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OPTIMIZATION OF THE FORCE MODE OF RANGE CHANGE IN AN ARTICULATED BOOM SYSTEM WITH A GEARED SECTOR AT A FIXED MODE OF CRANE ROTATION

V.S. Loveikin[1]
Yu.O. Romasevych[1]
D.A. Palamarchuk[2]
A.V. Loveikin[3]

[1]*National University of Life and Environmental Sciences of Ukraine*
[2]*Kyiv National University of Construction and Architecture*
[3]*Taras Shevchenko National University of Kyiv*

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To optimize the movement mode, a dynamic model of the boom system consisting of completely rigid links, except for a flexible load suspension, was used. The load on the flexible suspension carries out pendulum oscillations in the plane of the departure change. On the basis of the dynamic model, using Lagrange equations of the second kind, a mathematical model of the jib system of changing the departure of the cargo at a fixed mode of rotation was built.

The criterion for optimizing the movement mode of the boom system is the mean-square value of the driving torque of the mechanism for changing the departure during the start-up process, which is an integral functional.

Key words: turning mechanism, departure change mechanism, load swinging, movement mode, integral criterion, optimization, boom system of the crane.

Introduction

When operators work on cranes with articulated boom systems, they often combine several operations by simultaneously turning on two mechanisms. In gantry cranes, such a combination of simultaneous operations is most often observed during the operation of mechanisms for changing the reach of the jib system and turning the crane. Moreover, the combination of work is carried out when one mechanism works in a steady mode of movement, and the other, at this time, starts movement (starting is carried out) or ends it (braking is carried out).

In such cases, the load swings significantly on the flexible suspension and the dynamic loads in the elements of the drive mechanisms and the crane structure increase. Such factors negatively affect the efficiency of the crane, in particular, the productivity and reliability of the crane decreases, as well as its maneuverability and ergonomics deteriorate.

Analysis of publications

Analyzing literature sources for the study of the dynamics of crane movement, it is clear that most of such studies are conducted for overhead cranes. This is due to a wide range of fields of application of such cranes [1].

However, it is often impossible to project the dynamics equation of overhead cranes onto jib cranes [2, 3]. Because jib cranes have a rotation mechanism and a complete study of the dynamics of movement is inextricably linked to the operation of the rotation mechanism [3, 4].

This especially applies to cranes with a non-linear dependence of the movement of the boom and the drive mechanism. Such cranes include portal cranes with boom system [5, 6] and some hydraulic manipulators.

For an in-depth study of the dynamics of the movement of boom systems and establishing a sufficient convergence of theoretical and field studies, there is a need to identify and model the loads that the boom system experiences during operation. Such loads include: wind loads, frictional forces, loads from rocking loads with a shifted center of mass [5, 7].

Works [8, 9] are devoted to the study of the dynamics of the boom lifting mechanism during transient processes. This problem was solved using complex integral criteria [10].

The authors of the work [11] consider the influence of transient operating modes of crane mechanisms on drive electric motors.

The work [12] shows an example of studying the dynamics of the boom system during the combination of the two mechanisms.

Purpose and research task statement

The purpose of this study is to improve the efficiency of cranes with a hinged jib system due to the optimization of the mode of changing the departure of the load at the fixed mode of rotation of the lifting crane.

To realize the set goal, it is necessary to solve the following tasks: to develop a dynamic model of the combined movement of the mechanisms for changing the departure and turn and to make a mathematical model of the combined movement of these mechanisms. It is necessary to set a variational problem of optimizing the movement mode of the departure change mechanism at a fixed turning mode; justify the optimization criterion; solve the set optimization problem with the selected criterion; analyze the optimization results.

Research results

As an object of research, the jib system of a gantry crane with a toothed sector drive of the mechanism for changing the departure of the load and a planetary turning mechanism was chosen (Fig. 1).

When building a dynamic model of the boom system shown in Fig. 1, the following assumptions are used [5]:

– it is considered that all links of the system are solid bodies, except for the flexible suspension of the cargo;

– the load carries out pendular oscillations on the suspension, only in the vertical plane of the departure change;

– we neglect the fluctuations of the load on the flexible suspension in the plane of rotation, since the rotation of the crane is carried out with a constant angular speed ($\omega=const$);

– when the departure is changed, the load moves horizontally, because the cargo rope runs along the jib and the guy and does not change its own length when the departure is changed;

– it is considered that the boom system is completely balanced by a moving counterweight.

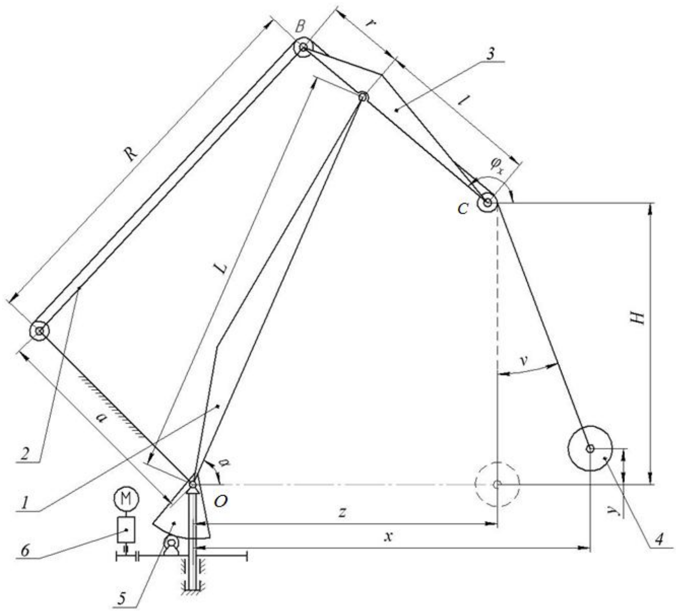

Fig. 1. Dynamic model of level-luffing boom system of the crane
1. Main jib; 2. Tieback; 3. Jib; 4. Load; 5. Outreach mechanism with gear sector; 6. Rotation mechanism

For such a dynamic model of the jib system, we will consider the joint movement of the mechanisms for changing the departure of the load and turning the crane. At the same time, we will present the boom system as a holonomic mechanical system with three degrees of freedom. For the generalized coordinates, we will take the horizontal coordinate of the center of mass of the load x in the plane of change of departure, the angular coordinate of rotation of the boom in the plane of change of departure α, as well as the angular coordinate of rotation of the crane φ in the horizontal plane. Since the angular speed of rotation of the crane is assumed to be a constant value $\dot{\varphi} = d\varphi / dt = \omega = const$, we have a boom system with two degrees of freedom, in which the generalized coordinates will be x and α. At the same time, the angular coordinate of the crane rotation changes according to the linear dependence $\varphi = \varphi_0 + \omega t$, where t is time, φ_0 is the coordinate of the initial position of the boom in the horizontal plane, and ω is the angular speed of the crane rotation.

