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COMPARATIVE ANALYSIS OF THE STABILITY AND NATURAL VIBRATIONS OF SHALLOW PANELS UNDER THE ACTION OF THERMOMECHANICAL LOADS

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The work is a continuation of research devoted to substantiating the reliability of solutions obtained by the finite element method for the analysis of nonlinear deformation, buckling and vibrations of thin elastic shells under the action of thermomechanical loads. The method is based on geometrically nonlinear relations of the three-dimensional theory of thermoelasticity and the principles of the moment finite element scheme. A thin elastic shell of an inhomogeneous structure is modeled by a universal spatial isoparametric finite element. The modal analysis of the shell is implemented at each step of the static thermomechanical load. The subspace iteration method is used to determine the spectrum of the lowest frequencies of natural vibrations of shells. A shallow spherical panel with a square plan is considered. The effect of preheating on the loss of stability and vibrations of an elastic isotropic shell under uniform pressure loading is investigated. The behavior of the shell weakened by two pairs of cross-channels is analyzed. The weakening of the panel by narrow and wide channels, which can be eccentrically located relative to the middle surface of the shell, is considered. The effectiveness and adequacy of the method is confirmed by a comparative analysis of solutions with results obtained using modern multifunctional software systems LIRA-SAPR and SCAD. The features of using the systems for solving the problems under consideration are given. Analysis of the results made it possible to evaluate the possibilities of using these software systems to substantiate the reliability of solutions to certain classes of problems of geometrically nonlinear deformation, buckling and vibrations of elastic shells.

Keywords: elastic shell, thermo mechanical loads, stability, modal analysis, universal 3D finite element, finite element moment scheme, comparative analysis.

Introduction. Improvement of existing and development of new methods and algorithms for the analysis of shell behavior is important for the effective use of thin-walled structures in various fields of engineering. During operation, real shell systems can be under the influence of loads of various nature, including mechanical and thermal. According to their functional purpose, shells can have different structural inhomogeneities. These include: ribs and overlays, reinforced and unreinforced holes, notches, channels, local thickening and thinning, mid-surface fractures, and other features.

Determination of its stability is important when calculating the shell [1-6]. Since the action of loads on the structure affects the distribution of movements and forces, therefore, when determining dynamic characteristics, such as self-oscillations, it is necessary to take into account mechanical and thermal effects [2-3, 7-10].

The primary task in the development of any method is to substantiate the reliability of the obtained solutions. Due to the insufficient number of test verification tasks, analytical solutions in the literature and the poorly researched class of shells for which they are developed, it is appropriate to conduct comparative analyzes using certified software (SW), which include domestic LIRA-SAPR SW [11,12] and SCAD [13,14].

This work is devoted to the further confirmation of the reliability of the solutions obtained by the developed method in the problems of stability and natural oscillations of elastic shells with various geometric features. The purpose of the work is also to study the possibilities of using modern LIRA-SAPR and SCAD SW to study the processes of nonlinear deformation and stability of thin shells, in particular of step-variable thickness, and to conduct a comparative analysis of solutions on this basis.

1. Solving geometrically nonlinear problems of deformation and buckling of inhomogeneous shells using the finite element moment scheme

The method of solving the problems of geometrically nonlinear deformation, stability, postcritical behavior, and self-oscillations of inhomogeneous shells under the action of thermomechanical loads is based on geometrically nonlinear relations of the three-dimensional theory of thermoelasticity [1-3, 15]. This approach is modern and effective [16-18].

The used model of a linear elastic continuous medium described by Hooke's law, which in the presence of a thermal field takes the form of the generalized Duhamel-Neumann law. The most common types of materials are used to describe the thermoelastic properties of the shell material: isotropic, transversely isotropic, and orthotropic. The study of the processes of nonlinear deformation of shells is based on the general Lagrangian formulation of the variational problem in increments. The finite-element relations were obtained by the variational method in curvilinear coordinates, taking into account all nonlinear terms, components of strain and stress tensors. A combination of the stepwise method of continuing the solution for the perturbation parameter with the procedure of the Newton-Kantorovich iterative method at the load step is used to construct the equilibrium trajectories of the structure. The created algorithm provides automation of the process of obtaining a solution to the problem regardless of the complexity of the "load-deflection" diagram and provides an opportunity to investigate the closed behavior of the shell. [1]. The algorithm provides, for example, the selection of the type of continuation of the solution parameter (loading or moving the characteristic node selected by the algorithm), adjusting the step value of the continuation of the solution parameter (decrease or increase), changing the accuracy of the solution of the system of nonlinear equations, and other actions for automated debugging operation of the algorithm in a mode close to optimal in terms of machine time consumption.

The application of the incremental approach provides an opportunity at each step of static thermomechanical loading to determine the modal characteristics of the inhomogeneous shell, taking into account the deformed and prestressed states, which significantly affect the spectrum of the structure's own vibrations.

The thin shell is considered as a three-dimensional body. Along the thickness, it is modeled by one 3D isoparametric finite element (FE) with polylinear shape functions:

$$\left\{x^{i'}:u^{i'}\right\}\left(x^1,x^2,x^3\right)=\sum_{s_1=\pm1}\sum_{s_2=\pm1}\sum_{s_3=\pm1}F\left(x^k,s_k\right)\left\{x^{i'}:u^{i'}\right\}_{s_1s_2s_3},\ F\left(x^k,s_k\right)=\prod_{k=1}^{3}\left(s_{(k)}x^{(k)}+\frac{1}{2}\right),$$

where $s_k=sign(x^k_{s_1s_2s_3})$ – conditional (grid) Lagrangian coordinates of FE nodes (Fig. 1 (b)); $x^k_{s_1s_2s_3}=\pm\frac{1}{2}$ – local normalized coordinates of FE nodes; $x^{i'}_{s_1s_2s_3}$ and $u^{i'}_{s_1s_2s_3}$ – given (known) values of the Cartesian coordinates of FE nodes and sought (unknown) values of displacements of these nodes, respectively; the denotation ":" corresponds to the logical operator "or", which means choosing to consider one of the components in curly brackets.

The developed 3D FE is universal. It is intended for the modeling of regions of the shell without constructive geometrical features by thickness (casing) as well as areas with such features. Thus, it is unique for a shell of step-variable thickness. Universal FE due to the introduction of new variable additional parameters (topological, geometric and physical-mechanical) and redefinition of the corresponding basic ones acquires the properties of a modified [1, 19].

Transformation of the casing finite element (CFE) (hexahedron $ABCDEFGH$ (Fig. 1 (a)) into FE with changed dimensions and location relative to the middle surface of the casing (hexahedron $\tilde{A}\tilde{B}\tilde{C}\tilde{D}\tilde{E}\tilde{F}\tilde{G}\tilde{H}$, (Fig. 1 (c), (d))) is performed along the local axis x^1, that is, along the thickness of the shell (Fig. 1). For ease of description, the element formed as a result of these changes will be called the "modified" finite element (MFE).

