UDC 624.074.04

STABILITY OF CYLINDRICAL ANISOTROPIC COMPOSITE SHELLS UNDER TORSION IN A THREE-DIMENSIONAL FORMULATION

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DOI: 10.32347/2410-2547.2023.111.74-86

The calculation of cylindrical anisotropic layered composite shells under the action of end torques in a spatial setting is considered. The considered anisotropy is characterized by one plane of the material's elastic characteristics. To derive three-dimensional systems of equations of subcritical equilibrium and stability of the spatial theory of elasticity, a modification of the Hu-Washizu variational principle was used. Solving the problems of the pre-critical stress-strain state and stability is carried out using the Bubnov-Galyorkin methods, discrete Fourier transforms and numerical discrete orthogonalization. The problem of stability of an anisotropic cylindrical thick-walled shell with an increase in the number of cross-reinforced layers is considered, depending on the angle of rotation of the main directions of elasticity of the material and the direction of torque application.

Key words: anisotropic cylindrical shell, three-dimensional setting, torsional stability.

Introduction

A small number of works are devoted to solving problems of the stability of shells made of composite materials, most of which are based on the use of two-dimensional classical or refined theories [1, 15, 16]. This leads to the fact that for thin shells, the low shear stiffness and in homogeneity of the material along the thickness are either not taken into account at all, or not taken into account to the full extent. On the other hand, the geometric parameters of shells made of modern materials do not always meet the conditions of applicability of both classical and refined versions of the theory of shells. Therefore, the study of the stability of composite shell structures in a three-dimensional setting [5, 6] is expedient and relevant.

Analysis of recent research and publications. In the works devoted to the calculation of the stability of shell structures in a spatial setting [5, 6, 7], attention is focused on isotropic and orthotropic shells. The use of materials with this degree of anisotropy narrows the class of application of such composite structures. Note that when forming shell systems from fibrous composites by winding them on mandrels, a discrepancy arises between the main directions of elasticity of the orthotropic material and the axes of the curvilinear coordinate system of the shells (Fig. 1). The material of such a structure in the axes of the shell must be considered as having one plane of elastic symmetry, which is parallel to the middle surface [1, 3, 4, 10÷15]. The lack of works devoted to a comprehensive analysis of the stability of shell structures made of materials whose elastic properties have one plane symmetry is associated with the difficulties that arise when compiling their solving models, which is caused by the interconnectedness of deformations of tension (compression), shear, bending and torsion. However, taking these features into account makes it possible to design shell systems from modern materials while ensuring the design bearing capacity.

The aim of the study. The presented paper shows approaches to obtaining three-dimensional equations of the subcritical stress-strain state and stability of cylindrical anisotropic layered cylindrical shells in the spatial formulation of the theory of elasticity based on the modification of the functional of the generalized Hu-Washizu principle. The solution of the system of equations of the stress-strain state is carried out by combining the numerical methods of direct and discrete orthogonalization, the system of stability equations is solved by the joint application of the Bubnov-Galyorkin method and numerical discrete orthogonalization. Coordination of subcritical stress components determined by the method of straight lines with the procedure of the Bubnov-Galyorkin method when solving the stability problem occurs using the method of discrete Fourier transformations. Using the presented methods, the stability of cylindrical thin anisotropic layered shells made of material with one plane of elastic symmetry under the action of end shear loads simulating external torques was investigated.

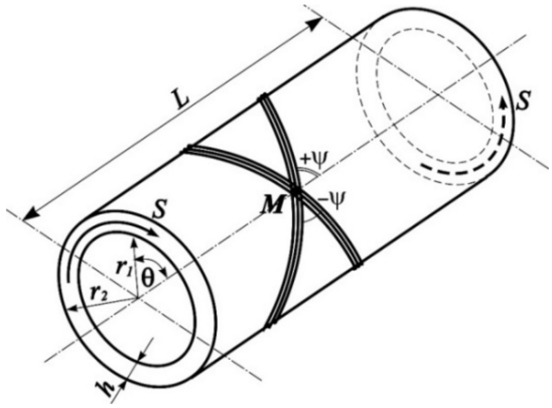


Fig. 1. Cylindrical non-thin anisotropic shell under torsion

1. Statement of the problem and method of solution

1.1. The problem of the subcritical stress-strain state

1.1.1. Hu-Washizu variational principle. In accordance with the variational principle of Hu-Washizu [17], the equilibrium equation, elasticity ratio (equation of state), geometric ratios and corresponding boundary conditions can be obtained from the condition of functional stationarity Π_1, which is defined with integral:

$$\Pi_1=\left\{\iiint\limits_V \left\{W(e_{ij})-T(u_i)+\Phi(u_i)-\sigma_{ij}\left[e_{ij}-\frac{1}{2}(u_{i;j}+u_{j;i})\right]\right\}dV+\iint\limits_{S_1}\Psi(u_i)dS-\iint\limits_{S_2}p_i(u_i-\bar{u}_i)\right\}dS. \quad (1)$$

Here, displacements u_i, deformations e_{ij}, stresses σ_{ii}, stresses p_i on the surface S_2 caused by displacements vary without additional conditions $\bar{u}_i$. Also in this functional $W(e_{ij})$ – potential energy of deformation, $T(u_i)$ – kinetic energy, $\Phi(u_i)$, $\Psi(u_i)$ – potentials of volumetric and surface loads, u_i – components of the displacement vector, a semicolon before the parameters i, j the covariant derivative along the coordinate with the corresponding index i, $j, k = 1, 2, 3$.

Potential energy of deformation in the vector-matrix representation is written as follows

$$W(e_{ij})=\frac{1}{2}\varepsilon^T B\varepsilon\ , \quad (2)$$

where $\varepsilon^T=(\varepsilon_{zz},\varepsilon_{\theta\theta},\varepsilon_{rr},2\varepsilon_{r\theta},2\varepsilon_{rz},2\varepsilon_{z\theta})$ is the vector of deformations, B is the matrix coefficients of elasticity.

If we enter the stress vector $\sigma^T=(\sigma_{zz},\sigma_{\theta\theta},\sigma_{rr},\tau_{r\theta},\tau_{rz},\tau_{z\theta})$ then from the condition of stationarity $\delta\Pi_1$, we get the following equations:

$$\sigma = B\varepsilon\ ; \quad (3)$$

$$\varepsilon=\varepsilon(u)\ ; \quad (4)$$

$$\sigma_{ij;j}+f_i=0 \quad (5)$$

and also boundary conditions $\sigma_{ij}n_j=\overline{F_i}$ on the surface S_1 and displacement $u_i=\bar{u}_i$ and stress $p_i=\sigma_{ij}n_j$ on the S_2 surface.

In ratios for deformations (4) the relationship between deformations and displacements is presented. Reversed to ratios elasticity (3) dependencies deformations from tensions let's introduce as

$$\varepsilon = A\sigma \, , \tag{6}$$

where matrix $A = B^{-1}$.

Coefficients matrices A let's mark through a_{ij}, a matrices B – b_{ij} $(i,\ j = \overline{1,6})$. Matrices A and B – symmetric, since $a_{ij} = a_{ji}$, $b_{ij} = b_{ji}$. In the future, the relationship between the matrices A and B is also established.