For this dynamic model of the jib system of the gantry crane, let us make its mathematical model. For this, the Lagrange equation of the second kind was used [5, 12]:

$$\begin{cases} \frac{d}{dt}\frac{\partial T}{\partial \dot{\alpha}} - \frac{\partial T}{\partial \alpha} = Q_{\alpha} - \frac{\partial \Pi}{\partial \alpha}; \\ \frac{d}{dt}\frac{\partial T}{\partial \dot{x}} - \frac{\partial T}{\partial x} = -\frac{\partial \Pi}{\partial x}, \end{cases} \tag{1}$$

where T – the kinetic energy of the system; Π – potential energy of the system; Q_{α} – generalized component of non-potential forces reduced to the coordinate α.

Determine the kinetic energy of the boom system with the combined movement of the mechanisms of change of departure and rotation of the crane

$$T = \frac{1}{2}\left(J_0 + J_C + m_X L^2\right)\dot{\alpha}^2 + \frac{1}{2}J_X\dot{\varphi}_X^2 - \frac{1}{2}m_X L\left(l-r\right)\dot{\alpha}\dot{\varphi}_X \cos\left(\varphi_X - \alpha\right) +$$
$$+\frac{1}{2}\Big\{J_X \cos^2\varphi_X + m_X\left[L^2\cos^2\alpha + L\left(l-r\right)\cos\alpha\cos\varphi_X\right] +$$
$$+J_B\cos^2\varphi_B - m_B a\cos\Theta\left(R\cos\varphi_B - a\cos\Theta\right) + J_P\Big\}\omega^2 +$$
$$+\frac{1}{2}m\left(\dot{x}^2 + \omega^2 x^2\right), \tag{2}$$

where m_X, m_B, m – respectively, the mass of the jib, tieback and cargo; J_0 – the moment of inertia of the drive elements of the departure change mechanism, which is reduced to the axis of rotation of the boom; J_P – moment of inertia of the drive of the turning mechanism, reduced to the axis of rotation of the crane; J_C, J_X, J_B – moments of inertia about their own axes of rotation, respectively, main jib, jib and tieback; L, R – respectively, the length of the main jib and the tieback; l, r – respectively the length of the jib and counter jib; a, Θ – respectively, the length of the tieback and its angle of inclination to the horizon; z – the horizontal coordinate of the position of the center of mass of the load relative to the lower hinge of the boom; φ_X, φ_B – angular coordinates of rotation, respectively, the jib and tieback.

The potential energy of a fully balanced boom system is determined by the potential energy of the load

$$\Pi = mgy = mgH\left(1-\cos\nu\right), \tag{3}$$

where g – the acceleration of free fall; H – height of the load suspension relative to the lower hinge of the boom; y – the vertical coordinate of the center of mass of the cargo; ν – the angular coordinate of the deviation of the flexible cargo suspension from the vertical.

This angular coordinate ν is determined by the following dependence

$$\nu = \left(x - z\right)/H. \tag{4}$$

After substituting dependence (4) into expression (3), we obtain an expression for determining the potential energy

$$\Pi = mgH\left(1-cos\left(\frac{x-z}{H}\right)\right). \tag{5}$$

The generalized force corresponding to the coordinate α has the following form

$$Q_\alpha = M = M_P u\eta, \tag{6}$$

where M – reduced to the axis of rotation of the crane driving moment of the rotation mechanism; M_P – driving torque on the motor shaft of the crane rotation mechanism; u – the gear ratio of the drive of the turning mechanism; η – the efficiency of the drive in the turning mechanism.

We differentiate expressions (2) and (5) according to the equation of system (1), as a result of which we will have:

$$\begin{aligned}\frac{\partial T}{\partial\alpha} &= J_X\dot{\alpha}^2\frac{\partial\varphi_X}{\partial\alpha}\frac{\partial^2\varphi_X}{\partial\alpha^2} - \frac{m_X L}{2}(l-r)\dot{\alpha}^2\times\\ &\times\left[\frac{\partial^2\varphi_X}{\partial\alpha^2}cos(\varphi_X-\alpha) - \frac{\partial\varphi_X}{\partial\alpha}\left(\frac{\partial\varphi_X}{\partial\alpha}-1\right)sin(\varphi_X-\alpha)\right] -\\ &-\frac{\omega^2}{2}\left\{J_X\frac{\partial\varphi_X}{\partial\alpha}sin\,2\varphi_X + m_X L\left[L\,sin\,2\alpha + (l-r)\times\right.\right.\\ &\left.\times\left(sin\,\alpha\,cos\,\varphi_X + \frac{\partial\varphi_X}{\partial\alpha}cos\,\alpha\,sin\,\varphi_X\right)\right]+\\ &\left.+J_B\frac{\partial\varphi_B}{\partial\alpha}sin\,2\varphi_B - m_B aR\,cos\,\Theta\frac{\partial\varphi_B}{\partial\alpha}sin\,\varphi_B\right\};\end{aligned} \tag{7}$$

$$\frac{\partial T}{\partial\dot{\alpha}} = \left[J_O + J_C + m_X L^2 + J_X\left(\frac{\partial\varphi_X}{\partial\alpha}\right)^2 - m_X L(l-r)\frac{\partial\varphi_X}{\partial\alpha}cos(\varphi_X-\alpha)\right]\dot{\alpha};$$

$$\begin{aligned}\frac{d}{dt}\frac{\partial T}{\partial\dot{\alpha}} &= \left[J_O + J_C + m_X L^2 + J_X\left(\frac{\partial\varphi_X}{\partial\alpha}\right)^2 - m_X L(l-r)\frac{\partial\varphi_X}{\partial\alpha}cos(\varphi_X-\alpha)\right]\times\\ &\times\ddot{\alpha} + \left\{2J_X\frac{\partial\varphi_X}{\partial\alpha}\frac{\partial^2\varphi_X}{\partial\alpha^2} - m_X L(l-r)\times\right.\\ &\left.\times\left[\frac{\partial^2\varphi_X}{\partial\alpha^2}cos(\varphi_X-\alpha) - \frac{\partial\varphi_X}{\partial\alpha}\left(\frac{\partial\varphi_X}{\partial\alpha}-1\right)sin(\varphi_X-\alpha)\right]\right\}\dot{\alpha}^2.\end{aligned} \tag{8}$$

$$\frac{\partial T}{\partial x} = m\omega^2 x;\quad \frac{\partial T}{\partial\dot{x}} = m\dot{x};\quad \frac{d}{dt}\frac{\partial T}{\partial\dot{x}} = m\ddot{x}; \tag{9}$$

$$\frac{\partial\Pi}{\partial\alpha} = mg\,sin\left(\frac{z-x}{H}\right)\frac{\partial z}{\partial\alpha};\quad \frac{\partial\Pi}{\partial x} = -mg\,sin\left(\frac{z-x}{H}\right). \tag{10}$$