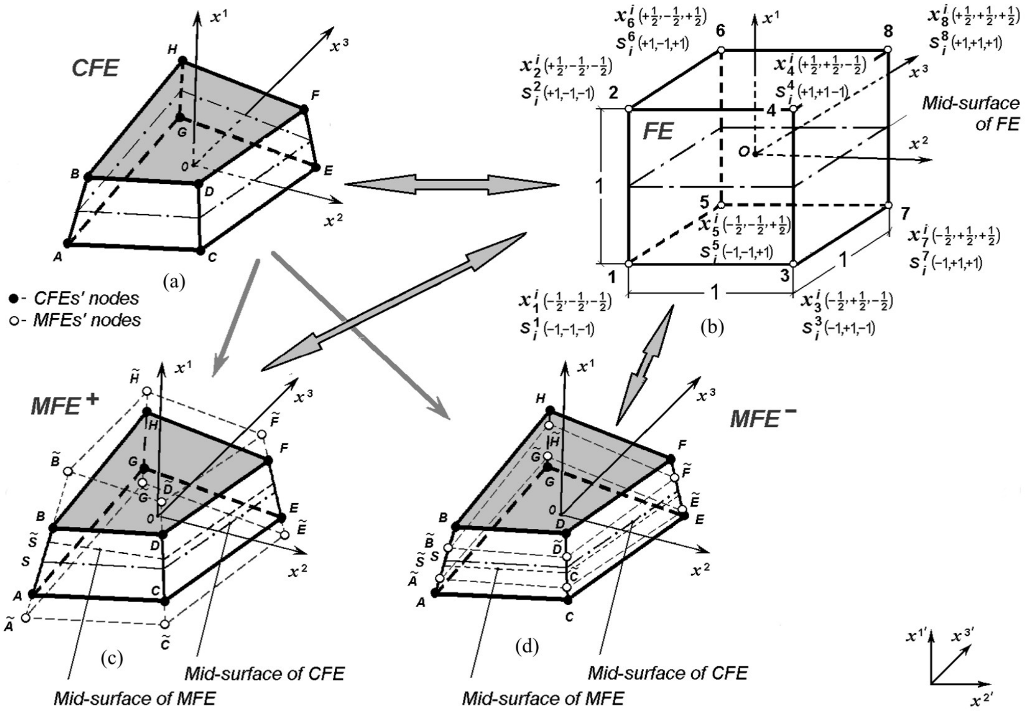


Fig. 1. Transformation of the CFE to the MFE (MFE $^{+}$ – with an edge, MFE $^{-}$ – with a hem)

Examples of shell regions with “ribs” (an area with a stepwise increasing thickness, Fig. 2 (a)), with “channels” (an area with a stepwise decreasing thickness, Fig. 2 (b)) and with extrusion (an area with only a shift in thickness, Fig. 2 (c)) schematically demonstrate the modeling of a shell of stepwise variable thickness using universal 3D FE [20].

Features of the stress-strain state (SSS) of a thin inhomogeneous shell are taken into account by kinematic and static non-classical hypotheses. According to the static hypothesis, the normal compression stresses of the fibers of the layers in the thickness direction are assumed to be constant $\partial \underset{n}{\sigma}^{11} \big/ \partial x^1 = 0$. The accepted hypothesis is weaker than the classical one $\sigma^{11} = 0$. The non-classical static hypothesis does not deprive the stressed state of the inhomogeneous shell of three-dimensional properties. The kinematic hypothesis is formulated as a deformable straight line hypothesis: a straight line in the direction of the thickness (not necessarily along the normal to the middle surface), shortening or lengthening, remains a straight line even after the deformation of the shell. The hypothesis provides a natural way to connect spatial elements in fractures and in areas of step-variable thickness without violating the compatibility of movements and coordinates in the process of deformation (Fig. 3).

Finite-element formulation was obtained using the finite-element moment scheme (FEMS) [1, 21]. The FEM relations are presented in the form of the displacements method: the nodal displacements of the FE in the global Cartesian coordinate system are taken as the sought unknowns $u^{i'}_{s_1 s_2 s_3}$. Usually, for thin shells, in order to improve the convergence of the obtained solutions, the translations of the nodal points on the mid-surface are taken as the sought functions $\upsilon^{i'}_{s_2 s_3}$ and generalized nodal rotations of FE edges $\nu^{i'}_{s_2 s_3}$ (for which differences in nodal displacements on its bounding surfaces are assumed): $\upsilon^{i'}_{s_2 s_3} = \dfrac{u^{i'}_{s_1=+1 s_2 s_3} + u^{i'}_{s_1=-1 s_2 s_3}}{2}$, $\nu^{i'}_{s_2 s_3} = u^{i'}_{s_1=+1 s_2 s_3} - u^{i'}_{s_1=-1 s_2 s_3}$.

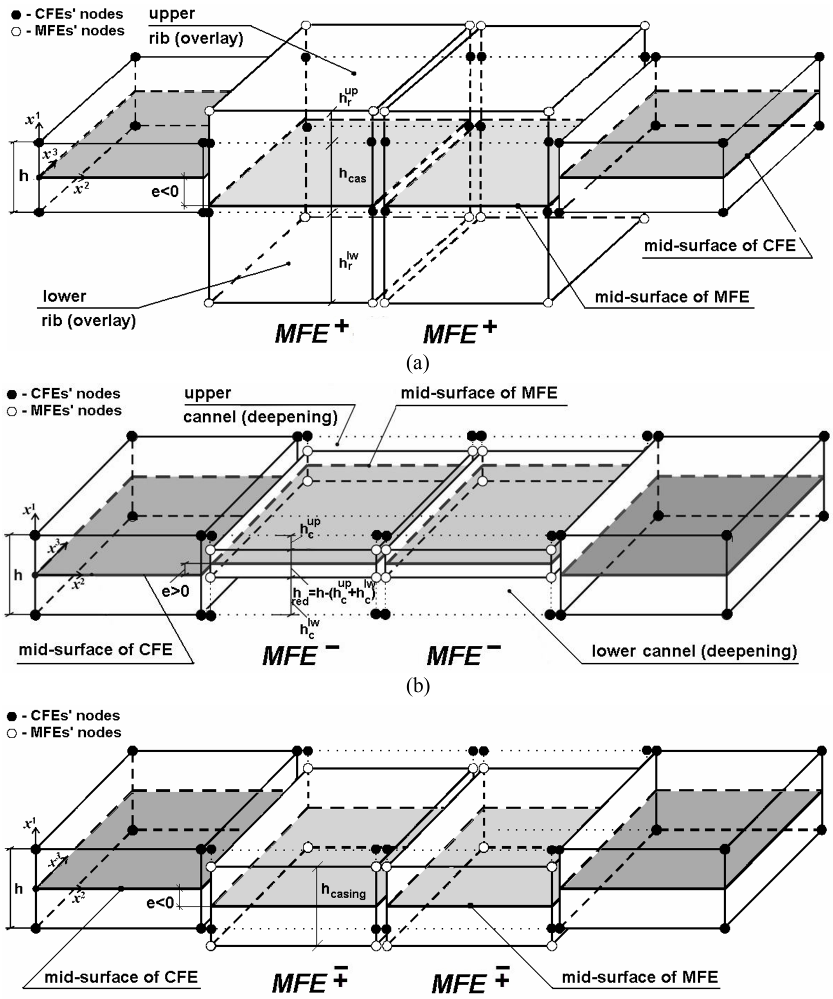


Fig. 2. Schematic representation of the modeling of the shell area with a rib (a), with a cannel (b), with extrusion - (c)