1.1.2. Modifiedmixed variationalprinciple. Let us follow the path presented in [13, 14, 16] to derive the modified Hu-Washizu mixed variational principle and divide vectors σ and ε on two parts, in order to

$$\sigma_1^T = (\sigma_{rr}, \tau_{r\theta}, \tau_{rz});\ \sigma_2^T = (\sigma_{zz}, \sigma_{\theta\theta}, \tau_{z\theta});\ \varepsilon_1^T = (\varepsilon_{rr}, \varepsilon_{r\theta}, \varepsilon_{rz});\ \varepsilon_2^T = (\varepsilon_{zz}, \varepsilon_{\theta\theta}, \varepsilon_{z\theta}). \tag{7}$$

To shorten ratio entries elasticity (6) will be record in matrix form

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix}, \tag{8}$$

where for blocks A_{ij}, according to the accepted division (7), with matrices A in (6) for an anisotropic material whose elastic properties are in one plane, we will get:

$$A_{11} = \begin{bmatrix} a_{33} & 0 & 0 \\ 0 & a_{44} & a_{45} \\ 0 & a_{45} & a_{55} \end{bmatrix};\ A_{12} = \begin{bmatrix} a_{31} & a_{32} & a_{36} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};\ A_{21} = \begin{bmatrix} a_{13} & 0 & 0 \\ a_{23} & 0 & 0 \\ a_{36} & 0 & 0 \end{bmatrix};\ A_{22} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix}. \tag{9}$$

After simple mathematical transformations the expression for $W(e_{ij})$ will be presented in the form:

$$W_1 = W(\sigma_1, \varepsilon_2) - \sigma_{ij}\left(\varepsilon_{ij} - \varepsilon_{ij}(u)\right) = -\frac{1}{2}\sigma_1^T B_{11}^{-1}\sigma_1 - \frac{1}{2}\varepsilon_2^T\left(B_{22} - B_{12}^T B_{11}^{-1} B_{12}\right)\varepsilon_2 +$$
$$+\left(\varepsilon_1^T(u) + \varepsilon_2^T(u) B_{12}^T B_{11}^{-1}\right)\sigma_1 + \varepsilon_2^T(u)\left(B_{22} - B_{12}^T B_{11}^{-1} B_{12}\right)\varepsilon_2. \tag{10}$$

In accordance with (1), we write down the potential of surface loads

$$\iint_{S_1} \Psi(u_i)\, dS_1 - \iint_{S_2} p_i\left(u_i - \bar{u}_i\right) dS_2 = \iint_{S_1}\Big[(q_r^- u_r + q_\theta^- u_\theta + q_z^- u_z,\, h_1, t) +$$
$$+\left(q_r^+ u_r + q_\theta^+ u_\theta + q_z^+ u_z,\, h_{n+1},\, t\right)\Big]\, dS_1 - \iint_{S_2} p_i\left(u_i - \bar{u}_i\right) dS_2 . \tag{11}$$

Here u_r, u_θ, u_z are displacements coinciding with the axes of the adopted cylindrical coordinate system (Fig. 1); h_1 and h_2 are the thicknesses of the first and $n+1$ shell layers.
We will also perform the variation of the potential of surface loads (11), after which we will obtain the variation of the work of external forces

$$\delta \iint_{S_1} \Psi(u_i)\, dS_1 = \iint_{S_1} \left(q_r \delta u_r + q_\theta \delta u_\theta + q_z \delta u_z\right) dS_1 + \iint_{S_2} \sum_{i=1}^{3} p_i\left(\delta u_i - \delta \bar{u}_i\right) dS_2 \, , \tag{12}$$

where $q_r = q_r^- + q_r^+$, $q_\theta = q_\theta^- + q_\theta^+$, $q_z = q_z^- + q_z^+$, and $p_i = 0$ for $i = \overline{1,3}$.

Let's write the final form of the functional Π_1 presented in (1) in the form

$$\Pi_1 = \iiint_V \left[W(\sigma_1, \varepsilon) - T(u_i)\right] dV - \iint_{S_1} \Psi(u_i)\, dS_1 - \iint_{S_2} p_i\left(u_i - \bar{u}_i\right) dS_2 \, . \tag{13}$$

The variation of the functional (13), due to the change in the components of the vector of displacements u and stresses σ_1, takes the form

$$\delta\Pi_1 = \iiint_V \left\{\left[-\frac{1}{2}\sigma_1^T B_{11}^{-1}\sigma_1 + \left(\varepsilon_1^T(u) + \varepsilon_2(u) B_{11}^T B_{12}^{-1}\right)\sigma_1 - \frac{1}{2} G_1^T B_{11}^{-1T}\sigma_1 - \frac{1}{2} G_1 B_{11}^{-1T}\sigma_1^T\right]\delta\sigma_1 - \right.$$

$$-\left[\frac{1}{2}\varepsilon_2^T\left(B_{22}-B_{12}^T B_{11}^{-1}B_{12}\right)\varepsilon_2\right]\delta\varepsilon_2+\left[\varepsilon_2^T(u)\left(B_{22}-B_{12}^T B_{11}^{-1}B_{12}\right)\varepsilon_2-\left(\varepsilon_1^T(u)+\varepsilon_2^T(u)B_{12}^T B_{11}^{-1^T}\right)G_1\right]\delta u-$$

$$-T(u)\delta u\}\,dV+\iint_{S_1}\left(\psi(u)\delta u\right)dS_1-\iint_{S_2}p_i\left(u-\bar{u}\right)\delta p dS_2\ . \qquad (14)$$

For further derivation, we will use linear geometric relations in the form [9]

$$e_{zz}^i=\frac{\partial u_z^i}{\partial z};\ e_{\theta\theta}^i=\frac{1}{r}u_r^i;\ e_{rr}^i=\frac{\partial u_r^i}{\partial r};\ e_{z\theta}^i=\frac{\partial u_\theta^i}{\partial z}+\frac{1}{r}\frac{\partial u_z^i}{\partial\theta};\ e_{rz}^i=\frac{\partial u_r^i}{\partial z}+\frac{\partial u_z^i}{\partial r};$$

$$e_{r\theta}^i=\frac{\partial u_\theta^i}{\partial r}-\frac{1}{r}u_\theta^i+\frac{1}{r}\frac{\partial u_r^i}{\partial\theta}\ . \qquad (15)$$

Here e_{zz}^i, $e_{\theta\theta}^i$, e_{rr}^i are relative linear deformations along the directions of the coordinate axes r, θ, z and $e_{z\theta}^i$, e_{rz}^i, $e_{r\theta}^i$ are relative shear deformations tangential to the corresponding coordinate surface, u_z^i, u_θ^i, u_r^i are linear displacements in the directions of the indicated axes, i – the shell layer number.