Taking into account that the angular coordinate $\nu=\dfrac{z-x}{H}$ does not exceed 12°, the dependencies (10) can be presented in the following form:

$$sin\nu\approx\nu;\ \ \frac{\partial\Pi}{\partial\alpha}=mg\frac{\partial z}{\partial\alpha}\frac{z-x}{H};\ \ \frac{\partial\Pi}{\partial x}=-mg\frac{z-x}{H}. \tag{11}$$

After substituting dependencies (7),...,(9) and (11) into system (1), we will obtain a mathematical model of the dynamics of the combined movement of the mechanisms for changing the departure of the cargo and the steady rotation mode of the gantry crane, which is a system of nonlinear differential equations of the second order:

$$\begin{cases}\left[J_O+J_C+m_XL^2+J_X\left(\dfrac{\partial\varphi_X}{\partial\alpha}\right)^2-m_XL\left(l-r\right)\dfrac{\partial\varphi_X}{\partial\alpha}cos\left(\varphi_X-\alpha\right)\right]\ddot{\alpha}+\\ +\left\{J_X\dfrac{\partial\varphi_X}{\partial\alpha}\dfrac{\partial^2\varphi_X}{\partial\alpha^2}-\dfrac{m_XL}{2}\left(l-r\right)\left[\dfrac{\partial^2\varphi_X}{\partial\alpha^2}cos\left(\varphi_X-\alpha\right)-\dfrac{\partial\varphi_X}{\partial\alpha}\left(\dfrac{\partial\varphi_X}{\partial\alpha}-1\right)\times\right.\right.\\ \left.\left.\times sin\left(\varphi_X-\alpha\right)\right]\right\}\dot{\alpha}^2+\left\{J_X\dfrac{\partial\varphi_X}{\partial\alpha}sin2\varphi_X+m_XL\times\right.\\ \times\left[L\,sin2\alpha+\left(l-r\right)\left(sin\,\alpha\,cos\,\varphi_X+\dfrac{\partial\varphi_X}{\partial\alpha}cos\,\alpha\,sin\,\varphi_X\right)\right]+\\ \left.+J_B\dfrac{\partial\varphi_B}{\partial\alpha}sin\,2\varphi_B-m_BaR\,cos\,\Theta\dfrac{\partial\varphi_B}{\partial\alpha}sin\,\varphi_B\right\}\dfrac{\omega^2}{2}=M-mg\dfrac{\partial z}{\partial\alpha}\dfrac{z-x}{H};\\ m\ddot{x}-m\omega^2x=mg\dfrac{z-x}{H}.\end{cases} \tag{12}$$

From the last equation of the system (12), we express the coordinate of the end point of the jib z through the coordinate of the load x and its time derivatives, as a result of which we get

$$z=\left(1-\frac{H}{g}\omega^2\right)x+\frac{H}{g}\ddot{x};\ \ \dot{z}=\left(1-\frac{H}{g}\omega^2\right)\dot{x}+\frac{H}{g}\dddot{x}. \tag{13}$$

At the same time, the coordinate z depends on the angular coordinate of the arrow α and is expressed by the following dependence:

$$z=L\,cos\,\alpha-l\,cos\,\varphi_X;\ \ \frac{\partial z}{\partial\alpha}=l\,sin\varphi_X\frac{\partial\varphi_X}{\partial\alpha}-L\,sin\alpha. \tag{14}$$

Let us write the angular coordinate of the boom φ_X through the arrow coordinate. To do this, we will use the condition that in the process of changing the departure, the end point of the jib C moves horizontally, which is at a distance H from the lower hinge of the boom O (Fig. 1). From this condition we have $L\,sin\alpha-l\,sin\,\varphi_X=H$.

From the resulting equation, we find that

$$sin\,\varphi_X = \frac{1}{l}\left(L\,sin\,\alpha - H\right);\quad \varphi_X = arcsin\frac{L\,sin\,\alpha - H}{l}. \tag{15}$$

From the first dependence (15), we find:

$$cos\,\varphi_X = \sqrt{1 - \frac{\left(L\,sin\,\alpha - H\right)^2}{l^2}}\,. \tag{16}$$

After substituting expression (16) into dependence (14), we obtain a quadratic equation relative to $cos\,\alpha$:

$$4L^2\left(H^2+z^2\right)cos^2\alpha - 4Lz\left(L^2+H^2-l^2+z^2\right)\times$$
$$\times cos\,\alpha + \left(L^2+H^2-l^2+z^2\right) - 4L^2H^2 = 0. \tag{17}$$

After solving the obtained equation, we will find the dependence of the angular coordinate of the arrow on the horizontal coordinate of the end point of the jib

$$cos\alpha_{1,2} = \frac{z\left(L^2+H^2-l^2+z^2\right) \pm H\sqrt{4L^2\left(H^2+z^2\right) - \left(L^2+H^2-l^2+z^2\right)^2}}{2L\left(H^2+z^2\right)}. \tag{18}$$

Taking into account the design features of the boom system, only the second solution (18) satisfies the requirements, therefore

$$\alpha = arccos\frac{z\left(L^2+H^2-l^2+z^2\right) - H\sqrt{4L^2\left(H^2+z^2\right) - \left(L^2+H^2-l^2+z^2\right)^2}}{2L\left(H^2+z^2\right)}. \tag{19}$$

Therefore, the dependence of the angular coordinate of the arrow α on the coordinate of the end point of the jib z, which in turn depends on the coordinate of the cargo x, is established.