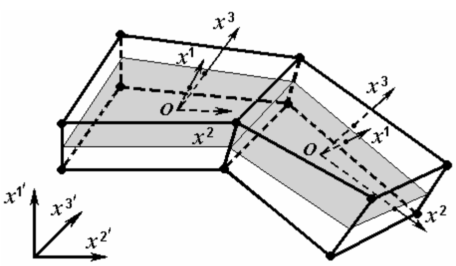


Fig. 3. Modeling of the sharp bend in a shell

The replacement of variables introduced in such a way is interpreted as a transition from an eight-node 3D FE with 3 nodal displacements to a four-node "shell" FE with 6 generalized displacements assigned to the nodes on the middle surface of the FE.

When obtaining the relations for the coefficients of reaction matrices, stiffness, geometric stiffness and equivalent temperature loads of the universal FE, its additional variable parameters are taken into account as necessary. When calculating the coefficients of these matrices, the output data for CFE or MFE are submitted to the corresponding dependences, for which their middle surfaces, which do not coincide with each other, are taken as the reference surfaces (Fig. 4). Since all dependencies are obtained for the general variant of the 3D FE (Fig. 1b), they are therefore universal. When forming a general system of solving equations for a shell finite-element model (SFEM) of step-variable thickness into a single ensemble, the MFE matrices are always adjusted with respect to the accepted reference surface - the middle surface of the shell (Fig. 4).

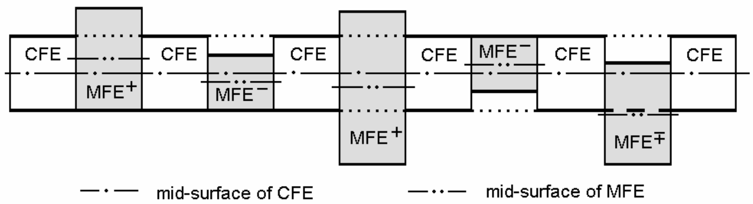


Fig. 4. An example of combining different FEs into a single ensemble (schematic presentation)

Due to the complexity of the formulation of the research problem, which is related both to the existing geometric features of thin elastic shells and to the processes under study, it is important to confirm the reliability of the obtained solutions. As a means of comparison, it is advisable to use the results of calculations that can be obtained with the help of SWs that have proven themselves well: SW LIRA-SAPR and SCAD.

2. Solution of geometrically nonlinear buckling problems using SW LIRA and SCAD

Nowadays specialized automated design systems (CAD) that implement FEM are widely used in the design and calculations of buildings and structures as the main tools of computer modeling and analysis. By purpose, CADs are divided into industrial and scientific. Industrial CADs often take the form of multifunctional SWs, which combine various modules for creating, calculating and analyzing a computer model of a structure. Industrial complexes are mainly focused on solving applied problems, the ultimate goal of which is to obtain the necessary data for checking its strength characteristics for further design of the structure. In turn, scientific complexes are mainly used for the study of complex phenomena and effects in the behavior of structures, in particular shell structures. These SWs use various specialized FEs from a developed library of elements. This makes it possible to obtain more accurate results in contrast to industrial SWs, but at the same time neglecting the project orientation. A large number of FEs in industrial SWs makes it difficult to choose the necessary element option and build a calculation scheme.

In addition, in most industrial SWs, algorithms for studying nonlinear deformation and stability of shell structures are not sufficiently developed. The study of this class of problems, due to their complexity and the possible ambiguity of the resulting solutions, is difficult to implement as a standard computational procedure.

In order to use SW LIRA and SCAD as means of solving stability problems, it is necessary to clarify the possibilities of their application: to study the underlying algorithms, to choose the most effective of them, to find out methods of modeling shells of smooth and step-variable thickness, to establish possible types of thermomechanical loads and their limitations. The assessment of the capabilities of the complexes was analyzed both for SW LIRA [4, 5] and for SW SCAD [6, 7]. Currently, there are later versions [22, 23] of these complexes, but the main approaches have been preserved.

2.1. Algorithms for solving a geometrically nonlinear problem used in SW LIRA and SCAD. Three algorithms for solving problems of geometrically nonlinear deformation and stability are implemented in both SWs. All of them use step-by-step procedure. The calculation is carried out according to the load parameter. In physical terms, this process is a gradual (step-by-step) increase in load from 0 to a given load value P.

The algorithms used in the SW LIRA-SAPR are based on [12, 22]:

1. Method of sequential loading (SL). The "Simple step" algorithm is implemented, which is a simple modification of the method of sequential loading. The solution is found as a broken line, since a linear problem is solved at each step. For this algorithm, it is necessary to manually set the number of steps and their size.

2. Method of sequential loading with automatic step selection (SLA). Unlike the previous algorithm, the number of steps and its size are automatically selected by the algorithm.

3. The Newton-Raphson (N-R) method is step-by-step procedure with the search for new forms of equilibrium, It implements the method of compensating loads. When implementing the algorithm, the loss of stability moment is fixed and a transition to a new stable branch of equilibrium is performed (as research has shown) with a significant error. The number of steps and its size are automatically selected by the algorithm.

According to all methods, the calculation is performed until the moment of degeneration of the stiffness matrix of the system. It is this moment that is interpreted as the loss of stability one. A branch point and a critical point are indistinguishable. The associated uncertainty does not allow qualitatively distinguishing the critical point from the bifurcation point. In this way, solving the problem of nonlinear deformation is realized either up to the branching point ($\overline{q}*$) or to the point of the upper critical load ($\overline{q}_{\text{cr}}^{\text{up}}$).

The SW SCAD uses such algorithms [14, 23]:

1. Method of sequential loading (SL). It uses the "Simple step" algorithm. A linearized problem is solved at each step. Transition to the next step of the nonlinear calculation is performed if solution of the linearized problem at the step is sufficiently accurate.

2. The stepwise Newton-Kantorovich method (N-K) with refinement of the approximation. At the current load step, the iterative refinement of the nonlinear solution is implemented based on the analysis of the imbalance of the equilibrium equations. Iterations are performed with unchanged coefficients of the linearized stiffness matrix calculated at the beginning of the current step. It is necessary to specify the number of load steps, the size of each step, and the number of iterations.

3. The stepwise Newton-Raphson method (N-R) with iterativte refinement. An iterative refinement of the solution is performed at each step with the use of the redefinition of the coefficients of the linearized stiffness matrix at each iteration. It is necessary to specify the number of load steps, the size of each step, and the number of iterations.

The possible appearance of a branch point is not analyzed in SW SCAD.