From the condition of stationarity (14), using expressions for stresses, displacements, geometric ratios (15), as well as variations in the work of external forces (12) and equating expressions for independent variations of stresses and displacements in the integral over the volume V to zero, we obtain system of differential equations in the form

$$\frac{\partial\sigma_{rr}^i}{\partial r}=-\frac{c_{23}^i+1}{r}\sigma_{rr}^i-\frac{\partial\tau_{rz}^i}{\partial z}-\frac{1}{r}\frac{\partial\tau_{r\theta}^i}{\partial\theta}+\frac{c_{22}^i}{r^2}u_r^i+\frac{c_{12}^i}{r}\frac{\partial u_z^i}{\partial z}+\frac{c_{26}^i}{r^2}\frac{\partial u_z^i}{\partial\theta}+\frac{c_{26}^i}{r}\frac{\partial u_\theta^i}{\partial z}+\frac{c_{22}^i}{r^2}\frac{\partial u_\theta^i}{\partial\theta}+q_r;$$

$$\frac{\partial\tau_{rz}^i}{\partial r}=c_{13}^i\frac{\partial\sigma_{rr}^i}{\partial z}-\frac{1}{r}\tau_{rz}^i-\frac{c_{12}^i}{r}\frac{\partial u_r^i}{\partial z}-c_{11}^i\frac{\partial^2u_z^i}{\partial z^2}-\frac{c_{66}^i}{r^2}\frac{\partial^2u_z^i}{\partial\theta^2}-\frac{c_{12}^i+c_{66}^i}{r}\frac{\partial^2u_\theta^i}{\partial z\partial\theta}+$$

$$+\frac{c_{36}^i}{r}\frac{\partial\sigma_{rr}^i}{\partial\theta}-\frac{c_{26}^i}{r^2}\frac{\partial u_r^i}{\partial\theta}-\frac{2c_{16}^i}{r}\frac{\partial^2u_z^i}{\partial z\partial\theta}-c_{16}^i\frac{\partial^2u_\theta^i}{\partial z^2}-\frac{c_{26}^i}{r^2}\frac{\partial^2u_\theta^i}{\partial\theta^2}+q_z;$$

$$\frac{\partial\tau_{r\theta}^i}{\partial r}=\frac{c_{23}^i}{r}\frac{\partial\sigma_{rr}^i}{\partial\theta}-\frac{2}{r}\tau_{r\theta}^i-\frac{c_{22}^i}{r^2}\frac{\partial u_r^i}{\partial\theta}-\frac{c_{12}^i+c_{66}^i}{r}\frac{\partial^2u_z^i}{\partial z\partial\theta}-c_{66}^i\frac{\partial^2u_\theta^i}{\partial z^2}-\frac{c_{22}^i}{r^2}\frac{\partial^2u_\theta^i}{\partial\theta^2}+$$

$$+c_{36}^i\frac{\partial\sigma_{rr}^i}{\partial z}-\frac{c_{26}^i}{r}\frac{\partial u_r^i}{\partial z}-c_{16}^i\frac{\partial^2u_z^i}{\partial z^2}-\frac{c_{26}^i}{r^2}\frac{\partial^2u_z^i}{\partial\theta^2}-\frac{2c_{26}^i}{r}\frac{\partial^2u_\theta^i}{\partial z\partial\theta}+q_\theta;$$

$$\frac{\partial u_r^i}{\partial r}=c_{33}^i\sigma_{rr}^i+\frac{c_{23}^i}{r}u_r^i+c_{13}^i\frac{\partial u_z^i}{\partial z}+\frac{c_{36}^i}{r}\frac{\partial u_z^i}{\partial\theta}+c_{36}^i\frac{\partial u_\theta^i}{\partial z}+\frac{c_{23}^i}{r}\frac{\partial u_\theta^i}{\partial\theta};$$

$$\frac{\partial u_z^i}{\partial r}=a_{55}^i\tau_{rz}^i+a_{45}^i\tau_{r\theta}^i-\frac{\partial u_r^i}{\partial z};\ \frac{\partial u_\theta^i}{\partial r}=a_{45}^i\tau_{rz}^i+a_{44}^i\tau_{r\theta}^i-\frac{1}{r}\frac{\partial u_r^i}{\partial\theta}+\frac{1}{r}u_\theta^i\ . \qquad (16)$$

Here r is the radius of the cylinder, which does not depend on the coordinates z and θ; σ_{rr}^i, τ_{rz}^i, $\tau_{r\theta}^i$ – stress tensor components (7); u_z^i, u_θ^i, u_r^i – movement of the shell in the directions of axes z, θ, r respectively. Steels c_{kl}^i (k, l =1, 2, 3, 6) are characteristics of the material of the shell layer, which are determined using mechanical constants a_{kl} [8]:

$$c_{11}^i=\frac{1}{\left|A_{22}^i\right|}\left(a_{22}^i a_{66}^i-a_{26}^{i\ 2}\right);\qquad c_{12}^i=\frac{1}{\left|A_{22}^i\right|}\left(a_{16}^i a_{26}^i-a_{12}^i a_{66}^i\right);$$

$$c_{22}^i=\frac{1}{\left|A_{22}^i\right|}\left(a_{11}^i a_{66}^i-a_{16}^{i\ 2}\right);\qquad c_{16}^i=\frac{1}{\left|A_{22}^i\right|}\left(a_{12}^i a_{26}^i-a_{22}^i a_{16}^i\right);$$

$$c_{26}^i = \frac{1}{\left|A_{22}^i\right|}\left(a_{12}^i a_{16}^i - a_{11}^i a_{26}^i\right); \qquad c_{66}^i = \frac{1}{\left|A_{22}^i\right|}\left(a_{11}^i a_{22}^i - {a_{12}^i}^2\right);$$

$$\left|A_{22}^i\right| = a_{66}^i\left(a_{11}^i a_{22}^i - {a_{12}^i}^2\right) + a_{26}^i\left(a_{12}^i a_{16}^i - a_{11}^i a_{26}^i\right) + a_{16}^i\left(a_{12}^i a_{26}^i - a_{22}^i a_{16}^i\right);$$

$$c_{13}^i = a_{13}^i c_{11}^i + a_{23}^i c_{12}^i + a_{36}^i c_{16}^i; \qquad c_{23}^i = a_{13}^i c_{12}^i + a_{23}^i c_{22}^i + a_{36}^i c_{26}^i;$$

$$c_{36}^i = a_{13}^i c_{16}^i + a_{23}^i c_{26}^i + a_{36}^i c_{66}^i; \qquad c_{33}^i = a_{33}^i - \left(a_{13}^i c_{13}^i + a_{23}^i c_{23}^i + a_{36}^i c_{36}^i\right). \qquad (17)$$

Thus, when using the variational equation (14), a heterogeneous three-dimensional system (16) is derived from six differential equations of equilibrium of the linear theory of elasticity. It is written in partial derivatives with respect to six components of the amplitude values of the vectors $\sigma_1^T = (\sigma_{rr}, \tau_{r\theta}, \tau_{rz})$ and $u^T = (u_r, u_\theta, u_z)$ and is used to study the stress-strain state of anisotropic non-thin composite cylindrical shells. To obtain it, the modified Hu-Washizu variational principle was used, which allows writing down the boundary conditions corresponding to the equations.

