpha_1\cdot t)\cdot\alpha_1\cdot\sin\alpha_1\cdot t+$$
$$+(2\cdot C_8-C_5\cdot\alpha_2+C_6\cdot\alpha_2\cdot t)\cdot\alpha_2\cdot\cos\alpha_2\cdot t-(2\cdot C_6+C_7\cdot\alpha_2+C_8\cdot\alpha_2\cdot t)\cdot\alpha_2\cdot\sin\alpha_2\cdot t+ \tag{30}$$
$$+6\cdot C_9\cdot t+2\cdot C_{10}\cdot t,\qquad 0\le t\le t_1\,.$$

Here C_1, C_2, …, C_{10} are the constants that should be determined by the boundary conditions of motion (12). Since the components of the boundary conditions of motion at $t=0$, $x=0$ and $\dot{x}=0$ have been already used in determining the constants C_{13} and C_{14} there 10 boundary conditions are left. Note, that $x_к$ is an unknown value and it must be found as well.

Solution (28) of the differential equation (17) contains 10 coefficients C_1, C_2, …, C_{10}. There is a lack of one condition due to the fact, that $x_к$ is an unknown value [13].

In order to determine the additional boundary condition, which will be used for calculation of $x_к$, we have found a variation of the functional (13). Taking into account expression (15) the result may be presented as follows:

$$\delta F_{ск}[x]=\int_0^{t_1}\left(a_0+a_1\cdot\ddot{x}+a_2\cdot\overset{IV}{x}+a_3\cdot\overset{VI}{x}\right)\cdot\delta\left(a_0+a_1\cdot\ddot{x}+a_2\cdot\overset{IV}{x}+a_3\cdot\overset{VI}{x}\right)dt=$$
$$=a_3\cdot\int_0^{t_1}\left(a_0+a_1\cdot\ddot{x}+a_2\cdot\overset{IV}{x}+a_3\cdot\overset{VI}{x}\right)\cdot\delta\overset{VI}{x}\,dt+a_2\cdot\int_0^{t_1}\left(a_0+a_1\cdot\ddot{x}+a_2\cdot\overset{IV}{x}+a_3\cdot\overset{VI}{x}\right)\cdot\delta\overset{IV}{x}\,dt+ \tag{31}$$
$$+a_1\cdot\int_0^{t_1}\left(a_0+a_1\cdot\ddot{x}+a_2\cdot\overset{IV}{x}+a_3\cdot\overset{VI}{x}\right)\cdot\delta\ddot{x}dt\,.$$

Calculation each of the integrals included in expression (31) in parts brings next result:

$$\dot{I}=\left.\left\{\begin{aligned}&a_3\cdot\left(a_0+a_1\cdot\ddot{x}+a_2\cdot\overset{IV}{x}+a_3\cdot\overset{VI}{x}\right)\cdot\delta\overset{V}{x}-a_3\cdot\left(a_1\cdot\dddot{x}+a_2\cdot\overset{V}{x}+a_3\cdot\overset{VII}{x}\right)\cdot\delta\overset{IV}{x}+\\&+\left[a_3^2\cdot\overset{VIII}{x}+2\cdot a_2\cdot a_3\cdot\overset{VI}{x}+\left(a_1\cdot a_3+a_2^2\right)\cdot\overset{IV}{x}+a_1\cdot a_2\cdot\ddot{x}+a_0\cdot a_2\right]\delta\dddot{x}-\\&-\left[a_3^2\cdot\overset{IX}{x}+2\cdot a_2\cdot a_3\cdot\overset{VII}{x}+\left(a_1\cdot a_3+a_2^2\right)\cdot\overset{V}{x}+a_1\cdot a_2\cdot\dddot{x}\right]\delta\ddot{x}+\\&+\left[a_3^2\cdot\overset{X}{x}+2\cdot a_2\cdot a_3\cdot\overset{VIII}{x}+\left(2\cdot a_1\cdot a_3+a_2^2\right)\cdot\overset{VI}{x}+2\cdot a_1\cdot a_2\cdot\overset{IV}{x}+a_1^2\cdot\ddot{x}+a_0\cdot a_1\right]\delta\dot{x}-\\&-\left[a_3^2\cdot\overset{XI}{x}+2\cdot a_2\cdot a_3\cdot\overset{IX}{x}+\left(2\cdot a_1\cdot a_3+a_2^2\right)\cdot\overset{VII}{x}+2\cdot a_1\cdot a_2\cdot\overset{V}{x}+a_1^2\cdot\dddot{x}\right]\delta x\end{aligned}\right\}\right|_0^{t_1}+$$
$$+\int_0^{t_1}\left[a_3^2\cdot\overset{XII}{x}+2\cdot a_2\cdot a_3\cdot\overset{X}{x}+\left(2\cdot a_1\cdot a_3+a_2^2\right)\cdot\overset{VIII}{x}+2\cdot a_1\cdot a_2\cdot\overset{VI}{x}+a_1^2\cdot\overset{IV}{x}\right]\delta x\,dt. \tag{32}$$

Expression (32) must be zero at the extremal $x(t)$ of the functional (13). Taking into account the expression of the integrand:

$$f = F_p^2 = \left(a_0 + a_1 \cdot \ddot{x} + a_2 \cdot \overset{IV}{x} + a_3 \cdot \overset{VI}{x} \right). \tag{33}$$

The minimum condition of the criterion (13) is the Poisson equation:

$$\sum_{j=2}^{6} (-1)^j \frac{d^j}{dt^j} \frac{\partial f}{\partial \overset{(j)}{x}} = 0 . \tag{34}$$