Due to insufficiently complete description of nonlinear algorithms presented in SW LIRA-SAPR and SCAD instructions, there occurs a need to solve problems using the above mentioned SWs in order to evaluate them and choose the most suitable algorithm for performing comparisons.

It should be noted that, from the point of view of engineering calculations performed by SW, it is more important to determine the deflections when the load is fixed in the process of nonlinear deformation of the structure, and not the moment of loss of stability, which corresponds to the upper critical load. The action of the load, which can cause the beginning of non-linear deformation and even more so the loss of stability, is usually not allowed. The post-critical behavior of the structure is considered in extremely rare cases. Therefore, the vast majority of modern SWs solve problems of nonlinear deformation of shells, limiting themselves to determining the value of the upper critical load. This is due to the use of a step algorithm for solving systems of nonlinear equations. Such stepwise or stepwise iterative algorithms traverse the load-deflection curve step by step. Without changing the step parameter from load to deflection, the passage of the curve from the upper critical load to the post-critical region along the descending branch is impossible.

2.2. Finite elements used in the SW LIRA-SAPR and SCAD. A number of special FEs are used in the SW LIRA and SCAD to calculate the stability of the shells taking into account its nonlinear deformation. In general, the type of FE, on the one hand, is determined by the element belonging to one or another section of the structure, and on the other hand, it depends on the geometric characteristics of the shell, in particular, whether its thickness is constant, linearly variable or step-variable.

2.2.1. Smooth shells and panels. Two types of geometrically nonlinear finite elements are used in SW LIRA-SAPR and SCAD to solve the problems of determining the SSS and stability of thin shallow geometrically nonlinear shells and plates. These are the triangular three-node FE shell No. 342 and the quadrangular four-node No. 344 (Fig. 5 (a), (b)). With the help of these elements, you can calculate thin-walled structures such as: shells – Karman equations are used; membranes – displacements are constant along thickness; shells with strong bending – the relations of the theory of thin shells are used. For the range of problems considered in this article, the last calculation option is used – a shell with a strong bending. Using these elements, it is possible to study the stability of thin elastic shells under geometrically nonlinear deformation.

Governing equations for FE are constructed in a physically linear formulation. The main feature is that the FE, which models a three-dimensional body, is a flat element with a constant thickness. The plane of the element models the mid-surface of the shell or plate. The nodes are located on the plane and have three displacements and three angles of rotation relative to the local axes (Figs. 5, 6).

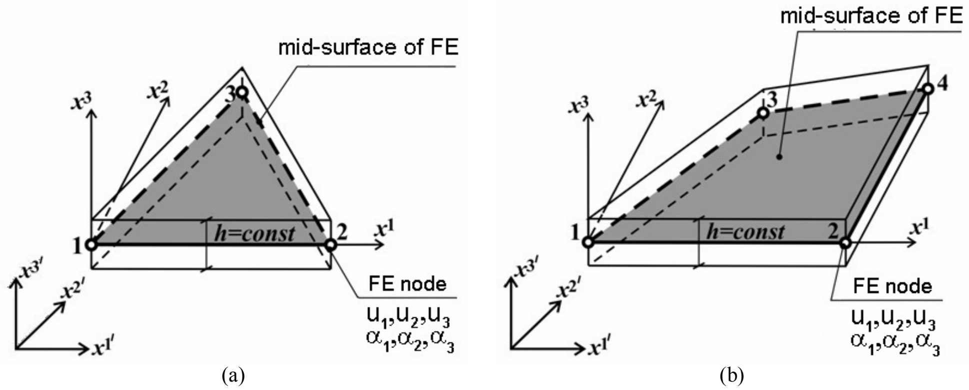


Fig. 5

Finite elements used in SW LIRA-SAPR and SCAD have somewhat limited applications due to the peculiarities of modeling thin shells. Since FEs are 2D elements of constant thickness, the approximation of the shell as a three-dimensional body is not accurate enough. These flat FEs cannot simulate mid-surface sharp bending without gaps and volume overlap (Fig. 6 (a)), unlike the spatial FE developed using the FEMS methodology (Fig. 6 (b)).

Due to the constant thickness of the FE the shell model of a linearly variable thickness in the SW LIRA-SAPR and SCAD is replaced by a step-variable model. At the same time, to obtain reliable results, it is necessary to use a sufficiently thick grid. In addition, all the nodes of one quadrilateral FE are located in the same plane and cannot form a hypar. Because of this, strongly uneven curvilinear surfaces must be modeled with triangular FEs. Such restrictions do not apply to the 3D universal isoparametric eight-node FE of MFES (Fig. 6 (b)).

2.2.2. Shells with ribs and channels. Shells and panels of symmetrical step-variable thickness (for example, with bilateral ribs or channels symmetrically located relative to the middle surface) are modeled by triangular three-node FE No. 342 or quadrilateral four-node FE No. 344 of the appropriate thickness.

Special elements are used in SW LIRA-SAPR and SCAD to model the geometric features of shells of variable thickness in the form of eccentrically located ribs, overlays, channels and recesses. In SW LIRA-SAPR, these are the so-called "absolutely rigid inserts" and "absolutely rigid bodies" [11, 12, 22]. In SW SCAD it is "absolutely rigid (solid) bodies" [13, 14, 23]. In both cases, these are artificial techniques that are used to approximate the gradual change in its thickness in the calculated FE model taking into account the eccentricity. The purpose of introducing these special elements is to specify the kinematic connection for the corresponding nodal displacements.

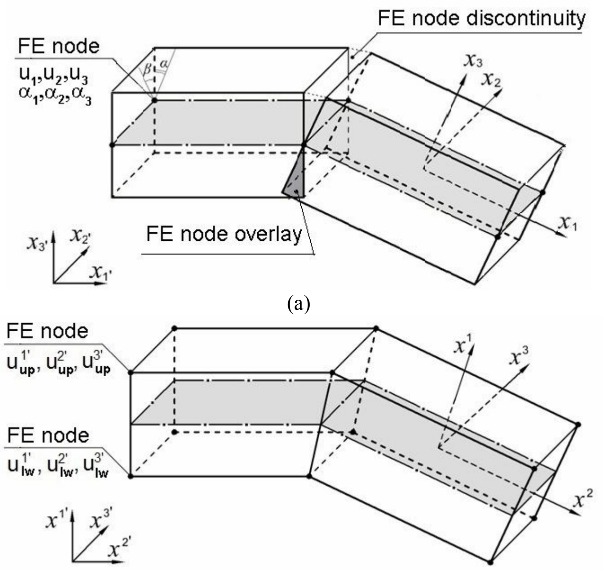


Fig. 6. Elements conection:
(a) – SW LIRA-SAPR and SCAD; (b) –FEMS

Absolutely rigid insert" (ARI) in SW LIRA is used to connect FE nodes in areas of step-variable thickness to the main nodes of the structure, located on its middle surface. The displacement (eccentricity) of the "elastic part" of the FE (the middle surface of a rib or a section with a notch) is modeled with the help of ARI. The "elastic part" of the insert is understood as a shell FE of the appropriate thickness, displaced relative to the middle surface of the structure. "Insertion nodes" are tied to the middle surface of the original shell using kinematic relations.