The generalized Hooke's law based on (3) and (6) and taking into account notations (7) and (17) can be written as:

$$\sigma_{zz}^i = c_{11}^i e_{zz}^i + c_{12}^i e_{\theta\theta}^i + c_{16}^i e_{z\theta}^i - c_{13}^i \sigma_{rr}^i;$$
$$\sigma_{\theta\theta}^i = c_{12}^i e_{zz}^i + c_{22}^i e_{\theta\theta}^i + c_{26}^i e_{z\theta}^i - c_{23}^i \sigma_{rr}^i;$$
$$\tau_{z\theta}^i = c_{16}^i e_{zz}^i + c_{26}^i e_{\theta\theta}^i + c_{66}^i e_{z\theta}^i - c_{36}^i \sigma_{rr}^i;$$
$$e_{rr}^i = c_{13}^i e_{zz}^i + c_{23}^i e_{\theta\theta}^i + c_{36}^i e_{z\theta}^i + c_{33}^i \sigma_{rr}^i;$$
$$e_{rz}^i = a_{45}^i \tau_{r\theta}^i + a_{55}^i \tau_{rz}^i; \quad e_{r\theta}^i = a_{44}^i \tau_{r\theta}^i + a_{45}^i \tau_{rz}^i. \qquad (18)$$

The solution of system (16), in the case of torsion, must meet the conditions on the lateral surfaces:
at $r = r_1$

$$\sigma_{rr}^0(r_1, z, \theta) = 0; \quad \tau_{rz}^0(r_1, z, \theta) = 0; \quad \tau_{r\theta}^0(r_1, z, \theta) = 0;$$

and $r = r_2$

$$\sigma_{rr}^n(r_2, z, \theta) = 0; \quad \tau_{rz}^n(r_2, z, \theta) = 0; \quad \tau_{r\theta}^n(r_2, z, \theta) = 0. \qquad (19)$$

Conditions at the ends at $z = 0$, $z = L$ (Fig. 1), for example

$$\tau_{z\theta} = \tau_{z\theta}^{/}, \quad \tau_{rz} = u_z = 0. \qquad (20)$$

Conditions for rigid contact of layers for stresses and displacements:

$$\sigma_{rr}^i(r_i) = \sigma_{rr}^{i+1}(r_i); \quad \tau_{rz}^i(r_i) = \tau_{rz}^{i+1}(r_i); \quad \tau_{r\theta}^i(r_i) = \tau_{r\theta}^{i+1}(r_i);$$
$$u_r^i(r_i) = u_r^{i+1}(r_i); \quad u_z^i(r_i) = u_z^{i+1}(r_i); \quad u_\theta^i(r_i) = u_\theta^{i+1}(r_i). \qquad (21)$$

Here i is the number of the shell layer, $\tau_{z\theta}^{/}$ the shear stress distributed on its ends corresponding to the applied twisting moment.

1.1.3. Research methodology. One of the numerical methods that allows reducing the dimensionality of system (16) is the method of straight lines [2, 4, 11]. Given that in the work we will consider only cases of axisymmetric deformation, we will reduce the thus obtained two-dimensional system of partial differential equations based on (16) to a one-dimensional system of ordinary differential equations by replacing the coordinate derivatives with difference relations z.

After simple mathematical operations [11], dependencies (16) are transformed into a one-dimensional system of the order $6n$ of ordinary differential equations with respect to the derivative on the coordinate r, which in abbreviated notation has the form

$$\frac{d\bar{y}}{dr}=T(r)\bar{y}\ , \tag{22}$$

where accepted $\bar{y}=\left\{\sigma_{rr}^1;\tau_{rz}^1;\tau_{r\theta}^1;u_r^1;u_z^1;u_\theta^1;...;\sigma_{rr}^{n-1};\tau_{rz}^{n-1};\tau_{r\theta}^{n-1};u_r^{n-1};u_z^{n-1};u_\theta^{n-1};\sigma_{rr}^n;\tau_{rz}^n;\tau_{r\theta}^n;u_r^n;u_z^n;u_\theta^n\right\}$ with boundary conditions (19); $T(r)$ − matrix of coefficients with unknown stress and displacement components, n – the number of equidistant straight lines (cross-sections) that divide the interval of the change of the derivatives by the coordinate along the generating line z.

The solution of the one-dimensional problem obtained in this way about the subcritical stress-strain state of an anisotropic non-thin layered cylindrical shell during torsion is carried out using the numerical method of discrete orthogonalization [1, 4].

1.2. The problem of stability

1.2.1. Modified mixed variational principle. To obtain the system of stability equations, we will use the elastic functional $W(e_{ij})$ (10) and use the following expansions in the form [9]:

$$\sigma_1=\sigma_1^0+\alpha\sigma_1^{(1)}+\alpha^2\sigma_1^{(2)}\ ;$$
$$\varepsilon_1=\varepsilon_1^0+\alpha\varepsilon_1^{(1)}+\alpha^2\varepsilon_1^{(2)}\ ;$$
$$\varepsilon_2=\varepsilon_2^0+\alpha\varepsilon_2^{(1)}+\alpha^2\varepsilon_2^{(2)}\ . \tag{23}$$

Here, the parameters of the stress-strain state with zero are subcritical values of strains and stresses; with indices (1) – disturbed; with indices (2) - also, only in a square; α - an infinitely small constant that is independent of coordinates.

Substituting (23) into (10) and performing the appropriate transformations, we obtain the following expression of the potential energy of deformation

$$\begin{aligned}W_1=&-\frac{1}{2}\left(\sigma_1^0+\alpha\sigma_1^{(1)}+\alpha^2\sigma_1^{(2)}\right)^T B_{11}^{-1}\left(\sigma_1^0+\alpha\sigma_1^{(1)}+\alpha^2\sigma_1^{(2)}\right)-\\&-\frac{1}{2}\left(\varepsilon_2^0+\alpha\varepsilon_2^{(1)}+\alpha^2\varepsilon_2^{(2)}\right)^T\left(B_{22}-B_{12}^TB_{11}^{-1}B_{12}\right)\left(\varepsilon_2^0+\alpha\varepsilon_2^{(1)}+\alpha^2\varepsilon_2^{(2)}\right)+\\&+\left[\left(\varepsilon_1^0+\alpha\varepsilon_1^{(1)}+\alpha^2\varepsilon_1^{(2)}\right)^T+\left(\varepsilon_2^0+\alpha\varepsilon_2^{(1)}+\alpha^2\varepsilon_2^{(2)}\right)^T B_{12}^TB_{11}^{-1}\right]\times\\&\times\left(\sigma_1^0+\alpha\sigma_1^{(1)}+\alpha^2\sigma_1^{(2)}\right)+\left(\varepsilon_2^0+\alpha\varepsilon_2^{(1)}+\alpha^2\varepsilon_2^{(2)}\right)^T\times\left(B_{22}-B_{12}^TB_{11}^{-1}B_{12}\right)\left(\varepsilon_2^0+\alpha\varepsilon_2^{(1)}+\alpha^2\varepsilon_2^{(2)}\right).\end{aligned} \tag{24}$$

After substituting (24) into (1) from the condition of stationarity of the variation of the functional (1) caused by the change in the components of the vector of displacements u and stresses σ_1, when using expressions for stresses $\sigma_1^T=(\sigma_{rr},\tau_{r\theta},\tau_{rz})$, displacements $u^T=(u_r,u_\theta,u_z)$, and geometric ratios in the form [9]

$$\varepsilon_{zz}^{(1)}=\frac{\partial u_z^{(1)}}{\partial z};\quad \varepsilon_{\theta\theta}^{(1)}=\frac{1}{r}\frac{\partial u_\theta^{(1)}}{\partial\theta}+\frac{1}{r}u_r^{(1)};\quad \varepsilon_{rr}^{(1)}=\frac{\partial u_r^{(1)}}{\partial r};\quad \varepsilon_{z\theta}^{(1)}=\frac{\partial u_\theta^{(1)}}{\partial z}+\frac{1}{r}\frac{\partial u_z^{(1)}}{\partial\theta};$$
$$\varepsilon_{rz}^{(1)}=\frac{\partial u_r^{(1)}}{\partial z}+\frac{\partial u_z^{(1)}}{\partial r};\quad \varepsilon_{r\theta}^{(1)}=\frac{\partial u_\theta^{(1)}}{\partial r}-\frac{1}{r}u_\theta^{(1)}+\frac{1}{r}\frac{\partial u_r^{(1)}}{\partial\theta}, \tag{25}$$