From the first equation of the system (12), we find the driving moment of the drive reduced to the axis of rotation of the boom. At the same time, in the obtained dependence, we accept $m_B = 0;\ J_B = 0$, since the tie has a negligible effect on the dynamics of the boom system movement, as a result of which we get

$$M = \left[J_O + J_C + m_X L^2 + J_X\frac{\partial\varphi_X}{\partial\alpha} - m_X L(l-r)\frac{\partial\varphi_X}{\partial\alpha}cos\left(\varphi_X - \alpha\right)\right]\ddot{\alpha} +$$
$$+\left\{J_X\frac{\partial\varphi_X}{\partial\alpha}\frac{\partial^2\varphi_X}{\partial\alpha^2} - \frac{m_X L}{2}(l-r)\left[\frac{\partial^2\varphi_X}{\partial\alpha^2}cos\left(\varphi_X-\alpha\right) - \frac{\partial\varphi_X}{\partial\alpha}\left(\frac{\partial\varphi_X}{\partial\alpha}-1\right)\times\right.\right.$$
$$\left.\left.\times sin\left(\varphi_X-\alpha\right)\right]\right\}\dot{\alpha}^2 + \left\{J_X\frac{\partial\varphi_X}{\partial\alpha}sin2\varphi_X + m_X L\times\right.$$
$$\left.\times\left[L sin2\alpha + (l-r)\left(sin\alpha\, cos\varphi_X + \frac{\partial\varphi_X}{\partial\alpha}cos\alpha\, sin\varphi_X\right)\right]\right\}\frac{\omega^2}{2} + mg\frac{\partial z}{\partial\alpha}\frac{z-x}{H}. \tag{20}$$

As a criterion for optimizing the mode of joint movement of the mechanism for changing the departure of the boom system and the established mode of rotation of the crane, the root mean square value of the drive torque of the mechanism for changing the departure during the start-up time is used, which is presented in the form of an integral functional and has the following form:

$$M_{CK} = \left[\frac{1}{t_1} \int_0^{t_1} M^2 dt \right]^{1/2}, \tag{21}$$

where t is time; t_1 is the duration of movement of the boom system.

To determine the optimal mode of joint movement of the mechanisms for changing the departure of the cargo and turning the crane, we will set a variational problem.

Find the dependence of cargo movement $x = x(t),\ 0 \le t \le t_1$, which ensures the extreme value of functional (21), taking into account expression (20), while ensuring the following boundary conditions of movement:

$$\begin{cases} t = 0: x = x_0,\ \dot{x} = 0,\ z = x_0,\ \dot{z} = 0; \\ t = t_1: x = x_0 + \dfrac{\upsilon t_1}{2},\ \dot{x} = \upsilon,\ z = x_0 + \dfrac{\upsilon t_1}{2},\ \dot{z} = \upsilon. \end{cases} \tag{22}$$

Here x_0 – the initial position of the load; υ – speed of steady movement of cargo [4].

Since, in the given variational problem, the functional (21) reflects undesirable properties (action of loads) of the boom system, so it must be minimized.

We present the variational problem (21), (22) in an equivalent form

$$\int_0^{t_1} M^2 dt \to min. \tag{23}$$

Boundary conditions (22), taking into account expressions (13), reduce to the load coordinate and its time derivatives, that is, we obtain:

$$\begin{cases} t = 0: x = x_0,\ \dot{x} = 0,\ \ddot{x} = \omega^2 x_0,\ \dddot{x} = 0; \\ t = t_1: x = x_0 + \dfrac{\upsilon t_1}{2},\ \dot{x} = \upsilon,\ \ddot{x} = \left(x_0 + \dfrac{\upsilon t_1}{2} \right)\omega^2,\ \dddot{x} = \omega^2 \upsilon. \end{cases} \tag{24}$$

We present the approximate solution of the given nonlinear variational problem in the form of a polynomial, which is represented by two terms

$$x(t) = x_1(t) + x_2(t),\ 0 \le t \le t_1. \tag{25}$$

Here, the first term $x_1(t)$ is a polynomial chosen from the condition of ensuring the boundary conditions of motion (24), and the second term $x_2(t)$ is a polynomial that includes free coefficients and satisfies zero boundary conditions, namely

$$\begin{cases} x_2(0) = 0,\ \dot{x}_2(0) = 0,\ \ddot{x}_2(0) = 0,\ \dddot{x}_2(0) = 0, \\ x_2(t_1) = 0,\ \dot{x}_2(t_1) = 0,\ \ddot{x}_2(t_1) = 0,\ \dddot{x}_2(t_1) = 0. \end{cases} \tag{26}$$

Let us choose the term $x_1(t)$ in the form of a polynomial of the 7th degree to ensure the given conditions (24):

$$x_1(t) = A_0 + A_1 t + A_2 t^2 + A_3 t^3 + A_4 t^4 + A_5 t^5 + A_6 t^6 + A_7 t^7,\ 0 \le t \le t_1. \tag{27}$$

We take time derivatives from expression (27) up to and including the third order, as a result of which we will have:

$$\dot{x}_1(t) = A_1 + 2A_2 t + 3A_3 t^2 + 4A_4 t^3 + 5A_5 t^4 + 6A_6 t^5 + 7A_7 t^6;$$

$$\ddot{x}_1(t) = 2A_2 + 6A_3 t + 12A_4 t^2 + 20A_5 t^3 + 30A_6 t^4 + 42A_7 t^5;$$

$$\dddot{x}_1(t) = 6A_3 + 24A_4 t + 60A_5 t^2 + 120A_6 t^3 + 210A_7 t^4. \quad (28)$$

As a result of substituting the initial conditions at t=0 from expression (24) in dependence (27) and (28), we find the constants A_0, A_1, A_2, A_3, which acquire the following values:

$$A_0 = x_0;\ A_1 = 0;\ A_2 = \frac{x_0\omega^2}{2};\ A_3 = 0. \quad (29)$$

After substituting the final conditions (at $t = t_1$) from (24) and constants (29) in dependence (27) and (28), we obtain a system of linear algebraic equations:

$$\begin{cases} A_4 + A_5 t_1 + A_6 t_1^2 + A_7 t_1^3 = \dfrac{1}{2}\dfrac{\left(\upsilon - x_0\omega^2 t_1\right)}{t_1^3}; \\ 4A_4 + 5A_5 t_1 + 6A_6 t_1^2 + 7A_7 t_1^3 = \dfrac{\left(\upsilon - x_0\omega^2 t_1\right)}{t_1^3}; \\ 12A_4 + 20A_5 t_1 + 30A_6 t_1^2 + 42A_7 t_1^3 = \dfrac{\upsilon\omega^2}{t_1}; \\ 24A_4 + 60A_5 t_1 + 120A_6 t_1^2 + 210A_7 t_1^3 = \dfrac{\upsilon\omega^2}{t_1}. \end{cases} \quad (30)$$

As a result of solving the system of equations (30), we obtain the following values of the constants

$$A_4 = \frac{1}{2t_1}\left[5\frac{\upsilon}{t_1^2} - 5\frac{x_0\omega^2}{t_1} + \frac{13}{6}\upsilon\omega^2\right];\ A_5 = \frac{-3}{t_1^2}\left[\frac{\upsilon}{t_1^2} - \frac{x_0\omega^2}{t_1} + \upsilon\omega^2\right];$$

$$A_6 = \frac{1}{t_1^3}\left[\frac{\upsilon}{t_1^2} - \frac{x_0\omega^2}{t_1} + \frac{11}{4}\upsilon\omega^2\right];\ A_7 = \frac{-5}{6}\frac{\upsilon\omega^2}{t_1^4}. \quad (31)$$

A polynomial $x_1(t)$ of the form (27) with coefficients (29) and (31) ensures the boundary conditions (24), and the polynomial $x_2(t)$ ensures the minimization of the integral functional (23) and is expressed by the following dependence

$$x_2(t) = t^4\left(t - t_1\right)^4 (C_0 + C_1 t + \ldots + C_n t^n),\ 0 \le t \le t_1. \quad (32)$$

The multiplier $t^4\left(t - t_1\right)^4$ guarantees the fulfillment of zero boundary conditions at any values of the coefficients $C_0, \ldots, C_n$. These coefficients remain free, and are used to find the minimum of the functionals (21) or (23).