"Absolutely rigid body" (ARB) in SW LIRA and SW SCAD is a conditional FE of great rigidity. ARB is additionally introduced into the calculation model for the connection of nodes of the middle surfaces of the cladding and the eccentric element. In general, ARB can only conditionally be attributed to the concept of a finite element, since it, in fact, does not have the classical attributes of a FE (basis functions, domain of a finite element, etc.) [11]. However, from the point of view of the implementation of ARB, it fits perfectly into the finite element procedure. When modeling FE shifts, the ARB is a rigid connection between nodes of eccentrically located elements. This FE does not have a number in SW LIRA-SAPR. In SW SCAD it is numbered #100.

2.3 Modeling of thermomechanical load. In LIRA-SAPR and SCAD software complexes, it is possible to set the following types of load:

- mechanical action: concentrated force and moment, uniformly-distributed load and moment, trapezoidal distributed load, uniform or trapezoidal load between two nodes of the plate, weight of the mass of the plate;
- temperature action: uniform and uneven heating (cooling) of the shell or plate through the thickness, a linear-variable law of temperature distribution over the thickness is allowed.

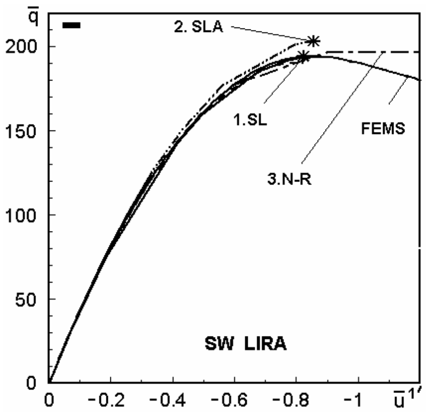
In SW LIRA-SAPR and SW SCAD there are some limitations when modeling the temperature load: in the plane of the element only a constant value of the temperature load can be set. Therefore, it is impossible to model the inhomogeneous temperature load of the middle surface of the shell within the framework of one FE. Setting a common thermomechanical load is also impossible. However, it is possible to adapt the algorithm to specify preheating of the shell followed by additional mechanical loading.

3. Analysis of the effectiveness of the different approaches in the problems of shell nonlinear deformation and buckling.

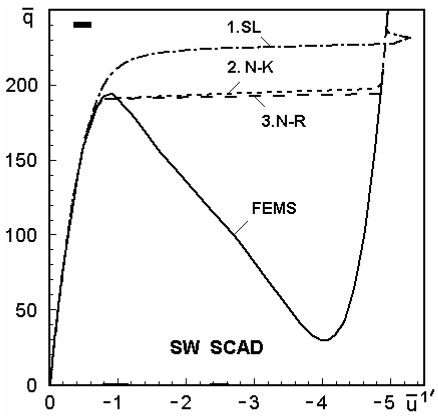
The main goal of the research is to compare the results to confirm the solutions obtained according to the FEMS and evaluate the capabilities of SW LIRA-SAPR and SCAD in solving complex problems of nonlinear deformation and buckling of thin inhomogeneous shells under the action of thermomechanical loads.

3.1. A square in plan (in the plane $x^{2'}x^{3'}$) smooth spherical panel, hingedly supported along the contour and loaded with a uniform normal pressure of intensity q, is investigated. The research results are presented using dimensionless parameters $\overline{q} = a^4 q/(Eh^4)$, $\overline{u}^{1'} = u^{1'}/h$. The curvature of the panel is determined by the parameter $K = 2a^2/(R\,h) = 32$. Accepted: $a = 60h$ is a panel size in plan, $R = 225h$ is radius, $h = 1$ cm is thickness, $E = 2.1 \cdot 10^6$ kg/cm^2, $\nu = 0.3$. The SFEM with a grid of 30×30 FE is adopted for calculation. As research has shown, such a grid ensures the convergence of solutions.

3.1.1. The SW LIRA an almost complete coincidence of the "load-deflection" curves ("$\overline{q}$ - $\overline{u}^{1'}$") has provided by all three approaches in the center of the panel to the upper critical point (Fig. 7 (a)). Both variants of the method of successive loads (1. SL and 2. SL) demonstrate the coincidence of the results in the pre-critical region and a slight difference between them in terms of the value of the upper critical load $\overline{q}_{\text{cr}}^{\text{up}}$ (Table 1). At this point (in the figure, the point is marked '*') the solution of the problem ends. The Newton-Raphson method (3. N-R) allows you to switch to the closed stable branch of the solution, but with a significant error (Fig. 7 (c)).



(a)



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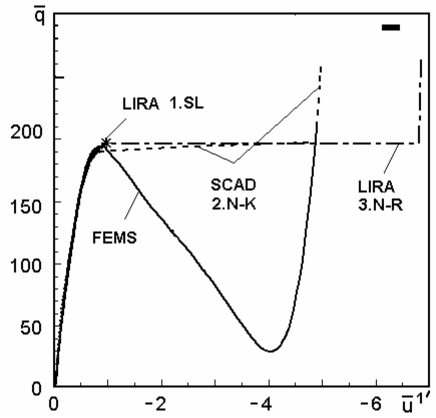
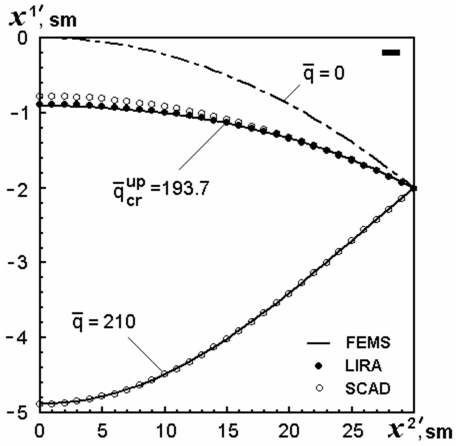


(c)



(d)

Fig. 7

All three algorithms of SW SCAD implement the transition to a new stable equilibrium branch (Fig. 7 (b), (c)). The calculation by the method of successive loads (1. SL) makes a transition to a closed stable branch with a large error in the value of the upper critical load $\bar{q}_{\rm cr}^{\rm up}$. According to the algorithms based on the application of the Newton-Kantorovich (2. N-K) and Newton-Raphson (3. N-R) methods, the problem of transition to a closed stable branch is solved quite precisely, the value of the upper critical load $\bar{q}_{\rm cr}^{\rm up}$ is the same (Table 1).