neglecting the dependences for the variation of kinetic energy and the potentials of surface and volume loads, equating the expressions for independent variations of stresses $\delta\sigma_{rr}$, $\delta\tau_{r\theta}$, $\delta\tau_{rz}$ and displacements δu_r, δu_θ, δu_z in the integral over the volume V to zero, we obtain the following system of stability equations in the spatial setting of anisotropic thin composite cylindrical shells :

$$\frac{\partial\sigma_{rr}}{\partial r}=-\frac{c_{23}+1}{r}\sigma_{rr}-\frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta}-\frac{\partial\tau_{rz}}{\partial z}+\frac{c_{12}}{r}\frac{\partial u_z}{\partial z}+\frac{c_{22}}{r^2}u_r+\frac{c_{22}}{r^2}\frac{\partial u_\theta}{\partial\theta}+\frac{c_{26}}{r^2}\frac{\partial u_z}{\partial\theta}+\frac{c_{26}}{r}\frac{\partial u_\theta}{\partial z}+$$

$$+\left(-\frac{\partial u_z}{\partial z}c_{13}-\frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta}+u_r\right)c_{23}-\left(\frac{\partial u_\theta}{\partial z}+\frac{1}{r}\frac{\partial u_z}{\partial \theta}\right)c_{36}-\sigma_{rr}c_{33}+r\frac{\partial^2 u_r}{\partial z^2}c_{13}+\frac{1}{r}\frac{\partial^2 u_r}{\partial \theta^2}c_{23}-\right.$$

$$\left.-\frac{2}{r}\frac{\partial u_\theta}{\partial \theta}c_{23}-\frac{1}{r}u_r c_{23}+2\frac{\partial^2 u_r}{\partial z\partial \theta}c_{36}-2\frac{\partial u_\theta}{\partial z}c_{36}\right)\sigma_{rr}^0+\left(-2r\left(\frac{\partial^2 u_z}{\partial z^2}c_{13}+\frac{1}{r}\left(\frac{\partial^2 u_\theta}{\partial z\partial \theta}+\frac{\partial u_r}{\partial z}\right)c_{23}+\right.\right.$$

$$\left.\left.+\left(\frac{\partial^2 u_\theta}{\partial z^2}+\frac{1}{r}\frac{\partial^2 u_z}{\partial z\partial \theta}\right)c_{36}+\frac{\partial \sigma_{rr}}{\partial z}c_{33}\right)-\frac{\partial u_r}{\partial z}\right)\tau_{rz}^0+\left(-2\left(\frac{\partial^2 u_z}{\partial z\partial \theta}c_{13}+\frac{1}{r}\left(\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{\partial u_r}{\partial \theta}\right)c_{23}+\right.\right.$$

$$\left.\left.+\left(\frac{\partial^2 u_\theta}{\partial z\partial \theta}+\frac{1}{r}\frac{\partial^2 u_z}{\partial \theta^2}\right)c_{36}+\frac{\partial \sigma_{rr}}{\partial \theta}c_{33}\right)-\frac{1}{r}\frac{\partial u_r}{\partial \theta}+\frac{1}{r}u_\theta\right)\tau_{r\theta}^0;$$

$$\frac{\partial \tau_{rz}}{\partial r}=\frac{\partial \sigma_{rr}}{\partial z}c_{13}+\frac{1}{r}\frac{\partial \sigma_{rr}}{\partial \theta}c_{36}-\frac{1}{r}\tau_{rz}-c_{11}\frac{\partial^2 u_z}{\partial z^2}-\frac{2c_{16}}{r}\frac{\partial^2 u_z}{\partial z\partial \theta}-\frac{c_{12}}{r}\frac{\partial u_r}{\partial z}-\frac{c_{66}}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}-\frac{c_{12}+c_{66}}{r}\frac{\partial^2 u_\theta}{\partial z\partial \theta}-$$

$$-\frac{c_{26}}{r^2}\frac{\partial u_r}{\partial \theta}-c_{16}\frac{\partial^2 u_\theta}{\partial z^2}-\frac{c_{26}}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\left(\frac{\partial u_r}{\partial z}-\tau_{r\theta}a_{45}-\tau_{rz}a_{55}+r\frac{\partial^2 u_z}{\partial z^2}c_{13}+\frac{1}{r}\frac{\partial^2 u_z}{\partial \theta^2}c_{23}+\right.$$

$$\left.+2\frac{\partial^2 u_z}{\partial \theta\partial z}c_{36}\right)\sigma_{rr}^0+\left(-2r\left(-\frac{\partial^2 u_r}{\partial z^2}+\frac{\partial \tau_{r\theta}}{\partial z}a_{45}+\frac{\partial \tau_{rz}}{\partial z}a_{55}\right)-\frac{\partial u_z}{\partial z}\right)\tau_{rz}^0+$$

$$+\left(-2\left(-\frac{\partial^2 u_r}{\partial z\partial \theta}+\frac{\partial \tau_{r\theta}}{\partial \theta}a_{45}+\frac{\partial \tau_{rz}}{\partial \theta}a_{55}\right)-\frac{1}{r}\frac{\partial u_z}{\partial \theta}\right)\tau_{r\theta}^0;$$

$$\frac{\partial \tau_{r\theta}}{\partial r}=\frac{1}{r}\frac{\partial \sigma_{rr}}{\partial \theta}c_{23}+\frac{\partial \sigma_{rr}}{\partial z}c_{36}-\frac{2}{r}\tau_{r\theta}-\frac{c_{12}+c_{66}}{r}\frac{\partial^2 u_z}{\partial z\partial \theta}-\frac{c_{22}}{r^2}\frac{\partial u_r}{\partial \theta}-c_{66}\frac{\partial^2 u_\theta}{\partial z^2}-\frac{c_{22}}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}-\frac{c_{26}}{r}\frac{\partial u_r}{\partial z}-$$

$$-c_{16}\frac{\partial^2 u_z}{\partial z^2}-\frac{c_{26}}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}-\frac{2c_{26}}{r}\frac{\partial^2 u_\theta}{\partial z\partial \theta}+\left(-\frac{1}{r}u_\theta+\frac{1}{r}\frac{\partial u_r}{\partial \theta}-\tau_{r\theta}a_{44}-\tau_{rz}a_{45}+r\frac{\partial^2 u_\theta}{\partial z^2}c_{13}+\frac{1}{r}\frac{\partial^2 u_\theta}{\partial \theta^2}c_{23}+\right.$$

$$\left.+\frac{2}{r}\frac{\partial u_r}{\partial \theta}c_{23}-\frac{1}{r}u_\theta c_{23}+2\frac{\partial^2 u_\theta}{\partial z\partial \theta}c_{36}+2\frac{\partial u_r}{\partial z}c_{36}\right)\sigma_{rr}^0+\left(-2r\left(\frac{1}{r}\frac{\partial u_\theta}{\partial z}-\frac{1}{r}\frac{\partial^2 u_r}{\partial z\partial \theta}+\frac{\partial \tau_{r\theta}}{\partial z}a_{44}+\frac{\partial \tau_{rz}}{\partial z}a_{45}\right)-\right.$$