After substituting expression (33) in equation (34) we obtain extended condition of the criterion (13) minimum:

$$a_3^2 \cdot \overset{XII}{x} + 2 \cdot a_2 \cdot a_3 \cdot \overset{X}{x} + \left(2 \cdot a_1 \cdot a_3 + a_2^2\right) \cdot \overset{VIII}{x} + 2 \cdot a_1 \cdot a_2 \cdot \overset{VI}{x} + a_1^2 \cdot \overset{IV}{x} = 0 . \tag{35}$$

Since the variations at the initial $t = 0$ and final moments $t = t_1$ are equal to zero and the variation of the function $\delta x(t)$ is arbitrary, expression (32) is zero under condition (35). The obtained equation (35) is the Poisson equation for the functional (13). Since the integral in the right part of the expression (32) becomes zero, the boundary expression included in (32) also equals to zero identically. Since $\delta x(0) = \delta \dot{x}(0) = \delta \ddot{x}(0) = \delta \dddot{x}(0) = \delta \overset{IV}{x}(0) = \delta \overset{V}{x}(0) = \delta \dot{x}(t_1) = \delta \ddot{x}(t_1) = \delta \dddot{x}(t_1) = \delta \overset{IV}{x}(t_1) = \delta \overset{V}{x}(t_1) = 0$, the following condition must be meet:

$$\left[a_3^2 \cdot \overset{XI}{x} + 2 \cdot a_2 \cdot a_3 \cdot \overset{IX}{x} + \left(2 \cdot a_1 \cdot a_3 + a_2^2\right) \cdot \overset{VII}{x} + 2 \cdot a_1 \cdot a_2 \cdot \overset{V}{x} + a_1^2 \cdot \dddot{x}(t_1) \right] \delta x(t_1) = 0 . \tag{36}$$

Since $\delta x(t_1)$ is arbitrary, based on the equation (36), we may obtain an additional condition for determining the integration constants in expressions (28)-(30), which are the solution of the differential equation (17):

$$a_3^2 \cdot \overset{XI}{x} + 2 \cdot a_2 \cdot a_3 \cdot \overset{IX}{x} + \left(2 \cdot a_1 \cdot a_3 + a_2^2\right) \cdot \overset{VII}{x} + 2 \cdot a_1 \cdot a_2 \cdot \overset{V}{x} + a_1^2 \cdot \dddot{x}(t_1) = 0 . \tag{37}$$

Condition (17) and boundary conditions (12) determine the minimum of the functional (13). Since conditions (32) and (37) include boundary values of time derivatives of the function (28) up to the 11-th order, we differentiate this function starting from the third derivative (the first and second derivatives were previously determined by dependences (29) and (30)):

$$\begin{aligned}
&\dddot{x}(t) = -\left(3 \cdot C_2 + C_3 \cdot \alpha_1 + C_4 \cdot \alpha_1 \cdot t\right) \cdot \alpha_1^2 \cdot \cos\alpha_1 t - \left(3 \cdot C_4 - C_1 \cdot \alpha_1 - C_2 \cdot \alpha_1 \cdot t\right) \cdot \alpha_1^2 \cdot \sin\alpha_1 t - \\
&-\left(3 \cdot C_6 + C_7 \cdot \alpha_2 + C_8 \cdot \alpha_2 \cdot t\right) \cdot \alpha_2^2 \cdot \cos\alpha_2 t - \left(3 \cdot C_8 - C_5 \cdot \alpha_2 - C_6 \cdot \alpha_2 \cdot t\right) \cdot \alpha_2^2 \cdot \sin\alpha_2 t + 6 \cdot C_9; \\
&\overset{IV}{x}(t) = -\left(4 \cdot C_4 - C_1 \cdot \alpha_1 - C_2 \cdot \alpha_1 \cdot t\right) \cdot \alpha_1^3 \cdot \cos\alpha_1 t + \left(4 \cdot C_2 + C_3 \cdot \alpha_1 + C_4 \cdot \alpha_1 \cdot t\right) \cdot \alpha_1^3 \cdot \sin\alpha_1 t - \\
&-\left(4 \cdot C_8 - C_5 \cdot \alpha_2 - C_6 \cdot \alpha_2 \cdot t\right) \cdot \alpha_2^3 \cdot \cos\alpha_2 t + \left(4 \cdot C_6 + C_7 \cdot \alpha_2 + C_8 \cdot \alpha_2 \cdot t\right) \cdot \alpha_2^3 \cdot \sin\alpha_2 t; \\
&\overset{V}{x}(t) = \left(5 \cdot C_2 + C_3 \cdot \alpha_1 + C_4 \cdot \alpha_1 \cdot t\right) \cdot \alpha_1^4 \cdot \cos\alpha_1 t - \left(5 \cdot C_4 - C_1 \cdot \alpha_1 - C_2 \cdot \alpha_1 \cdot t\right) \cdot \alpha_1^4 \cdot \sin\alpha_1 t + \\
&+\left(5 \cdot C_6 + C_7 \cdot \alpha_2 + C_8 \cdot \alpha_2 \cdot t\right) \cdot \alpha_2^4 \cdot \cos\alpha_2 t + \left(5 \cdot C_8 - C_5 \cdot \alpha_2 - C_6 \cdot \alpha_2 \cdot t\right) \cdot \alpha_2^4 \cdot \sin\alpha_2 t; \\
&\overset{VI}{x}(t) = \left(6 \cdot C_4 - C_1 \cdot \alpha_1 - C_2 \cdot \alpha_1 \cdot t\right) \cdot \alpha_1^5 \cdot \cos\alpha_1 t - \left(6 \cdot C_2 + C_3 \cdot \alpha_1 + C_4 \cdot \alpha_1 \cdot t\right) \cdot \alpha_1^5 \cdot \sin\alpha_1 t + \\
&+\left(6 \cdot C_8 - C_5 \cdot \alpha_2 - C_6 \cdot \alpha_2 \cdot t\right) \cdot \alpha_2^5 \cdot \cos\alpha_2 t - \left(6 \cdot C_6 + C_7 \cdot \alpha_2 + C_8 \cdot \alpha_2 \cdot t\right) \cdot \alpha_2^5 \cdot \sin\alpha_2 t; \\
&\overset{VII}{x}(t) = -\left(7 \cdot C_2 + C_3 \cdot \alpha_1 + C_4 \cdot \alpha_1 \cdot t\right) \cdot \alpha_1^6 \cdot \cos\alpha_1 t - \left(7 \cdot C_4 - C_1 \cdot \alpha_1 - C_2 \cdot \alpha_1 \cdot t\right) \cdot \alpha_1^6 \cdot \sin\alpha_1 t - \\
&-\left(7 \cdot C_6 + C_7 \cdot \alpha_2 + C_8 \cdot \alpha_2 \cdot t\right) \cdot \alpha_2^6 \cdot \cos\alpha_2 t - \left(7 \cdot C_8 - C_5 \cdot \alpha_2 - C_6 \cdot \alpha_2 \cdot t\right) \cdot \alpha_2^6 \cdot \sin\alpha_2 t; \\
&\overset{VIII}{x}(t) = -\left(8 \cdot C_4 - C_1 \cdot \alpha_1 - C_2 \cdot \alpha_1 \cdot t\right) \cdot \alpha_1^7 \cdot \cos\alpha_1 t + \left(8 \cdot C_2 + C_3 \cdot \alpha_1 + C_4 \cdot \alpha_1 \cdot t\right) \cdot \alpha_1^7 \cdot \sin\alpha_1 t - \\
&-\left(8 \cdot C_8 - C_5 \cdot \alpha_2 - C_6 \cdot \alpha_2 \cdot t\right) \cdot \alpha_2^7 \cdot \cos\alpha_2 t + \left(8 \cdot C_6 + C_7 \cdot \alpha_2 + C_8 \cdot \alpha_2 \cdot t\right) \cdot \alpha_2^7 \cdot \sin\alpha_2 t;
\end{aligned} \tag{38}$$