Table 1

SW	Algorithm	$\bar{q}_{\rm cr}^{\rm up}$	Δ, %	$\bar{u}_{\rm cr}^{1'\rm up}$	Δ, %
FEMS	Newton-Kantorovich method (FEMS)	193.7	0	0.9125	0
LIRA	1. Sequential loading method (SL)	194.1	0.20	0.8796	-3.60
	2. Sequential loading method with automatic step selection (SLA)	202.8	4.70	0.8580	-5.97
	3. Newton-Raphson method (N–R)	196.4	1.40	0.9013	-1.23
SCAD	1. Sequential loading method (SL)	-	-	-	-
	2. Newton-Kantorovich method (N–K)	190.2	-1.80	0.7729	-15.30
	3. Newton-Raphson method (N–R)	190.2	-1.80	0.7730	-15.29

In the pre-critical region for both SWs, we have an almost complete coincidence of the curves with the diagram obtained on the basis of the use of FEMS. The equilibrium shapes of the deformed panels in the subcritical and supercritical regions have a simple appearance and match well. (Fig. 7 (d)) shows the modes of deformation of the middle surface of the shells in the area of the critical load and at the point of transition to a stable closed branch. Deformation of the panel is characterized by a deflection in its central area.

In further research, when performing calculations, we will use method 2. Successive loads with automatic step selection (SLA) for SW LIRA-SOFT, and method 2. Newton-Kantorovich (N-K) for SW SCAD, as the most accurate and efficient.

3.1.2 The influence of the combined effect of preliminary uniform heating with subsequent pressure loading on the loss of panel stability is considered (Fig. 8). Heating (cooling) is performed on $T = \pm 20^\circ \rm C$.

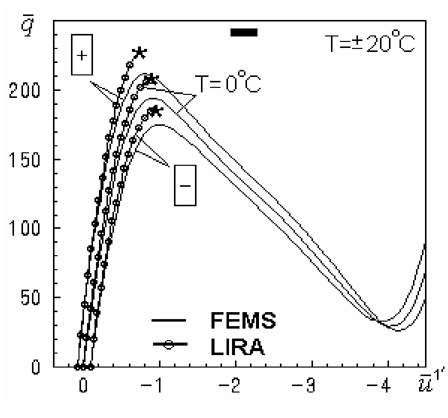


Fig. 8

A uniform temperature increase of 20 degrees leads to an almost identical corresponding increase in the upper critical load $\bar{q}_{\rm cr}^{\rm up}$ =175,0; 193,7; 212,2 and a uniform decrease of the lower one $\bar{q}_{\rm cr}^{\rm lw}$ =32,82; 29,78; 26,53 (Fig. 8).

A comparison of the results obtained by the FEMS with the calculations made by SW LIRA showed a fairly good match between them. The corresponding diagrams almost completely coincide in the pre-critical region. We have a slight run-up in terms of values $\bar{q}_{\rm cr}^{\rm up}$ та $\bar{u}_{\rm cr}^{1'\rm up}$ at the upper critical point (Table 2).

Table 2

Loading (preheating)	Calculation method					
	FEMS		SW LIRA			
	$\overline{q}_{cr}^{up}$	$\overline{u}_{cr}^{1'up}$	$\overline{q}_{cr}^{up}$	Δ, %	$\overline{u}_{cr}^{1'up}$	Δ, %
$T=-20^\circ$ C	175,0	-0,9613	185,9	6,23	-1,0000	4,03
$T=\ \ 0^\circ$ C	193,7	-0,9125	202,8	4,70	-0,8580	-5,97
$T=+20^\circ$ C	212,2	-0,7984	228,4	7,63	-0,7509	-5,95

3.1.3. Modal analysis of the shell shows the following. The first four forms of natural oscillations of the panel, obtained by the method based on the use of MFES and by SW LIRA-SAPR, are given for the initial unloaded state $\overline{q}^{i=0}$ (Fig. 9). The difference in frequencies is within 1%. The forms of natural oscillations are identical. When calculating according to the FEMS ($T=0^\circ$ C) it was obtained that at all loads the first frequencies are multiples, $\omega_1=\omega_2$.

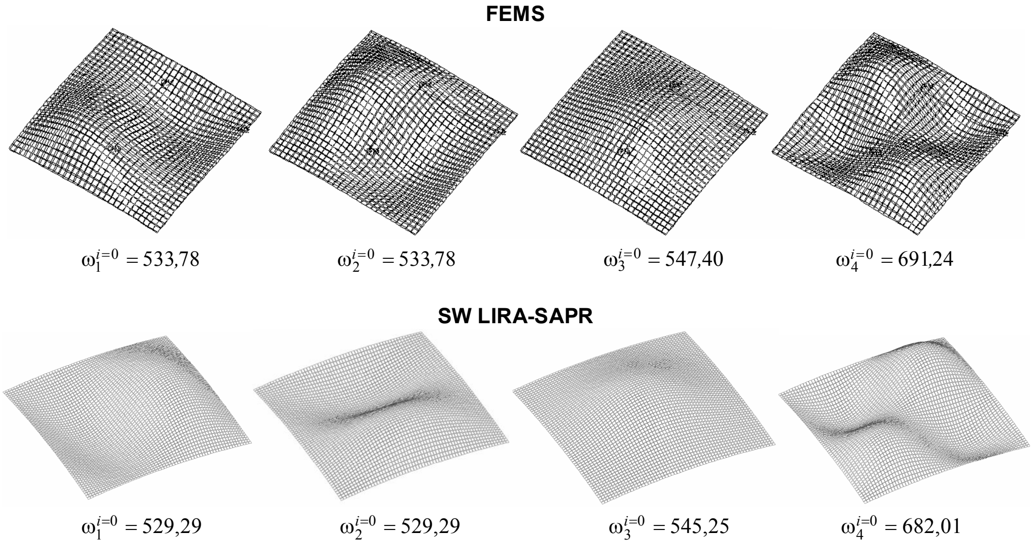


Fig. 9. Natural modes ($\omega_k^{i=0}$, Hz) of the unloaded shell, $\overline{q}^{i=0}$

The conducted modal analysis makes it possible to investigate the influence of the prestressed state of the shell on the frequencies and forms of natural oscillations of the deforming structure (Fig. 10). In the figure, the resulting dependence is presented as a "load-lower frequency" ω_1" ("$\overline{q}-\omega$") diagram. Modal analysis is performed until the zero (negative) value of the fundamental tone appears. This approach corresponds to the dynamic criterion of the loss of shell stability [24] and allows determining the stability of the panel simultaneously according to static and dynamic criteria. There are no branching points of the solutions in the pre-critical region on the "$\overline{q}$ - $\overline{u}$" curves. Therefore, according to both criteria,

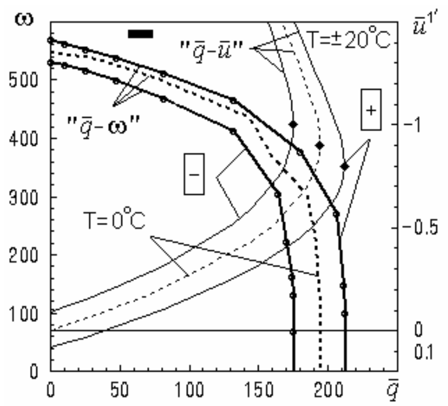


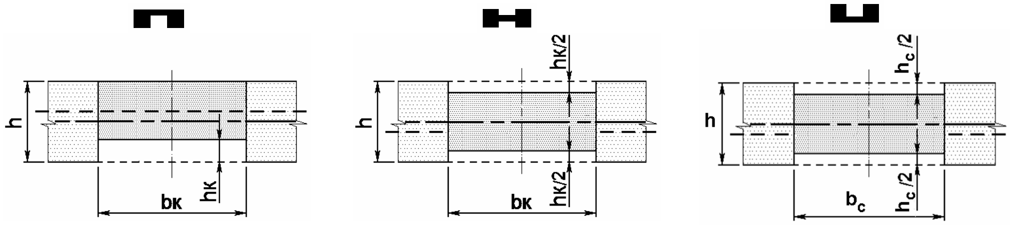
Fig. 10

almost the same corresponding values of critical loads were obtained.