As a result of substituting dependencies (27) and (28) with coefficients (29) and (31) and dependence (32) in expression (25), we will get a clear form of function $x(t)$, which includes free coefficients $C_0, ..., C_n$. Moreover, the obtained function $x(t)$ provides the boundary conditions (24) with an arbitrary choice of coefficients $C_0, ..., C_n$.

From the explicit form of function $x(t)$, we find the form of function $z(t)$ and its time derivative using expressions (12). At the same time, the function $z(t)$ also includes free coefficients $C_0, ..., C_n$. Using dependence (18), the angular coordinate of the arrow $\alpha(t)$ is determined, which is a function of $z(t)$ and also includes free coefficients $C_0, ..., C_n$.

According to dependence (15), the angular coordinate of the jib $\varphi_x(t)$ is determined, which depends on the angular coordinate of the boom $\alpha(t)$ and also includes free coefficients $C_0, ..., C_n$.

By substituting the functions $x(t)$, $z(t)$, $\alpha(t)$, $\varphi_x(t)$ and their derivatives into expression (20), we find the function of the driving moment of the drive reduced to the axis of rotation of the boom. The expression of the driving moment of the drive also includes free coefficients $C_0, ..., C_n$, which, in turn, is included in the integral expression of the functional (21). It reflects the mean square value of the driving moment M_{CK}.

After integration, in expression (21) the functional M_{CK} is a function of the arguments $C_0, ..., C_n$. Considering what has been said, the approximate solution of the variational problem (21) with the boundary conditions (24) and taking into account the dependencies (13), ..., (20) is reduced to finding the minimum of the function of many variables. At the same time, one of the approximate methods [12] can be used.

In these studies, an applied software package was used to solve the given variational problem. In which methods based on the simplex method are used to find the minimum of a function of many variables.

The derivatives $\dot{\alpha}, \ddot{\alpha}, \frac{\partial z}{\partial \alpha}, \frac{\partial \varphi_X}{\partial \alpha}, \frac{\partial^2 \varphi_X}{\partial \alpha^2}$ included in the expressions (13), (14), (20) are determined by the approximate formulas of numerical differentiation, and the approximate calculation of the integral (21) is carried out by the trapezium formula.

Calculations were made for the sought functions $x(t)$, $z(t)$, $\alpha(t)$, $\varphi_x(t)$ and their derivatives, as well as for the driving moment M (20), at the maximum value of the exponent $n=5$, which is included in the dependence (32).

These calculations were performed for the crane boom system with the following parameters [5]: $L = 25{,}76$ m, $l = 10{,}16$ m, $r = 2{,}51$ m, $H = 14{,}7$ m, $g = 9{,}81$ m/s^2, $m = 20000$ kg, $m_X = 5453$ kg, $J_C = 2{,}856 \cdot 10^6$ kg$\cdot$m^2,

$J_X = 1{,}189 \cdot 10^5$ kg $\cdot$ m^2, $J_P = 6{,}338 \cdot 10^5$ kg $\cdot$ m^2, $\omega = 0{,}157$ radian/s, $\upsilon = 1{,}05$ m/s, $x_0 = 15$ m, $t_1 = 4$ s.

As a result of solving the problem for the coefficients $C_0, \ldots, C_n$ (n=5), the following approximate values were obtained: $C_0 = 0{,}1188 \cdot 10^{-4}$, $C_1 = 0{,}0446 \cdot 10^{-4}$, $C_2 = -0{,}0122 \cdot 10^{-4}$, $C_3 = 0{,}0062 \cdot 10^{-4}$, $C_4 = -0{,}0017 \cdot 10^{-4}$, $C_5 = 0{,}0002 \cdot 10^{-4}$. At the same time, the minimum value of the root mean square moment M_{CK} is equal to $(M_{CK})_{min} = 4{,}1187 \times 10^4$ N$\times$m.

The results of the calculations of the kinematic, power and energy characteristics of the boom system in the process of starting the mechanism for changing the departure at the established mode of rotation of the crane are shown in fig. 2...16.

The kinematic characteristics of the boom system, which characterize the amount of swinging of the load, include the linear movements of the load x and the end point of the jib z (Fig. 2), their linear velocities, respectively (Fig. 3), as well as the acceleration of the load (Fig. 4) and the end point jib (Fig. 5).

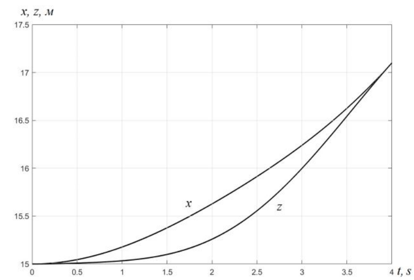

Fig. 2. The graph of changes in the horizontalcoordinate of the load (x) and the end pointof the jib (z)

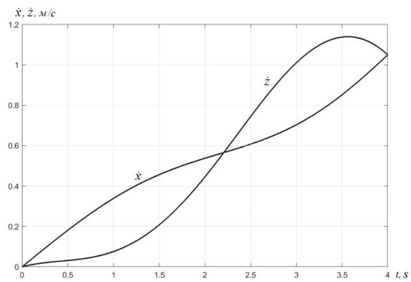

Fig. 3. Graph of changes in the linear speed of the cargo ($\dot{x}$) and the end point of the jib ($\dot{z}$)

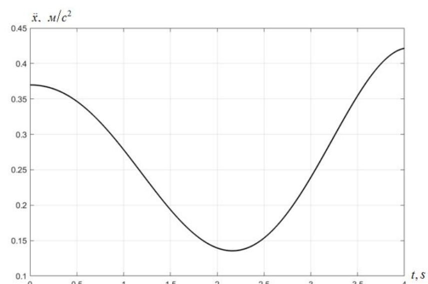

Fig. 4. Graph of changes in load acceleration

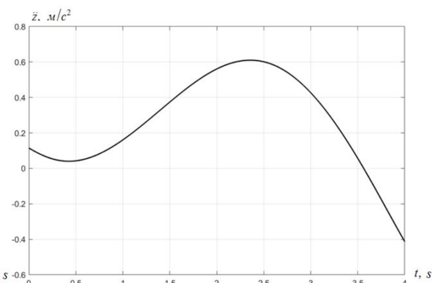

Fig. 5. Graph of the change in the acceleration of the end point of the jib

Angular coordinate, speed and acceleration of the boom $\alpha, \dot{\alpha}, \ddot{\alpha}$ are important kinematic parameters that significantly affect the dynamics of the movement of the boom system (Fig. 6...8).