$$\overset{IX}{x}(t)=(9\cdot C_2+C_3\cdot\alpha_1+C_4\cdot\alpha_1\cdot t)\cdot\alpha_1^8\cdot\cos\alpha_1 t+(9\cdot C_4-C_1\cdot\alpha_1-C_2\cdot\alpha_1\cdot t)\cdot\alpha_1^8\cdot\sin\alpha_1 t+$$
$$+(9\cdot C_6+C_7\cdot\alpha_2+C_8\cdot\alpha_2\cdot t)\cdot\alpha_2^8\cdot\cos\alpha_2 t+(9\cdot C_8-C_5\cdot\alpha_2-C_6\cdot\alpha_2\cdot t)\cdot\alpha_2^8\cdot\sin\alpha_2 t;$$
$$\overset{X}{x}(t)=(10\cdot C_4-C_1\cdot\alpha_1-C_2\cdot\alpha_1\cdot t)\cdot\alpha_1^9\cdot\cos\alpha_1 t-(10\cdot C_2+C_3\cdot\alpha_1+C_4\cdot\alpha_1\cdot t)\cdot\alpha_1^9\cdot\sin\alpha_1 t+$$
$$+(10\cdot C_8-C_5\cdot\alpha_2-C_6\cdot\alpha_2\cdot t)\cdot\alpha_2^9\cdot\cos\alpha_2 t-(10\cdot C_6+C_7\cdot\alpha_2+C_8\cdot\alpha_2\cdot t)\cdot\alpha_2^9\cdot\sin\alpha_2 t;$$
$$\overset{XI}{x}(t)=-(11\cdot C_2+C_3\cdot\alpha_1+C_4\cdot\alpha_1\cdot t)\cdot\alpha_1^{10}\cdot\cos\alpha_1 t-(11\cdot C_4-C_1\cdot\alpha_1-C_2\cdot\alpha_1\cdot t)\cdot\alpha_1^{10}\cdot\sin\alpha_1 t-$$
$$-(11\cdot C_6+C_7\cdot\alpha_2+C_8\cdot\alpha_2\cdot t)\cdot\alpha_2^{10}\cdot\cos\alpha_2 t-(11\cdot C_8-C_5\cdot\alpha_2-C_6\cdot\alpha_2\cdot t)\cdot\alpha_2^{10}\cdot\sin\alpha_2 t;$$
$$0\le t\le t_1.$$

Substituting the boundary conditions (12) into equations (28)-(30) and (38) and using condition (37) brings a system of linear equations for determining the integration constants C_1, C_2, …, C_{10} and the value x_κ. Note, that we have used two of the initial conditions (position and velocity of the third mass) previously. As a result, we obtain the following system of equations:

$$(2\cdot C_4-C_1\cdot\alpha_1)\cdot\alpha_1+(2\cdot C_8-C_5\cdot\alpha_2)+2\cdot C_{10}-a_0/a_1=-F_3/m_3;$$
$$-(3\cdot C_2+C_3\cdot\alpha_1)\cdot\alpha_1^2-(3\cdot C_6+C_7\cdot\alpha_2)\cdot\alpha_2^2+6\cdot C_9=0;$$
$$-(4\cdot C_4-C_1\cdot\alpha_1)\cdot\alpha_1^3-(4\cdot C_8-C_5\cdot\alpha_2)\cdot\alpha_2^3=\frac{C}{m_2\cdot m_3}\cdot\left(\frac{m_2}{m_3}\cdot F_3-F_2\right);$$
$$(5\cdot C_2+C_3\cdot\alpha_1)\cdot\alpha_1^4+(5\cdot C_6+C_5\cdot\alpha_2)\cdot\alpha_2^4=0;$$
$$(C_1+C_2\cdot t_1)\cdot\cos\alpha_1 t_1+(C_3+C_4\cdot t_1)\cdot\sin\alpha_1 t_1+(C_5+C_6\cdot t_1)\cdot\cos\alpha_2 t_1+(C_7+C_8\cdot t_1)\cdot\sin\alpha_2 t_1+$$
$$+C_9\cdot t_1^3+\left(C_{10}-\frac{a_0}{2\cdot a_1}\right)\cdot t_1^2-(C_2+C_3\cdot\alpha_1+C_6+C_7\cdot\alpha_2)\cdot t_1-C_1-C_5=x_\kappa;$$
$$(C_2+C_3\cdot\alpha_1+C_4\cdot\alpha_1\cdot t_1)\cdot\cos\alpha_1 t_1+(C_4-C_1\cdot\alpha_1-C_2\cdot\alpha_1\cdot t_1)\cdot\sin\alpha_1 t_1+(C_6+C_7\cdot\alpha_2+C_8\cdot\alpha_2\cdot t_1)\cdot\cos\alpha_2 t_1+$$
$$+(C_8-C_5\cdot\alpha_2-C_6\cdot\alpha_2\cdot t_1)\cdot\sin\alpha_2 t_1+3\cdot C_9\cdot t_1^2+\left(2\cdot C_{10}-\frac{a_0}{a_1}\right)\cdot t_1-C_2-C_3\cdot\alpha_1-C_6-C_7\cdot\alpha_2=v;$$
$$(2\cdot C_4-C_1\cdot\alpha_1-C_2\cdot\alpha_1\cdot t_1)\cdot\alpha_1\cdot\cos\alpha_1 t_1-(2\cdot C_2+C_3\cdot\alpha_1+C_4\cdot\alpha_1\cdot t_1)\cdot\alpha_1\cdot\sin\alpha_1 t_1+$$
$$+(2\cdot C_8-C_5\cdot\alpha_2-C_6\cdot\alpha_2\cdot t_1)\cdot\alpha_2\cdot\cos\alpha_2 t_1-(2\cdot C_6+C_7\cdot\alpha_2+C_8\cdot\alpha_2\cdot t_1)\cdot\alpha_2\cdot\sin\alpha_2 t_1+$$
$$+6\cdot C_9\cdot t_1+2\cdot C_{10}-a_0/a_1=-F_3/m_3;$$
$$-(3\cdot C_2+C_3\cdot\alpha_1+C_4\cdot\alpha_1\cdot t_1)\cdot\alpha_1^2\cdot\cos\alpha_1 t_1-(3\cdot C_4-C_1\cdot\alpha_1-C_2\cdot\alpha_1\cdot t_1)\cdot\alpha_1^2\cdot\sin\alpha_1 t_1-$$
$$-(3\cdot C_6+C_7\cdot\alpha_2+C_2\cdot\alpha_2\cdot t_1)\cdot\alpha_2^2\cdot\cos\alpha_2 t_1-(3\cdot C_8-C_5\cdot\alpha_2-C_6\cdot\alpha_2\cdot t_1)\cdot\alpha_2^2\cdot\sin\alpha_2 t_1+6\cdot C_9=0; \qquad (39)$$
$$-(4\cdot C_4-C_1\cdot\alpha_1-C_2\cdot\alpha_1\cdot t_1)\cdot\alpha_1^3\cdot\cos\alpha_1 t_1+(4\cdot C_2+C_3\cdot\alpha_1+C_4\cdot\alpha_1\cdot t_1)\cdot\alpha_1^3\cdot\sin\alpha_1 t_1-$$
$$-(4\cdot C_8-C_5\cdot\alpha_2-C_6\cdot\alpha_2\cdot t_1)\cdot\alpha_2^3\cdot\cos\alpha_2 t_1+(4\cdot C_6+C_7\cdot\alpha_2+C_8\cdot\alpha_2\cdot t_1)\cdot\alpha_2^3\cdot\sin\alpha_2 t_1=$$
$$=\frac{C}{m_2\cdot m_3}\cdot\left(\frac{m_2}{m_3}\cdot F_3-F_2\right);$$
$$(5\cdot C_2+C_3\cdot\alpha_1+C_4\cdot\alpha_1\cdot t_1)\cdot\alpha_1^4\cdot\cos\alpha_1 t_1+(5\cdot C_4-C_1\cdot\alpha_1-C_2\cdot\alpha_1\cdot t_1)\cdot\alpha_1^4\cdot\sin\alpha_1 t_1+$$
$$+(5\cdot C_6+C_7\cdot\alpha_2+C_8\cdot\alpha_2\cdot t_1)\cdot\alpha_2^4\cdot\cos\alpha_2 t_1+(5\cdot C_8-C_5\cdot\alpha_2-C_6\cdot\alpha_2\cdot t_1)\cdot\alpha_2^4\cdot\sin\alpha_2 t_1=0;$$
$$-a_3^2\begin{bmatrix}(11C_2+C_3\alpha_1+C_4\alpha_1\cdot t_1)\cdot\alpha_1^{10}\cdot\cos\alpha_1 t_1+(11C_4-C_1\alpha_1-C_2\alpha_1\cdot t_1)\cdot\alpha_1^{10}\cdot\sin\alpha_1 t_1+\\+(11C_6+C_7\alpha_2+C_8\alpha_2\cdot t_1)\cdot\alpha_2^{10}\cdot\cos\alpha_2 t_1+(11C_8-C_5\alpha_2-C_6\alpha_2\cdot t_1)\cdot\alpha_2^{10}\cdot\sin\alpha_2 t_1\end{bmatrix}+$$