$$\left.-\frac{\partial u_\theta}{\partial z}\right)\tau_{rz}^0+\left(-2\left(\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}-\frac{1}{r}\frac{\partial^2 u_r}{\partial \theta^2}+\frac{\partial \tau_{r\theta}}{\partial \theta}a_{44}+\frac{\partial \tau_{rz}}{\partial \theta}a_{45}\right)-\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}-\frac{1}{r}u_r\right)\tau_{r\theta}^0;$$

$$\frac{\partial u_r}{\partial r}=\frac{\partial u_z}{\partial z}c_{13}+\frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta}+u_r\right)c_{23}+\left(\frac{\partial u_\theta}{\partial z}+\frac{1}{r}\frac{\partial u_z}{\partial \theta}\right)c_{36}+\sigma_{rr}c_{33};$$

$$\frac{\partial u_z}{\partial r}=-\frac{\partial u_r}{\partial z}+\tau_{r\theta}a_{45}+\tau_{rz}a_{55};$$

$$\frac{\partial u_\theta}{\partial r}=\frac{1}{r}u_\theta-\frac{1}{r}\frac{\partial u_r}{\partial \theta}+\tau_{r\theta}a_{44}+\tau_{rz}a_{45}. \qquad (26)$$

In (26) r – the radius of the cylinder (Fig. 1) is independent of the coordinates z and θ; σ_{rr}, τ_{rz}, $\tau_{r\theta}$ – stress vector components (3); u_z, u_θ, u_r – moving the shell according to the directions of the corresponding axes z, θ, r. Stresses $\sigma_{rr}^0, \tau_{rz}^0$ and $\tau_{r\theta}^0$ are determined by solving the problem of the subcritical stress-strain state (22).

Thus, using the modified Hu-Washizu variational principle, a three-dimensional system of six homogeneous differential equations of stability in partial derivatives with respect to the components of the vectors $\sigma_1^T=(\sigma_{rr}, \tau_{r\theta}, \tau_{rz})$ and is obtained $u^T=(u_r, u_\theta, u_z)$.

The given system of stability equations (26) must meet the conditions on the side surfaces of the shell of type (19), conditions at the ends (20) and conditions of rigid contact of layers for stresses and displacements (21).

1.2.2. Research methodology. We will reduce the dimensionality of the three-dimensional system of stability equations (26) using the procedure of the Bubnov-Galyorkin method. Let us decompose the functions describing stresses and strains (26) into double trigonometric series so that conditions (20) are satisfied along the generator z and take into account the periodicity in the circular direction θ:

$$\sigma_{rr}(r,z,\theta)=\sum_{m=1}^{\infty}\sum_{k=0}^{\infty}\left[y_{1,pk}(r)\cos k\theta+y'_{1,mk}(r)\sin k\theta\right]\sin l_m z\,;$$

$$\tau_{rz}(r,z,\theta)=\sum_{m=0}^{\infty}\sum_{k=0}^{\infty}\left[y_{2,pk}(r)\cos k\theta+y'_{2,mk}(r)\sin k\theta\right]\cos l_m z\,;$$

$$\tau_{r\theta}(r,z,\theta)=\sum_{m=1}^{\infty}\sum_{k=0}^{\infty}\left[y_{3,pk}(r)\sin k\theta+y'_{3,mk}(r)\cos k\theta\right]\sin l_m z\,;$$

$$u_r(r,z,\theta)=\sum_{m=1}^{\infty}\sum_{k=0}^{\infty}\left[y_{4,pk}(r)\cos k\theta+y'_{4,mk}(r)\sin k\theta\right]\sin l_m z\,;$$

$$u_z(r,z,\theta)=\sum_{m=0}^{\infty}\sum_{k=0}^{\infty}\left[y_{5,pk}(r)\cos k\theta+y'_{5,mk}(r)\sin k\theta\right]\cos l_m z\,;$$

$$u_\theta(r,z,\theta)=\sum_{m=1}^{\infty}\sum_{k=0}^{\infty}\left[y_{6,pk}(r)\sin k\theta+y'_{6,mk}(r)\cos k\theta\right]\sin l_m z\,. \tag{27}$$

In (27) $y_{i,pk}$, $y'_{i,mk}$ (i=1÷6) are components of stress components σ_{rr}, τ_{rz}, $\tau_{r\theta}$ and displacements u_r, u_z, u_θ decomposed by trigonometric Fourier series, p, m, k are wave numbers in the series. Parameter $l_m = m\pi/L$, where L is the length of the generating cylinder (Fig. 1).

To take into account the variability of stresses for the length of the envelope, we use the discrete Fourier transform operation. In accordance with it, we present the distribution along the z axis subcritical values $\sigma^0_{rr}, \tau^0_{rz}$ and $\tau^0_{r\theta}$, obtained using the straight line method, in the form of series:

$$\sigma^0_{rr}(z)=\frac{a_0^{\sigma^0_{rr}}}{2}+\sum_{i=1}^{n-1}a_i^{\sigma^0_{rr}}\cdot\cos\frac{2\pi i}{N\cdot z_{od}}z+\sum_{i=1}^{n-1}b_i^{\sigma^0_{rr}}\cdot\sin\frac{2\pi i}{N\cdot z_{od}}z\,;$$

$$\tau^0_{rz}(z)=\frac{a_0^{\tau^0_{rz}}}{2}+\sum_{i=1}^{n-1}a_i^{\tau^0_{rz}}\cdot\cos\frac{2\pi i}{N\cdot z_{od}}z+\sum_{i=1}^{n-1}b_i^{\tau^0_{rz}}\cdot\sin\frac{2\pi i}{N\cdot z_{od}}z\,;$$

$$\tau^0_{r\theta}(z)=\frac{a_0^{\tau^0_{r\theta}}}{2}+\sum_{i=1}^{n-1}a_i^{\tau^0_{r\theta}}\cdot\cos\frac{2\pi i}{N\cdot z_{od}}z+\sum_{i=1}^{n-1}b_i^{\tau^0_{r\theta}}\cdot\sin\frac{2\pi i}{N\cdot z_{od}}z\,, \tag{28}$$

where the following notations are introduced: i – the number of members of the series $i=\overline{1,n-1}$; $n=(N+1)/2$; N is the number of equidistant points by which the shell is broken along the generating cylinder when solving the problem of the subcritical stress-strain state; z_{od} – the distance between these points along the z coordinate in the cylindrical coordinate system (Fig. 1); $a_0^{\sigma^0_{rr}}$, $a_0^{\tau^0_{rz}}$, $a_0^{\tau^0_{r\theta}}$, $a_i^{\sigma^0_{rr}}$, $a_i^{\tau^0_{rz}}$, $a_i^{\tau^0_{r\theta}}$, $b_i^{\sigma^0_{rr}}$, $b_i^{\tau^0_{rz}}$, $b_i^{\tau^0_{r\theta}}$ – coefficients of the trigonometric Fourier series into which the corresponding components of the stress state are decomposed $\sigma^0_{rr,j}$, $\tau^0_{rz,j}$, $\tau^0_{r\theta,j}$, $j=1\div N$.