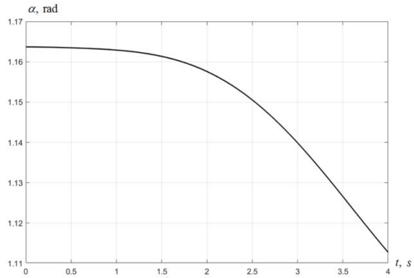

Fig. 6. Graph of the change in the angular coordinate of the boom

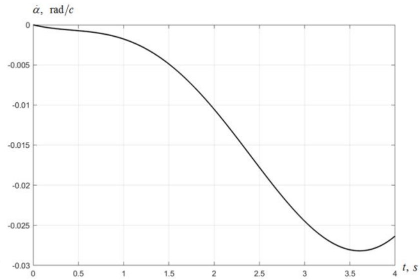

Fig. 7. Graph of the change in the angular speed of the boom

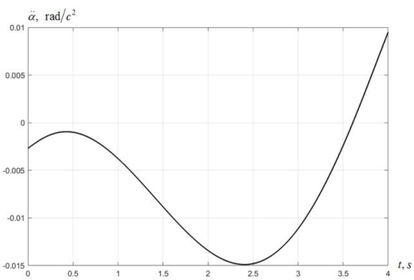

Fig. 8. Graph of changes in the angular acceleration of the boom

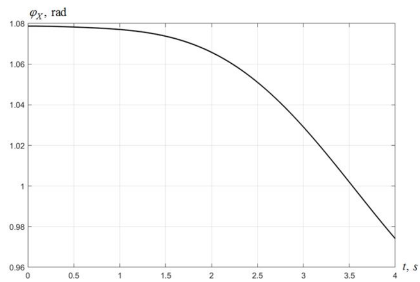

Fig. 9. Graph of the change in the angular coordinate of the jib

Also, the angular coordinate, speed and acceleration of the jib $\varphi_X, \dot{\varphi}_X, \ddot{\varphi}_X$ are important kinematic parameters affecting the operation of the crane (Fig. 9...11).

In addition, graphs of the driving moment of the drive of the departure change mechanism $M = M(t)$ (Fig. 12) and the power of the same drive $P = P(t)$ (Fig. 13) are depicted.

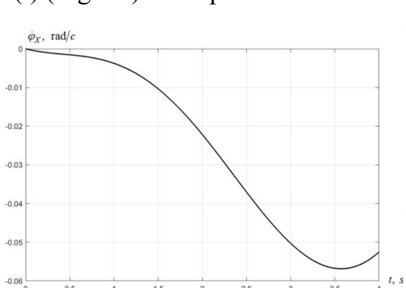

Fig. 10. Graph of change in the angular speed of the jib

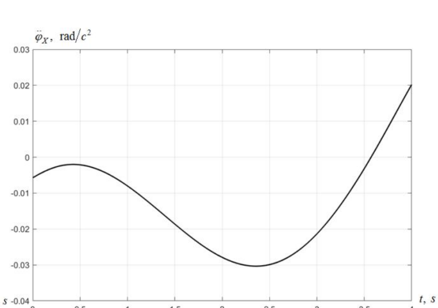

Fig. 11. Graph of changes in angular acceleration of the jib

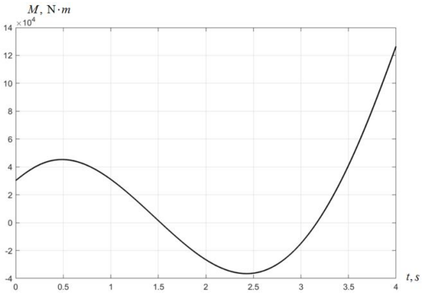

Fig. 12. Graph of the change in the driving moment of the departure change mechanism

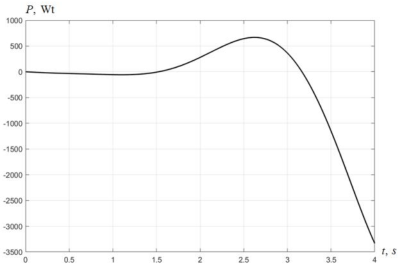

Fig. 13. Graph of the power change of the departure change mechanism

It can be seen from the graphic dependences of the movements of the center of mass of the cargo and the end point of the jib (Fig. 2) that the dependences have a smooth character of change, but at the corresponding moments of time their coordinates have certain shifts. This indicates that in the process of starting the mechanism of departure changes, there are fluctuations of the load relative to the point of flexible suspension of the load. Dependencies of the linear velocities of the center of mass of the cargo and the end point of the jib (Fig. 3) also have a smooth character of change. At the same time, the coordinates of these velocities at the corresponding moments of time have an offset, which also indicates fluctuations in the speed of the cargo relative to the end point of the jib, which is the point of suspension of the cargo. The acceleration of the center of mass of the cargo (Fig. 4), as well as the end point of the jib (Fig. 5), have convexities in opposite directions with shifted coordinates at the corresponding moments of time. This mode of change of accelerations indicates the presence of fluctuations in the acceleration of the center of mass of the cargo relative to the acceleration of the end point of the jib [13].

The angular coordinate of the boom (Fig. 6) smoothly decreases during the start-up process as the load movement coordinate increases (graph x in Fig. 2), that is, these coordinates have opposite directions of change. The angular velocity of the boom (Fig. 7) gradually decreases over a larger part of the launch, and has a slight increase towards the end of the launch. At the same time, the angular velocity of the arrow takes negative values throughout the launch area. The angular acceleration of the boom (Fig. 8) during the start-up process has an oscillatory character: at first it increased slightly, then decreased, and at the end of the start-up a sharp increase is observed. Angular displacements (Fig. 9), speed (Fig. 10) and acceleration (Fig. 11) of the jib are similar in nature to angular displacements (Fig. 6), speed (Fig. 7) and acceleration (Fig. 8) of the boom .

Reduced to the axis of rotation of the boom, the driving moment of the drive of the departure change mechanism (Fig. 12) is close to the nature of the change in the angular acceleration of the boom (Fig. 8). This can be explained by the fact that the inertial properties of the boom system when the pitch changes are determined by the parameters of the drive mechanism and the boom. The drive power of the departure change mechanism (Fig. 13) at the beginning of the start-up practically does not change, then it increases slightly, and at the end of the start-up it drops sharply. During most of the launch, the drive power of the flight change mechanism takes negative values. This is caused by the fact that the angular velocity of the arrow is also negative and it determines the sign of the power. In reality, the power value is taken as an absolute value and does not depend on the direction of movement of the links.