$$+2a_2a_3\begin{bmatrix}(9C_2+C_3\alpha_1+C_4\alpha_1\cdot t_1)\cdot\alpha_1^8\cdot\cos\alpha_1t_1+(9C_4-C_1\alpha_1-C_2\alpha_1\cdot t_1)\cdot\alpha_1^8\cdot\sin\alpha_1t_1+\\+(9C_6+C_7\alpha_2+C_8\alpha_2\cdot t_1)\cdot\alpha_2^8\cdot\cos\alpha_2t_1+(11C_8-C_5\alpha_2-C_6\alpha_2\cdot t_1)\cdot\alpha_2^8\cdot\sin\alpha_2t_1\end{bmatrix}-$$
$$-\left(2a_1a_2+a_2^2\right)\begin{bmatrix}(7C_2+C_3\alpha_1+C_4\alpha_1t_1)\alpha_1^6\cdot\cos\alpha_1t_1+(7C_4-C_1\alpha_1-C_2\alpha_1t_1)\cdot\alpha_1^6\cdot\sin\alpha_1t_1+\\+(7C_6+C_7\alpha_2+C_8\alpha_2t_1)\alpha_2^6\cdot\cos\alpha_2t_1+(7C_8-C_5\alpha_2-C_6\alpha_2t_1)\alpha_2^6\cdot\sin\alpha_2t_1\end{bmatrix}+$$
$$+2a_1a_2\begin{bmatrix}(5C_2+C_3\cdot\alpha_1+C_4\cdot\alpha_1\cdot t_1)\cdot\alpha_1^4+(5\cdot C_4-C_1\cdot\alpha_1-C_2\cdot\alpha_1\cdot t_1)\cdot\alpha_1^4\cdot\sin\alpha_1t_1+\\+(5C_6+C_7\alpha_2+C_8\alpha_2\cdot t_1)\cdot\alpha_2^4\cdot\cos\alpha_2t_1+(5C_8-C_5\alpha_2-C_6\alpha_2\cdot t_1)\cdot\alpha_2^4\cdot\sin\alpha_2t_1\end{bmatrix}-$$
$$-a_1^2\begin{bmatrix}(3C_2+C_3\alpha_1+C_4\alpha_1\cdot t_1)\cdot\alpha_1^2\cdot\cos\alpha_1t_1+(3C_4-C_1\alpha_1-C_2\alpha_1\cdot t_1)\cdot\alpha_1^2\cdot\sin\alpha_1t_1+\\+(3C_6+C_7\alpha_2+C_8\alpha_2t_1)\alpha_2^2\cdot\cos\alpha_2t_1+(3C_8-C_5\alpha_2-C_6\alpha_2t_1)\alpha_2^2\cdot\sin\alpha_2t_1-6C_9\end{bmatrix}=0.$$

Execution proper calculation leads to the solution of the system of equation (39): expressions of the constants C_1, C_2, …, C_{10} and finite position of the third mass $x_к$, $C_1=0.042$; $C_2=-0.002$; $C_3=0.006$; $C_4=-0.009$; $C_5=-0.007$; $C_6=0.004$; $C_7=-0.002$; $C_8=-0.001$; $C_9=8.517$; $C_{10}=0.725$; $x_к=2{,}353$. As a result of substituting the found expressions into expressions (28)-(30) and (38), we determine the law of motion of the third mass $x(t)=x_3(t)$ and its time derivatives. Through the law of motion of the third mass according to the formulas (2)-(7) the law of motion of the second and first masses of the dynamic model of the belt conveyor were determined.

Determining the motion law of all three masses, we find the optimal laws of:
1) the driving force (according to formula (8) and taking into account expressions (9), (30) and (38));
2) force in the belt
$$F_{12}=C\cdot(x_1-x_2);\tag{40}$$
3) the force in the belt when leaving the take-up drum
$$F_{23}=C\cdot(x_2-x_3);\tag{41}$$
4) the drive mechanism power
$$P=F\cdot\dot{x}_1.\tag{42}$$

Based on the obtained dependences, the plots of kinematic (fig. 2-4), dynamic (fig. 5-8), and energy (fig. 9) characteristics of the belt conveyor during optimal starting mode were built.

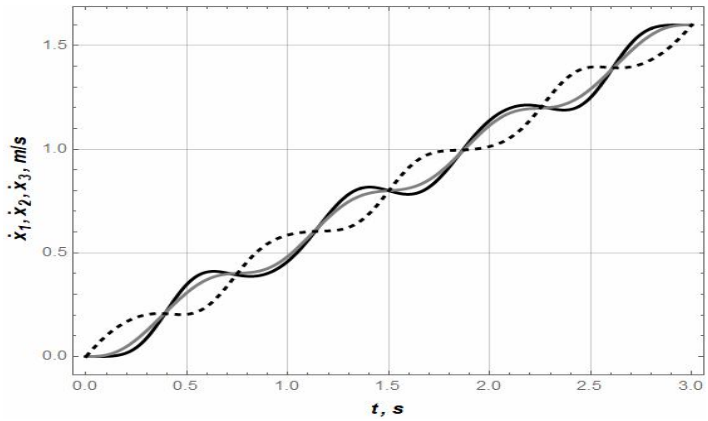

Fig. 2. Plots of the masses velocities

Here and further the following designations are accepted: black solid line refers to the third mass characteristics; gray solid line – to the second mass characteristics; black dashed line – to the first mass characteristics.

From the plots it is seen that the steady velocity reach all of three masses. Moreover, the velocities of the second and third masses at the beginning of the movement increase smoothly. A similar feature is observed when these masses reach a steady velocity. The velocity of the first mass at the beginning and at the end of the optimal starting the movement increases more intensively compared to the second and third masses. The same phenomenon is observed when the first mass reaches a steady speed.

The change in the velocity of the first mass at the beginning and at the end of the starting processes may be explained by the fact that the driving force is applied to this mass. The smoothness of second and third masses movement at the beginning and the end of a starting is provided by the existence of stiffness of the belt (it modelled with *C* coefficient).