By separating the variables in equations (26) using dependencies (27), while taking into account relation (28), we obtain an infinite one-dimensional system of ordinary homogeneous differential equations of stability of a cylindrical shell in the normal Cauchy form

$$\frac{d\bar{y}}{dr} = T(r,\lambda)\bar{y}, \quad T(r,\lambda) = t_{i,j}(r,\lambda),\ i = \overline{1,\infty},\ j = \overline{1,\infty}. \tag{29}$$

In (29) $\bar{y} = \{y_{1,pk}; y_{2,pk}; y_{3,pk}; y_{4,pk}; y_{5,pk}; y_{6,pk}; y'_{1,mk}; y'_{2,mk}; y'_{3,mk}; y'_{4,mk}; y'_{5,mk}; y'_{6,mk}\}$ the solving vector function $T(r,\lambda)$ is a matrix with variable coefficients that depends on the argument r and the load parameter λ.

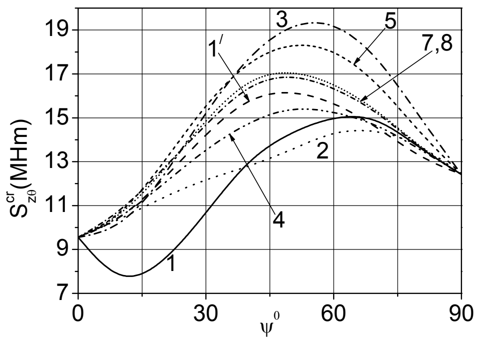
The system of stability equations (29) under the conditions on the surfaces (19) is solved using the numerical method of discrete orthogonalization [1, 4].

The presented algorithm is implemented in the form of packages of application programs for a PC, where the setting of the parameters of the subcritical stress-strain state and the solution of stability problems of non-thin anisotropic cylindrical shells subjected to torsion are combined in a single process.

2. Implementation of the proposed method of setting critical loads of a composite anisotropic cylindrical shell. Let's investigate the effect of changing the number of layers of an anisotropic shell on the values of its critical loads in the case of torsion. To do this, consider the stability of a cylindrical shell with a length of L = 1,2 m; radii of the inner r_1 =0,585m and outer surfaces r_2 =0,615m. The shell is formed by reinforcing the composite at angles $\pm\psi$ to the z axis. Fiberglass with the following physical and mechanical characteristics was selected as a composite material: E_{zz}=44,5E_0, $E_{\theta\theta}$=E_{rr}=10,7E_0, $G_{z\theta}$=$G_{r\theta}$=4,18E_0, G_{rz}=8,48E_0, $\nu_{\theta z}$=0,26, $\nu_{z\theta}$=0,0628, E_0=1000MPa .

In fig. 2 presents graphs describing the dependence of the critical values of shear loads (torques) $S_{z\theta}^{cr}$ on the angle of rotation ψ of the main directions of elasticity of the composite material and the number of cross-reinforced $\pm\psi$ layers for the cases of application of the end torsional moment in positive (Fig. 2, a) and negative (Fig. 2, b) directions.

In fig. 2 (a), (b), plotted in the $S_{z\theta}^{cr}$ − axes ψ, the numbering of the curves corresponds to the number of layers reinforced at angles $\pm\psi$ to the resulting cylindrical shell, the curve $1'$ (dashed) represents the results of calculating the stability problem of an anisotropic cylinder according to the orthotropic approach when the mechanical c_{16} characteristics c_{26}, c_{36}, a_{45} of the accepted generalized Hooke's law (18) have zero values.



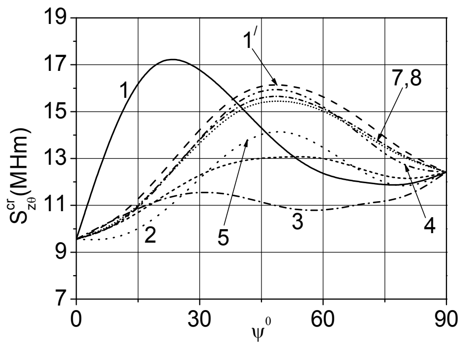


Fig. 2. Values of critical shear loads $S_{z\theta}^{cr}$ for one- (1), two- (2), three- (3), four- (4), five- (5), seven- (7), eight-layer (8) shells and the results are obtained on the basis of the orthotropic approach ($1'$) for the direction of application of the twisting moment: (a) positive; (b) negative

From the analysis of the results presented in fig. 2 it is possible to draw the following conclusions. The critical values of shear loads depend on the angle of rotation of the main directions of elasticity of the material, the number of cross-reinforced layers, and the direction of application of the twisting moment. We also note that the critical loads $S_{z\theta}^{cr}$ determined for the anisotropic shell according to the orthotropic approach (curves $1^{/}$) do not depend on the number of layers with cross reinforcement and remain constant. At the same time, an increase in the number of layers with cross-reinforcement leads to an approximation of the critical loads $S_{z\theta}^{cr}$ determined taking into account all the constants of the generalized Hooke's law of the considered material to those obtained according to the orthotropic approach. If the maximum discrepancies between the results for a single-layer anisotropic cylinder (curve 1) and the critical loads obtained for the orthotropic equivalent shell (curve $1^{/}$) are 46% and 69%, respectively, from positive (Fig. 2 (a)) and negative (Fig. 2 (b)) applied loads, then for the two-layer (curve 2) in comparison with graph $1^{/}$ the differences decrease to 25% and 16%, respectively, according to the signs of the loads. A further increase in the number of layers with cross-reinforcement leads to the fact that with seven to eight layers, the discrepancy between the anisotropic and orthotropic approaches to the calculation decreases to a maximum of 5%. At the same time, we note that the critical loads $S_{z\theta}^{cr}$ obtained for anisotropic cylinders with the number of layers seven to eight (curves 7, 8) from the twisting moment applied in the positive direction (Fig. 2 (a)) are slightly greater than those $S_{z\theta}^{cr}$ determined according to the orthotropic approach (curve $1^{/}$), and from the negative, on the contrary, smaller (Fig. 2 (b)).

In general, from the analysis of the results, it can be seen that for the considered anisotropic cylindrical shells, increasing the layers of the cross-reinforced package to seven to eight or more leads to the possibility of calculating such shells according to the orthotropic approach, which confirms the results given in [1].

Conclusions

The paper proposes an approach to obtaining and solving three-dimensional systems of inhomogeneous equations of the subcritical stress-strain state and homogeneous partial differential stability equations for anisotropic thin cylindrical shells based on the modification of the Hu-Washizu variational principle. To reduce the obtained systems to one-dimensional, the methods of straight lines for the stress-strain state problem and decomposition into double trigonometric series with approximation of stress components and displacements in the direction of the source using the procedure of the Bubnov-Galyorkin method and taking into account the periodicity of the solving functions in the circular direction for the stability problem were used. The solution of the obtained one-dimensional systems in the direction normal to the middle surface of the shell was carried out using the numerical method of discrete orthogonalization. The proposed approach makes it possible to solve problems of stability of cylindrical shells at different angles of reinforcement of the construction material relative to the structure.

The problem of stability of a non-thin composite anisotropic shell against end torsional loads is solved, depending on the number of layers and the angles of rotation of their main directions of elasticity, according to the proposed approach and using the orthotropic model for the calculation of anisotropic shells. A comparison of the obtained results was carried out and a conclusion was drawn about the use of the proposed approach.