Conclusions

1. In the proposed article, the optimization task of researching the joint movement of the mechanisms for changing the boom system departure and rotation in a gantry crane is set. At the same time, the crane is turned in a steady state at a constant angular velocity of the electric motor shaft. The change in the departure of the load occurs during the start-up process, when the shaft of the electric motor changes its angular velocity from zero to a fixed value.
2. The optimization problem includes a mathematical model of the joint movement of the mechanisms for changing the range and rotation of the crane, the optimization criterion, which is the root mean square value of the drive torque of the mechanism for changing the range during the start-up process and the boundary conditions of the movement. These driving conditions ensure the elimination of load fluctuations on the flexible suspension after the end of the transition process.
3. The nonlinear optimization problem is solved by an approximate method, where the solution is presented in the form of a polynomial with unknown coefficients, which are determined using a package of application programs based on the simplex method.
4. As a result of the solution of the optimization problem, graphic dependences of the kinematic characteristics of the boom system and the load, as well as the driving torque and power of the drive of the departure change mechanism during the start-up process, were constructed. The obtained optimal mode of changing the departure of the load during the start-up process with the established mode of rotation of the crane made it possible to eliminate the oscillation of the load on the flexible suspension and to minimize the dynamic loads in the drive mechanism.
5. Recommendations are provided regarding the possible application of the determined optimal mode of simultaneous movement of the mechanisms for changing the departure and rotation of the boom system of the crane in practice under limited operating conditions.

REFERENCES

1. *Loveikin V. S., Romasevych Yu. O.* Optimization of bridge crane movement control. Science and technique. International scientific and technical journal. 17 (S), P. 413-420. https://doi.org/10.21122/2227-1031-2018-17-5-413-420. Web of science.
2. *Naidenko O. V., Makhortova D. O.* Keruvannya elektropryvodom mekhanizmiv obertannya z urakhuvannyam pidvishenoho vantazhu (Control of the electric drive of mechanisms of rotation taking into account the suspended load). Electrical and computer systems, 2010. No 01 (77). P. 17-26.

3. *Loveikin V. S., Chovnyuk Yu. V., Kadikalo I.O.* Optymizatsiya rezhymiv rukhu mekhanizmiv obertannya vantazhopidyomnykh kraniv (Optimization of modes of movement of mechanisms of rotation of cranes). Scientific Bulletin of the National University of Life and Environmental Sciences of Ukraine. Series: machinery and energy of agro-industrial complex. Kyiv. 2017. Vol. 262. P. 177-190.
4. *Loveykin V. S., Romasevich Yu. O., Kadykalo I. O.* Obhruntuvannia kraiovykh umov rukhu v zadachi optymyzatsii rezhymu povorotu strilovoho krana (Justification of the boundary conditions of movement in the problem of optimizing the rotation mode of a jib crane). Lifting and transport equipment, No. 2 (61), 2019. P. 45-59.
5. *Loveikin V. S., Palamarchuk D. A.* Optymizatsiya rezhymiv rukhu sharnirno-zchlenovanoyi strilovoyi systemy krana (Optimization of modes of movement of the articulated boom system of the crane). – Kyiv : Publisher TsP «KOMPRINT», 2015. 224 p.
6. *Keqin LI, Cuxiang Jiang* Inverse design of a new double-link luffing mechanism and realization on MATLAB. Proceedings of the 3rd ICMEM International conference on mechanical engineering and mechanics. October 21−23, 2009. Beijing, P. R. China. P. 301-304.
7. *Bargazov E., Bortyakov D., Uzunov T., Alipiev O., Antonov S.* Optimization research of the cargo pendulum and units displacements of the gantry cranes level luffing jib system. International scientific journal "Machines. Technologies. Materials", 2018. Year XII, Issue 10. P. 386-391.
8. *V. S. Loveikin, Yu. O. Romasevych, I. Kadykalo, A. Liashko* Optimization of the swinging mode of the boom crane upon a complex integral criterion. Journal of theoretical and applied mechanics, Sofia, Vol. 49 (2019), P. 285-296. Interdisciplinary topics (Scopus).
9. *Zairulazha Bin Zainal* Modeling and Vibration Control of a Gantry Crane / Zairulazha Bin Zainal. – Faculty of Electrical Engineering Universiti Teknologi Malaysia, 2005. – 160 p.
10. Dyukarev Yu.M., Litvinova O.G. Differential and integral equations and calculus of variations: Study guide. – Kharkiv: Karazin KhNU, 2010. 138 p.
11. *Limonov L. G., Netesa A. N., Kreslavsky A. I., Teslitsky A. N.* Elektroprivody peremennogo toka osnovnykh mekhanizmov portal'nogo krana s greyfernym i kryukovym zakhvatom (AC electric drives of the main mechanisms of the gantry crane with grab and hook gripper). Electrical engineering and electrical equipment, 2006. No 66. P. 138–140.
12. *Loveikin V. S., Palamarchuk D. A., Romasevych Yu. O., Loveykin A. V.* Optimization of rotate mode at constant change of departure in the level-luffing crane with geared sector. Strength of Materials and Theory of Structures. Kyiv. 2021. Vol. 106. P. 221-235. https://doi.org/10.32347/2410-2547.2021.106.221-235. Web of science.
13. *Loveykin V. S., Palamarchuk D. A.* Doslidzhennia rushiinykh syl v mekhanizmi zminy vylotu strilovoi systemy krana (Research of the driving forces in the mechanism of changing the travel of the crane boom system). Mining, construction, road and melioration machines, 2014. No 84. P. 39-45.

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Ловейкін В. С., Ромасевич Ю. О., Паламарчук Д. А., Ловейкін А. В.
ОПТИМІЗАЦІЯ СИЛОВОГО РЕЖИМУ ЗМІНИ ВИЛЬОТУ ШАРНІРНО-ЗЧЛЕНОВАНОЇ СТРІЛОВОЇ СИСТЕМИ ІЗ ЗУБЧАСТИМ СЕКТОРОМ ПРИ УСТАЛЕНОМУ РЕЖИМІ ПОВОРОТУ КРАНА

Для оптимізації режиму зміни вильоту стрілової системи використані методи варіаційного числення. При цьому, поставлено варіаційну задачу, яка включає диференціальні рівняння руху стрілової системи, критерій оптимізації та крайові умови руху при зміні вильоту та повороту крана. За критерій оптимізації режиму руху стрілової системи обрано середньо-квадратичне значення рушійного моменту приводу механізму зміни вильоту в процесі пуску, який являє собою інтегральний функціонал. Крайовими умовами обрано кінематичні характеристики механізму зміни вильоту стрілової системи від стану спокою до досягнення усталеної швидкості вантажу, при усталеній кутовій швидкості механізму повороту. Такі крайові умови усувають коливання вантажу на гнучкому підвісі після закінчення процесу пуску при зміні вильоту

В результаті чисельного розв’язку поставленої варіаційної задачі побудовані графічні залежності оптимальних кінематичних, силових і енергетичних характеристик в процесі пуску механізму зміни вильоту, при усталеному режимі повороту крана. Отриманий оптимальний режим руху механізму зміни вильоту усуває коливання вантажу на гнучкому підвісі і мінімізує динамічні навантаження.