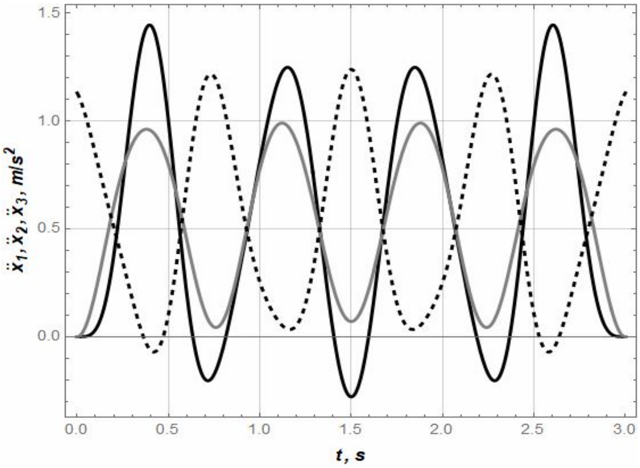

Fig. 3. Plots of acceleration of the masses

Analysis of the plots, that are presented in Fig. 3, shows that the fluctuations of accelerations are more pronounced. The largest maximum value of acceleration ($1.45\,\mathrm{m/s^2}$) corresponds to the third mass, and the smallest one – $1.0\,\mathrm{m/s^2}$ – to the second mass. These masses at the beginning and end of the starting have zero acceleration. However, the first mass at the beginning and end of the movement (starting) has non-zero acceleration, which is one of the main reasons for the emergence of intense oscillations of the masses. They continue until the end of the optimal starting.

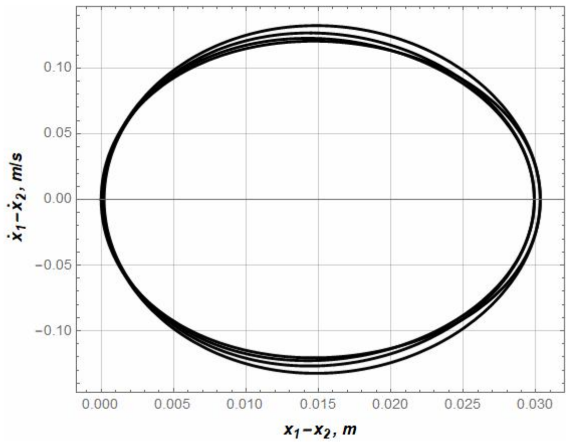

Fig. 4. Phase portrait of the oscillations of the second mass relative to the first one

From the phase portrait of the second mass oscillations (relative to the first one) (Fig. 4), it is seen that in the process of the optimal conveyor starting there are almost harmonic oscillations with slight deformations of the belt (connecting element with coefficient of stiffness *C*) and small velocity deviations of the masses m_1 and m_2. Phase portrait characterises the undamped oscillations that occur near the equilibrium (relative to each other) position of the masses.

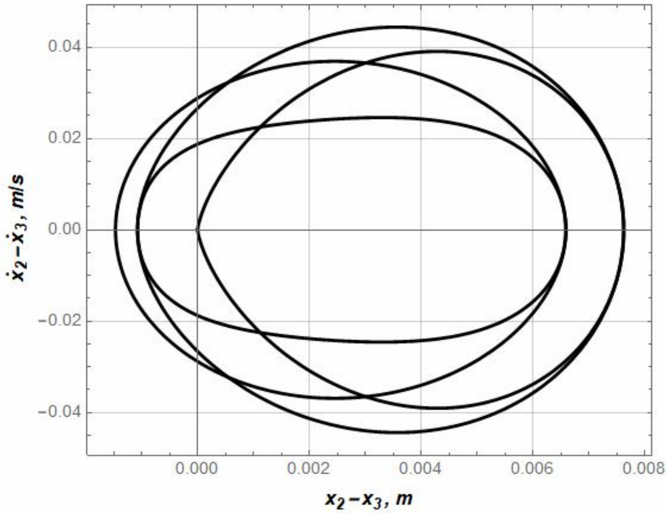

Fig. 5. Phase portrait of oscillations of the third mass relative to the second mass

Analysis of the phase portrait of the oscillations of the third mass relative to the second one (Fig. 5) shows the presence of oscillations with much larger relative deformations of the connection element and velocities deviations between masses (compared to the phase portrait on Fig. 4). The above phase portrait also indicates undamped oscillations between the third and second masses.

Fig. 6 and 7 shows that the elastic force between the first and second masses varies according to the harmonic aperiodic law. Its maximum value equals to 565 N, the minimum is zero. This law of elastic force leads to a permanent change of stresses in the belt and, as a consequence, provides its fatigue failure.

The elastic force between the second and third masses varies according to the periodic law with a small negative component (-25 N). Its maximum value equals to 137 N. In this part of the conveyor less force is applied, but they are also variable and lead to fatigue failure of the belt.

The driving force (Fig. 8) has the oscillating feature. It varies from minimum (1380 N) to maximum (1510 N) values. The average value during starting is 1445 N. The presence of the driving force at the beginning and end of the starting of the driving force (for both of the cases it equal to 1457 N) leads to oscillations in the mechanical system (belt, metal structure) of the belt conveyor.