In the existing versions of SW LIRA-SAPR and SCAD, it is not possible to analyze the natural vibrations of shells taking into account the prestressed state.

3.2 The stability analysis of shells with stepwise variable thickness is illustrated using the example of the panel discussed above. The panel is weakened by four criss-crossed channels placed on the surface of the shell in three ways. The channels are located eccentrically on the inner (Fig. 11 (a)) or outer (Fig. 11 (c)) surfaces of the shell and symmetrically on its inner and outer surfaces (Fig. 11 (b)).

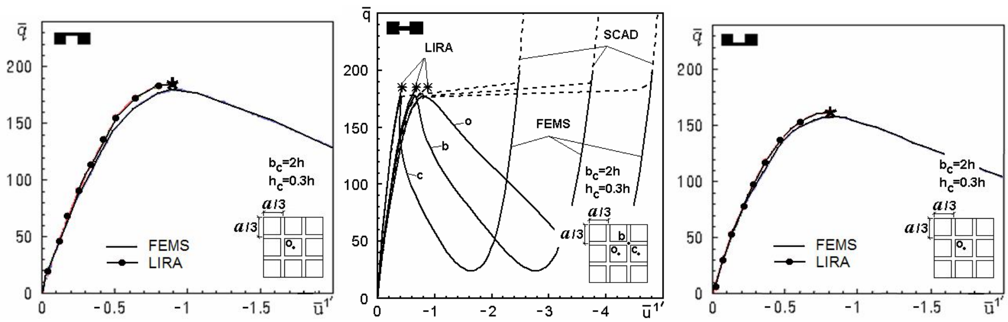
The SFEM with a sufficiently dense mesh of 30×30 FE was adopted as the calculation model, which ensures the convergence of solutions. To approximate channels, the SW LIRA-SAPR uses ‘absolutely rigid insertions’, and the SW SCAD uses ‘absolutely rigid bodies’.



(a) (b) (c)

Fig. 11

3.2.1. We consider a shell with “narrow” channels having the same parameters: length a, width $b_c = 2h$ and total depth $h_c = 0.3h$. For all algorithms there is a good agreement between the ‘ $\bar{q}$ - $\bar{u}$ ’ curves in the subcritical region (Fig. 12). The calculation performed using the SW LIRA-SAPR stops at the upper critical point. In the pictures this point is marked '*'. The solution for a panel with a symmetrical arrangement of channels obtained using the SW SCAD accurately implements the transition to a supercritical stable branch (Fig. 12 (b)). For this symmetrical weakening, the ‘ $\bar{q}$ - $\bar{u}$ ’ curves are compared at three characteristic points.



(a) (b) (c)

Fig. 12

A comparison of solutions at the upper critical point obtained using the MFES, the SW LIRA-SAPR and SCAD shows that for all types of weakening, the difference in load $\bar{q}_{\mathrm{cr}}^{\mathrm{up}}$ and deflection $\bar{u}_{\mathrm{cr}}^{1'\mathrm{up}}$ in the center of the panel does not exceed 4% (Table 3).

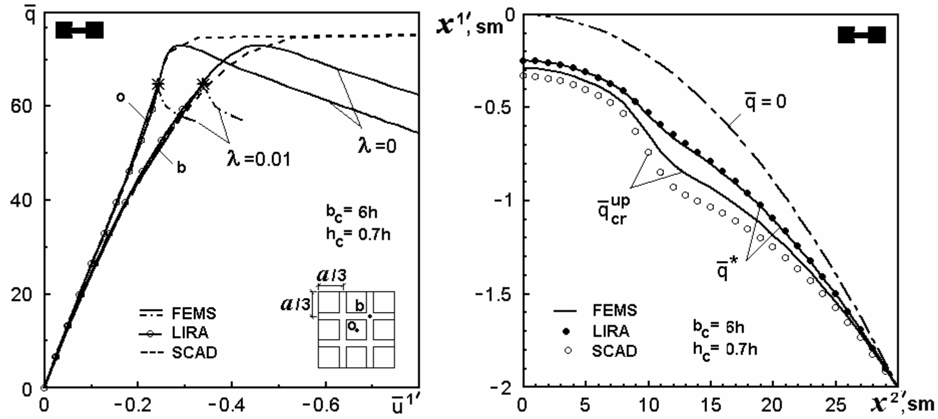
For narrow channels, their location has little effect on the value of the critical load $\bar{q}_{\mathrm{cr}}^{\mathrm{up}}$. The greatest reduction in load (compared to a smooth panel) caused weakening on the outer side of the shell, $e < 0$. The critical load of this shell is reduced by 18.7%.

Table 3

Panel type	$\bar{q}_{cr}^{up}$			$\bar{u}_{cr}^{1'up}$ (in the center)		
	FEMS	SW LIRA Δ, %	SW SCAD Δ, %	FEMS	SW LIRA Δ, %	SW SCAD Δ, %
$e > 0$	178,8	178,98 0,10%	-“-	-0,889	-0,923 3,8%	-“-
$e = 0$	177,3	184,3 3,95%	176,21 -0,61%	-0,842	-0,819 -2,7%	-0,839 0,36%
$e < 0$	157,4	162,40 2,82%	-“-	-0,781	-0,805 3,1%	-“-
Гладка	193,7	202,8 4,7%	190,2 -1,8%	-0,9125	-0,8580 -5,97%	-0,7729 -15,30%

3.2.2. The effect of weakening in the form of “wide” channels on shell buckling is considered. The channels are located symmetrically relative to the shell mid-surface and has such parameters: $b_c = 6h$, $h_c = 0.7h$.

The ‘$\bar{q}$ -$\bar{u}^{1'}$’ curves obtained using the FEMS, the SW LIRA-SAPR and SCAD have been plotted for deflections at different points of the shell (Fig. 13 (a)). The resulting curves completely coincide with each other in the subcritical region and near the upper critical load $\bar{q}_{cr}^{up}$.