Ключові слова: механізм повороту, механізм зміни вильоту, розгойдування вантажу, режим руху, інтегральний критерій, оптимізація, стрілова система крана.

Loveykin V.S., Romasevich Yu.O., Palamarchuk D.A., Loveykin A.V.

OPTIMIZATION OF THE FORCE MODE OF RANGE CHANGE IN AN ARTICULATED BOOM SYSTEM WITH A GEARED SECTOR AT A FIXED MODE OF CRANE ROTATION

Variational calculus methods were used to optimize the boom system's flight change mode. For this, a variational problem is set, which includes the differential equations of motion of the jib system, the optimization criterion, and the boundary conditions of the movement when the departure and rotation of the crane are changed. The root-mean-square value of the drive torque of the flight change mechanism during the start-up process, which is an integral function, was chosen as the criterion for optimizing the movement mode of the boom system. The kinematic characteristics of the mechanism for changing the departure of the boom system from the state of rest to reaching the steady speed of the load, at the steady angular speed of the turning mechanism, were chosen as boundary conditions. Such boundary conditions eliminate fluctuations of the load on the flexible suspension after the end of the start-up process when the departure is changed

As a result of the numerical solution of the given variational problem, graphical dependencies of the optimal kinematic, power and energy characteristics in the process of starting the flight change mechanism, with a fixed mode of rotation of the crane, were constructed. The obtained optimal mode of movement of the departure change mechanism eliminates load fluctuations on the flexible suspension and minimizes dynamic loads.

Key words: turning mechanism, departure change mechanism, load swinging, movement mode, integral criterion, optimization, boom system of the crane.

УДК 621.87

Ловейкін В. С., Ромасевич Ю. О., Паламарчук Д. А., Ловейкін А. В. **Оптимізація силового режиму зміни вильоту шарнірно-зчленованої стрілової системи із зубчастим сектором при усталеному режимі повороту крана** // Опір матеріалів і теорія споруд: наук.-тех. збірник. – К.: КНУБА, 2023. – Вип. 110. – С. 404-420.

Наведено послідовність проведення та результати оптимізації режиму зміни вильоту шарнірно-зчленованої стрілової системи із зубчастим сектором на ділянці пуску при усталеному режимі повороту крана. За об’єкт дослідження взято стрілову систему крана з секторним приводом механізму зміни вильоту. За критерій оптимізації режиму руху стрілової системи обрано середньо-квадратичне значення рушійного моменту приводу механізму зміни вильоту. Отриманий оптимальний режим руху механізму зміни вильоту дозволяє мінімізувати динамічні навантаження під час пуску.

Іл. 13. Бібліогр. 13 назв.

UDC 621.87

Loveykin V. S., Romasevich Yu. O., Palamarchuk D. A., Loveykin A.V. **Optimization of the force mode of range change in an articulated boom system with a geared sector at a fixed mode of crane rotation** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles – Kyiv: KNUCA, 2023. – Issue 100. – P. 404-420.

The sequence of execution and optimization results for the mode of change of departure of the hinged boom system with a geared sector in the starting area at the established mode of crane rotation are presented. The boom system of the crane with the sector drive of the mechanism for changing the departure is taken as the object of the study. The criterion for optimizing the movement mode of the boom system is the root-mean-square value of the drive torque of the flight change mechanism. The obtained optimal mode of movement of the departure change mechanism allows to minimize dynamic loads during start-up.

Il. 13. Ref. 13.

Автор (науковий ступінь, вчене звання, посада): доктор технічних наук, професор, завідувач кафедри конструювання машин і обладнання Національного університету біоресурсів і природокористування України ЛОВЕЙКІН Вячеслав Сергійович
Адреса робоча: 03041 Україна, м. Київ, вул. Героїв Оборони, 12, Національний університет біоресурсів і природокористування України, кафедра конструювання машин і обладнання, ЛОВЕЙКІНУ Вячеславу Сергійовичу.
Робочий тел.: +38(044) 527-87-34
Мобільний тел.: +38(097) 349-14-53
E-mail: lovvs@ukr.net
ORCID ID: https://orcid.org/0000-0003-4259-3900

Автор (науковий ступінь, вчене звання, посада): доктор технічних наук, доцент, професор кафедри конструювання машин і обладнання Національного університету біоресурсів і природокористування України РОМАСЕВИЧ Юрій Олександрович.
Адреса робоча: 03041 Україна, м. Київ, вул. Героїв Оборони, 12, Національний університет біоресурсів і природокористування України, кафедра конструювання машин і обладнання, РОМАСЕВИЧУ Юрію Олександровичу.
Робочий тел.: +38(044) 527-87-34
E-mail: romasevichyuriy@ukr.net
ORCID ID: https://orcid.org/0000-0001-5069-5929

Автор (науковий ступінь, вчене звання, посада): кандидат технічних наук, доцент, доцент кафедри теоретичної механіки КНУБА ПАЛАМАРЧУК Дмитро Анатолійович.
Адреса робоча: 03037 Україна, м. Київ, Повітрофлотський проспект 31, КНУБА, кафедра теоретичної механіки, ПАЛАМАРЧУКУ Дмитру Анатолійовичу.
Мобільний тел.: +38(097) 825-73-35
E-mail: palamarchuk-dima@ukr.net
ORCID ID: https://orcid.org/0000-0002-8019-9659

Автор (науковий ступінь, вчене звання, посада): кандидат фізико-математичних наук, доцент, доцент кафедри математичної фізики Київського національного університету імені Тараса Шевченка ЛОВЕЙКІН Андрій Вячеславович
Адреса робоча: 03022, Україна, м. Київ, проспект академіка Глушкова, 4е ,корпус механіко-математичного факультету, Київський національний університет імені Тараса Шевченка, кафедра математичної фізики, ЛОВЕЙКІНУ Андрію Вячеславовичу
Мобільний тел.: +38(097) 350-91-23
E-mail: anlov74@gmail.com
ORCID ID: https://orcid.org/0000-0002-7988-8350