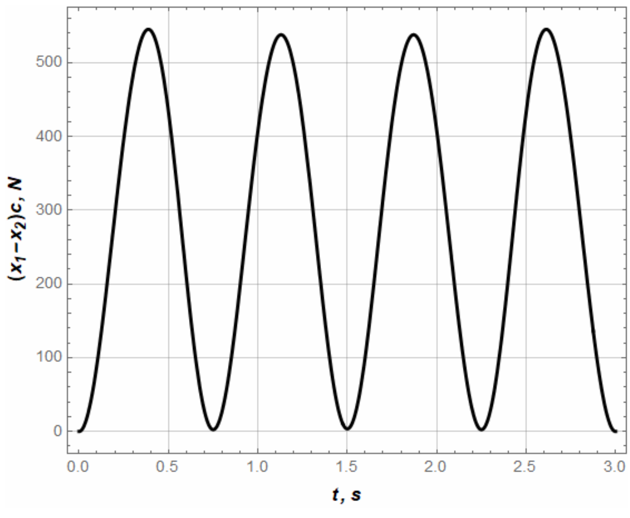

Fig. 6. Plot of the elastic force in the belt between the first and second masses

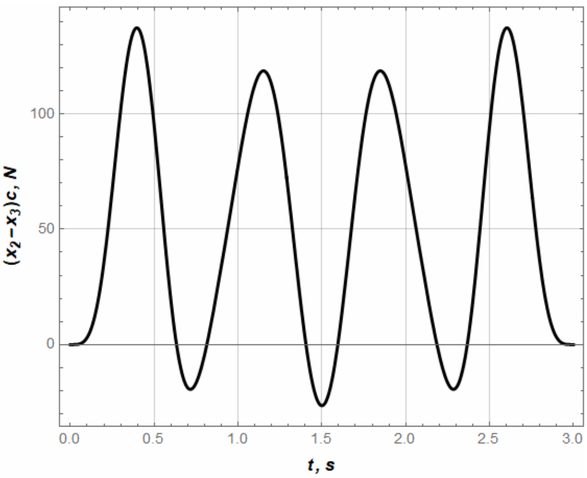

Fig. 7. Plot of the elastic force in the belt between the second and third masses

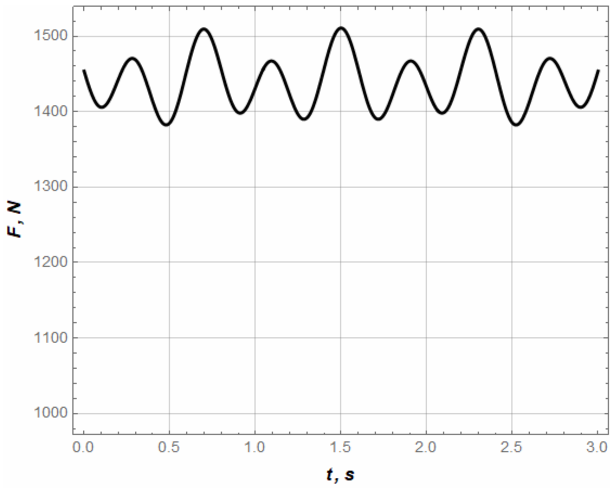

Fig. 8. Plot of the driving force of the belt conveyor drive mechanism

The power of the drive mechanism (Fig. 9) has an oscillating feature during the starting process. It varies from zero to a maximum value (2.35 kW) at the end of the optimal starting.

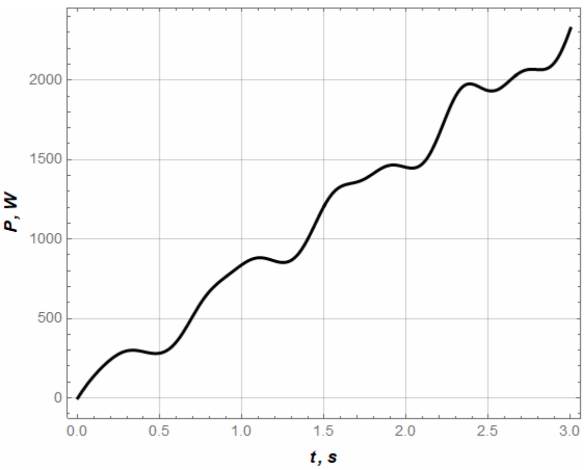

Fig. 9. Plot of consumed power of the belt conveyor drive mechanism

Conclusions. In the presented article the variational problem of optimization starting mode of a belt conveyor is stated and analytically solved. In order to solve it a conveyor was modelled as dynamical system with three (connected in a chain manner with elastic elements) masses dynamic model. Based on the equations, which describe their movement, the optimization criterion was formed. It is the root mean square value of the driving force during starting mode. Finding the minimum of the optimization criterion (integral functional) with satisfying the boundary conditions of motion presents the sense of a variational problem. In order to solve it the Euler-Poisson equation was applied.

The determination of the position of the conveyor masses at the end of the starting was carried out by calculation of variation of the integral criterion at the begin and at the end of the law of motion. As a result, the system of algebraic equations was obtained. Their solution brought all the necessary components of the optimal control problem solution: values of constants of integration and position of masses at the end of starting.

For the optimal starting mode, the kinematic, dynamic, and energy characteristics of the conveyor are determined. They show the presence of oscillations in the conveyor elements during the starting and absence of the oscillations at the end of the starting mode. However, carried analysis supports the statement about the need of further development of optimal modes of movement. One of the negative feature of the obtained optimal law of motion – is quit big amplitudes of forces in the belt during starting. Their minimization is the problem for further studies.

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ОПТИМІЗАЦІЯ СИЛОВОГО РЕЖИМУ ПУСКУ СТРІЧКОВОГО КОНВЕЄРА

Робота направлена на підвищення ефективності стрічкових конвеєрів шляхом вибору режиму руху приводного механізму.

Під час експлуатації стрічкових конвеєрів в елементах приводного механізму та тягового органу виникають суттєві енергетичні та динамічні навантаження. Наявні навантаження суттєво впливають на енергетичні втрати та надійність роботи приводного механізму і тягового органу. Особливо небезпечними є енергетичні та динамічні навантаження під час проходження перехідних процесів (пуск, гальмування, зміна швидкості руху чи зміна продуктивності та аварійна зупинка). В цей момент зароджуються високочастотні коливання як тягового органу так і елементів приводу. При таких коливаннях у тяговому органі (стрічці) конвеєра, виникають значні розривні зусилля, які створюють в ньому додаткові не бажані напруження та як наслідок сприяють передчасному руйнуванню. Окрім того, відбувається нагрівання статорних та роторних обмоток електродвигуна, що пришвидшує їх зношування та вихід його з ладу.