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Стаття надійшла 16.10.2023

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СТІЙКІСТЬ НЕТОНКИХ ЦИЛІНДРИЧНИХ АНІЗОТРОПНИХ ОБОЛОНОК ПІД ДІЄЮ КРУЧЕННЯ В ТРИВИМІРНІЙ ПОСТАНОВЦІ

В статті у просторовій постановці приведено розрахунок на стійкість нетонких циліндричних анізотропних шаруватих композитних оболонок під дією торцевих крутних моментів. Анізотропія використовуваного матеріалу характеризується однією площиною пружної симетрії його характеристик. Це викликано не співпадінням головних напрямків пружності волокнистого композитного ортотропного матеріалу та осями криволінійної циліндричної системи координат.

Тривимірна неоднорідна система диференціальних рівнянь у частинних похідних, що описує, в межах лінійної теорії пружності, докритичний напружено-деформований стан виведена при використанні варіаційного принципу Ху-Васідзу. Зменшення розмірності розглядуваної задачі з тривимірної до одновимірної проводиться при урахуванні осьової симетрії деформування циліндричної оболонки та використанням, у вздовж твірної, методу прямих.

Спираючись на модифікований варіаційний принцип Ху-Васідзу, виведено тривимірну систему однорідних диференціальних рівнянь стійкості у частинних похідних в рамках просторової теорії пружності. Приведення тривимірної системи до одновимірної здійснюється у вздовж твірної та за коловим напрямком - шляхом розкладення компонентів напружень і переміщень у подвійні тригонометричні ряди при застосуванні процедури методу Бубнова-Гальоркіна, а також з урахуванням періодичності розв'язуючих функцій.

Розроблено алгоритм, який реалізований у вигляді пакетів прикладних програм для ПК. В ньому в єдиному обчислювальному процесі, за використання чисельного методу дискретної ортогоналізації у напрямку нормальному до серединної поверхні оболонки, поєднуються встановлення параметрів докритичного напружено-деформованого стану та розв'язку на цій основі задач стійкості нетонких анізотропних циліндричних оболонок, що знаходяться під дією кручення.

Розглянута задача про вплив на стійкість анізотропної циліндричної нетонкої оболонки збільшення кількості перехресно-армованих шарів в залежності від кута повороту головних напрямів пружності матеріалу та напрямку прикладання крутного моменту. Проведене співставлення отриманих результатів розрахунків на стійкість згідно запропонованого підходу із критичними навантаженнями кручення, що вирахувані при використанні ортотропної моделі розрахунку анізотропних оболонок. Показано, що для одношарових циліндричних оболонок розходження між порівнюваними результатами сягає 69%. Збільшення

кількості перехресно-армованих шарів веде до зменшення цієї розбіжності та при семи-восьми шарах різниця між критичними навантаженнями отриманими за описаним підходом та ортотропної моделлю знаходиться в межах 5%. Такий результат цілком узгоджуються з тими, що отримані при використанні класичних чи уточнених теорій розрахунків як тонких, так і нетонких анізотропних циліндричних оболонок.

Ключові слова: анізотропна циліндрична оболонка, тривимірна постановка, стійкість при крученні.

Trach V.M., Podvornyi A.V.

STABILITY OF CYLINDRICAL ANISOTROPIC COMPOSITE SHELLS UNDER TORSION IN A THREE-DIMENSIONAL FORMULATION

The article presents a calculation of the stability of non-thin cylindrical anisotropic layered shells under the action of end torsional moments in a spatial formulation. The anisotropy of the used material is characterized by one plane of elastic symmetry of characteristics. This is caused by the mismatch between the main elastic directions of the composite fibrous orthotropic material and the axes of the curvilinear cylindrical coordinate system.

A three-dimensional inhomogeneous system of partial differential equations describing the subcritical stress-strain state within the linear theory of elasticity is derived using the Hu-Washizu variational principle. Reducing the dimension of the problem under consideration from three-dimensional to one-dimensional is carried out by taking into account the axial symmetry of the deformation of the cylindrical shell and using the method of straight lines along the generatrix.

Based on the modified Hu-Washizu variational principle, a three-dimensional system of homogeneous partial differential stability equations is derived within the framework of the spatial theory of elasticity. The reduction of a three-dimensional system to a one-dimensional one is carried out along the generatrix and in the circular direction - by expanding the components of stresses and displacements into double trigonometric series when applying the procedure of the Bubnov-Galorkin method, as well as taking into account the periodicity of the resolving functions.

An algorithm has been developed, implemented in the form of application software packages for PCs. In it, in a single computational process using the numerical method of discrete orthogonalization in the direction normal to the middle surface of the shell, the establishment of the parameters of the subcritical stress-strain state and the solution on this basis of stability problems for non-thin anisotropic cylindrical shells under the influence of torsion are combined.

The problem of the influence on the stability of an anisotropic cylindrical non-thin shell of an increase in the number of cross-reinforced layers depending on the angle of rotation of the main directions of elasticity of the material and the direction of application of torque is considered. The obtained results of stability calculations according to the proposed approach were compared with critical torsion loads calculated using an orthotropic model for calculating anisotropic shells. It is shown that for single-layer cylindrical shells the difference between the compared results reaches 69%. An increase in the number of cross-reinforced layers leads to a decrease in this discrepancy, and with seven to eight layers, the difference between the critical loads obtained using the described approach and the orthotropic model is within 5%. This result is consistent with those obtained using classical or refined theories of calculations of both thin and non-thin anisotropic cylindrical shells.

Key words: anisotropic cylindrical shell, three-dimensional setting, torsional stability.

УДК 624.074.04

Трач В.М., Подворний А.В. **Стійкість нетонких циліндричних анізотропних оболонок під дією кручення в тривимірній постановці** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2023. – Вип. 111. – С. 74-86.

Розглядається розрахунок нетонких циліндричних анізотропних шаруватих оболонок під дією торцевих скручуючих моментів у просторовій постановці. Розглядувана анізотропія характеризується однією площиною пружних характеристик матеріалу. Для отримання тривимірних систем рівнянь докритичної рівноваги та стійкості просторової теорії пружності, використано модифікацію варіаційного принципу Ху-Васідзу. Чисельний розв'язок поставленої задачі проводиться при використанні методів Бубнова-Гальоркіна, дискретних перетворень Фур'є та дискретної ортогоналізації. Розглянута задача стійкості анізотропної циліндричної нетонкої оболонки при збільшенні кількості перехресно-армованих шарів в залежності від кута повороту головних напрямів пружності матеріалу та напрямку прикладання скручуючого моменту.

Табл. -. Іл. 2. Бібліогр. 17 назв.

UDC 539.3

Trach V.M., Podvornyi A.V. **Stability of cylindrical anisotropic composite shells under torsion in a three-dimensional formulation** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – Kyiv: KNUBA, 2023. – Issue 111. – P. 74-86.

The calculation of cylindrical anisotropic layered composite shells under the action of end torques in a spatial setting is considered. The considered anisotropy is characterized by one plane of the material's elastic characteristics. To derive three-dimensional systems of equations of subcritical equilibrium and stability of the spatial theory of elasticity, a modification of the Hu-Washizu variational principle was used. Solving the problems of the pre-critical stress-strain state and stability is carried out using the Bubnov-Galyorkin methods, discrete Fourier transforms and numerical discrete orthogonalization. The problem of stability of an anisotropic cylindrical thick-walled shell with an increase in the number of cross-reinforced layers is considered, depending on the angle of rotation of the main directions of elasticity of the material and the direction of torque application.

Tabl. -. Fig. 2. Ref. 17

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