(a) (b)

Fig. 13

Table 4

Calculation method	$\dfrac{\bar{q}_{cr}^{up}}{\bar{u}_{cr}^{1'up}}$	$\dfrac{\Delta^{q}, \%}{\Delta^{u}, \%}$	$\dfrac{\bar{q}^{*}}{\bar{u}^{1'*}}$	$\dfrac{\Delta^{q*}, \%}{\Delta^{u*}, \%}$
FEMS	$\dfrac{72{,}94}{-0{,}2876}$	$\dfrac{0}{0}$	$\dfrac{63{,}34}{-0{,}2410}$	$\dfrac{0}{0}$
SW LIRA	–“–	–“–	$\dfrac{64{,}79}{-0{,}2451}$	$\dfrac{2{,}89}{1{,}70}$
SW SCAD	$\dfrac{74{,}71}{-0{,}3289}$	$\dfrac{2{,}43}{12{,}81}$	–“–	–“–

In contrast to the solution for a panel with ‘narrow’ channels, the SW LIRA stopped calculations at the branching point at load $\overline{q}^* = 64{,}79$, taking the branching point as the upper critical load (Fig. 13 (a)). This point is marked '*' in the figure. The branch point has been also discovered in calculations using the MFES. The MFES algorithm allows us to more accurately determine the load value at the branch point $\overline{q}^*$ (Table 4). By introducing a small ($\lambda = 0{,}01$) asymmetrical perturbation into the initial shell shape, a specially developed technique allows us to turn the branch point '*' into a critical one ($\overline{q}^* = 63{,}34$) and reach a new branch of the solution (dash-dotted curve). The resulting load $\overline{q}^*$ is 13.16% less than the critical load $\overline{q}_{cr}^{up}$.

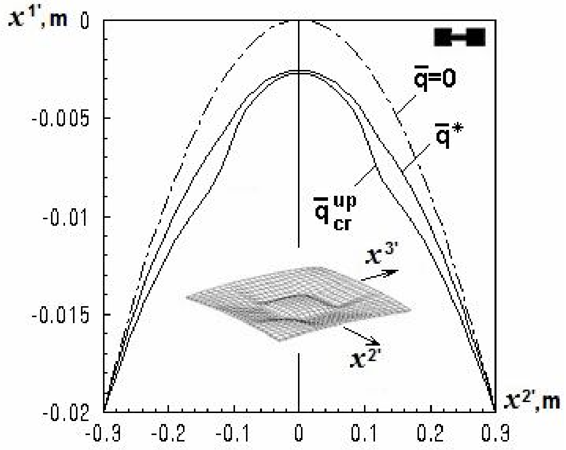


Fig. 14

The weakening of the shell by wide channels led to a significant decrease in the value of the upper critical load (by 62.34%) compared to that corresponding to a smooth panel.

The shell deformation shapes obtained using different algorithms coincide well with each other (Fig. 13, b). The deflection in the center of the panel at the moment of buckling is less than in the weakening zone (Fig. 14).

Conclusions

The effectiveness of the finite element method for studying geometrically nonlinear deformation, buckling, post-buckling behavior and vibrations of thin elastic shells under the static action of thermomechanical loads is analyzed and the reliability of the obtained solutions is confirmed. The research method is based on the three-dimensional approach of thermoelasticity theory and the use of a finite element moment scheme. Comparisons are carried out with the results of calculations performed using domestic software LIRA-SAPR and SCAD.

The possibilities of using these programs to study the processes under consideration have been investigated and identified. The approaches used in them to finite element modeling of shells, in particular of step-variable thickness, are described.

On this basis, a comparative analysis of solutions obtained using three software packages for nonlinear deformation, stability and vibration of shallow panels under the action of thermomechanical loads is carried out. Good agreement between solutions is obtained.

The research makes it possible to conclude that the LIRA-SAPR and SCAD can be used, within certain limits, as a means of confirming the reliability of the results obtained when it is studying the geometrically nonlinear behavior of thin flexible elastic shells.

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ПОРІВНЯЛЬНИЙ АНАЛІЗ СТІЙКОСТІ І ВЛАСНИХ КОЛИВАНЬ ПОЛОГИХ ПАНЕЛЕЙ ПРИ ДІЇ ТЕРМОСИЛОВИХ НАВАНТАЖЕНЬ

Робота є продовженням низки досліджень, присвячених обґрунтуванню достовірності розв'язків, що отримуються за скінченно-елементною методикою дослідження нелінійного деформування, стійкості та коливань тонких пружних оболонок при дії термосилових навантажень. Методика базується на геометрично нелінійних співвідношеннях тривимірної теорії термопружності та положеннях моментної схеми скінченних елементів. Тонка пружна оболонка неоднорідної структури моделюється універсальним просторовим ізопараметричним скінченним елементом. Модальний аналіз оболонки реалізується на кожному кроці статичного термосилового навантаження. Для визначення спектру нижчих частот власних коливань оболонок застосовується метод ітерації підпростору. Розглядається квадратна за планом полога сферична панель. Досліджується вплив попереднього нагріву на стійкість і коливання пружної ізотропної оболонки при навантаженні рівномірним тиском. Аналізується поведінка оболонки, що послаблена двома парами перехресних каналів. Розглядається послаблення панелі вузькими і широкими каналами, які можуть бути ексцентрично розташованими відносно серединної поверхні оболонки. Ефективність та адекватність методу підтверджується порівняльним аналізом розв'язків з результатами, що отримані з використанням сучасних багатофункціональних програмних комплексів ЛІРА-САПР та SCAD. Наведено особливості застосування комплексів до розв'язання розглядуваних задач. Аналіз результатів розрахунків дозволив оцінити межі та можливості використання цих програмних комплексів для обґрунтування достовірності розв'язків певних класів задач геометрично нелінійного деформування, втрати стійкості та коливань пружних оболонок.

Ключові слова: пружна оболонка, термосилове навантаження, стійкість, модальний аналіз, універсальний просторовий скінченний елемент, моментна схема скінченних елементів, порівняльний аналіз.

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Кривенко О.П., Лізунов П.П., Ворона Ю.В., Калашніков О.Б. **Порівняльний аналіз стійкості і власних коливань пологих панелей при дії термосилових навантажень** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – Київ: КНУБА, 2023. – Вип. 111. – С. 49-64.
Проведено порівняльний аналіз розв'язків щодо нелінійного деформування, стійкості та коливань тонких пружних оболонок при дії термосилових навантажень, що отримані за моментною схемою скінченних елементів та з використанням програмних комплексів ЛІРА-САПР та SCAD. Наведено особливості застосування комплексів до розв'язання розглядуваних задач.
Табл. 3. Іл. 14. Бібліогр. 24 назв.

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Krivenko O.P., Lizunov P.P., Vorona Yu.V., Kalashnikov O.B. **Comparative analysis of the stability and natural vibrations of shallow panels under the action of thermomechanical loads //** Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – Kyiv: KNUBA, 2023. – Issue 111. – P. 49-64.
A comparative analysis of solutions for nonlinear deformation, buckling and vibrations of thin elastic shells under the action of thermomechenical loads, obtained using the finite element moment scheme and using the LIRA-SAPR and SCAD software packages, has been carried out. The features of using complexes for solving the problems under consideration are given.
Tabl. 3. Fig. 14. Ref. 24.

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