Для зменшення динамічних навантажень запропоновано здійснити оптимізацію режиму руху приводного механізму. Для цього конвеєр представлено у вигляді тримасової динамічної моделі, на основі якої складено математичну модель. З проведеного динамічного аналізу стрічкового конвеєру встановлено, що під час процесу пуску в елементах приводу та тягового органу мають місце значні силові та енергетичні перевантаження, які залежать від рушійного зусилля приводу. Крім того спостерігаються високочастотні коливання стрічки. Оскільки не бажані властивості стрічкового конвеєра в значній мірі залежать від величини рушійної сили приводу, тому вона повинна складати основу критерію оптимізації. Крім того рушійне зусилля приводу повинно бути відображене в критерії оптимізації протягом всього процесу руху, тобто критерій оптимізації має бути представлений в інтегральному виді. Для усунення можливої компенсації дії на конвеєр від'ємних та додатніх значень рушійного зусилля, останнє повинно представлятись в інтегральному критерії в квадратичному вигляді. Тому за критерій оптимізації режиму руху стрічкового конвеєра обрано середньоквадратичне значення рушійного зусилля приводу за час пуску.

Оптимізацію режиму руху конвеєра здійснено шляхом мінімізації інтегрального динамічного критерію. В результаті проведеної оптимізації визначено режим пуску стрічкового конвеєра, який до мінімуму зводить дію динамічних навантажень. Такий режим пуску дозволяє підвищувати надійність роботи конвеєра і зменшити енергетичні витрати приводу.

Ключові слова: динамічна модель, режим руху, стрічковий конвеєр, приводний механізм, динамічні навантаження.

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BELT CONVEYOR STARTING MODE OPTIMIZATION

The work is aimed at increasing the efficiency of belt conveyors by choosing the drive mechanism movement mode.

During the operation of belt conveyors, significant energy and dynamic loads occur in the elements of the drive mechanism and traction body. The available loads significantly affect the energy losses and the reliability of the drive mechanism and the traction body. Energy and dynamic loads during transient processes (starting, braking, changing speed or performance and emergency stop) are especially dangerous. At this moment, high-frequency oscillations of both the traction body and the drive elements arise. With such fluctuations in the traction body (belt) of the conveyor, significant breaking forces arise, which create additional unwanted stresses in it and, as a result, contribute to premature destruction. In addition, the stator and rotor windings of the electric motor are heated, which accelerates their wear and failure.

In order to reduce dynamic loads, it is proposed to optimize the movement mode of the drive mechanism. For this purpose, the conveyor is presented in the form of a three-mass dynamic model, based on which a mathematical model was created. From the conducted dynamic analysis of the belt conveyor, it was established that during the start-up process, significant power and energy overloads occur in the elements of the drive and the traction body, which depend on the driving force of the drive. In addition, high-frequency oscillations of the tape are observed. Since the undesirable properties of the belt conveyor largely depend on the magnitude of the driving force of the drive, it should form the basis of the optimization criterion. In addition, the driving force of the drive must be reflected in the optimization criterion during the entire movement process, that is, the optimization criterion must be presented in an integral form. To eliminate the possible compensation of negative and positive values of the driving force on the conveyor, the latter should be represented in the integral criterion in quadratic form. Therefore, the rms value of the driving force of the drive during the start-up time was chosen as the criterion for optimizing the motion mode of the belt conveyor.

Optimization of the conveyor movement mode was carried out by minimizing the integral dynamic criterion. As a result of the optimization, the start-up mode of the belt conveyor was determined, which minimizes the effect of dynamic loads. This start-up mode makes it possible to increase the reliability of the conveyor and reduce the energy consumption of the drive.

Keywords: dynamic model, motion mode, belt conveyor, drive mechanism, dynamic loads.

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У представленій статті поставлено та аналітично розв'язано варіаційну задачу оптимізації пускового режиму стрічкового конвеєра. Для її вирішення було змодельовано конвеєр як динамічну систему з трьома (з'єднаними ланцюгом пружними елементами) масами динамічної моделі. На основі рівнянь, що описують їх рух, сформовано критерій оптимізації. Це середньоквадратичне значення рушійної сили під час пускового режиму. Знаходження мінімуму критерію оптимізації (інтегрального функціонала) із задоволенням граничних умов руху має зміст варіаційної задачі. Для її вирішення застосовано рівняння Ейлера-Пуассона. Отриманий оптимальний режим роботи підвищив продуктивність, надійність та енергоефективність стрічкового конвеєра.

Іл. 9. Бібліогр. 14 назв.

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Loveikin V.S., Romasevich Yu.O., Kulpin R.A., Pochka K.I. **Belt conveyor starting mode optimization** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles – Kyiv: KNUBA, 2024. – Issue 112. – P. 170-184.
In the presented article the variational problem of optimization starting mode of a belt conveyor is stated and analytically solved. In order to solve it a conveyor was modelled as dynamical system with three (connected in a chain manner with elastic elements) masses dynamic model. Based on the equations, which describe their movement, the optimization criterion was formed. It is the root mean square value of the driving force during starting mode. Finding the minimum of the optimization criterion (integral functional) with satisfying the boundary conditions of motion presents the sense of a variational problem. In order to solve it the Euler-Poisson equation was applied.The obtained optimal operation mode improved the productivity, reliability and energy efficiency of the belt conveyor
Fig. 9. Ref. 14.

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