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THE FRACTAL SCALE-INVARIANT STRUCTURE OF A TEMPORAL HIERARCHY IN THE RELAXATION AND ENERGY DISSIPATION PROCESSES IN A VISCO-ELASTIC/CAPILLARY-POROUS MEDIUM

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The phenomena of elastic aftereffects during loading/unloading of viscoelastic and capillary-porous bodies, relaxation of their stresses is accompanied by the energy accumulation and dissipation to be taken into account in the theory of oscillations which also considers the behavior of materials when the force is applied to them. The elastic aftereffect and stress relaxation forms ostensibly opposite energy processes. In the first case, under constant load deformation, the work increases in course of time, and in the second case, under constant load deformation, the work (energy) decreases. While researching on the energy dissipation in the conditions of oscillations application, i.e. within the frame of internal friction theories, one can find that some theories are based on the dependence of friction on the oscillations' velocity, other ones establish the dependence of friction on the amplitude. Research papers are based on the hypothesis of M.M. Davydenkov, according to which the energy when subjected to oscillations depends on the amplitude and does not depend on the velocity. According to E.S. Sorokin, the theory of internal friction is poorly consistent with the theories describing the inherited properties of materials (viscoelastic and capillary-porous ones). A tendency is observed: the better a theory reflects hereditary properties, the worse this theory is adapted to describe energy losses due to oscillations.In this paper, an attempt has been made to harmonize both these theories and numerous experiments on the destruction of materials described in the academic literature. It turns out that in order to remove contradictions, it is necessary to take into account the dependence of body deformation changing in the course of time.It is shown that the hierarchy of times determining shear and bulk relaxation in viscoelastic/capillary-porous medium has a fractal (scale-invariant) structure. It was observed that the presence of time fractality eases the modeling of viscoelastic/capillary-porous bodies resulting in the universal relaxation function of a rather simple kind. In particular, for the scale-invariant distribution of relaxation characteristics medium, the application of algebraic relaxation law for viscoelastic/capillary-porous materials is possible: this resulting in rheological models and state equations with the derivatives of fractional order.

Key words: fractality, scale-invariant structure, temporal hierarchies, processes, relaxation, aftereffect, energy dissipation, internal friction,viscoelasticity, capillary-porous medium

1. Introduction

Hereditary properties of materials have long been studied by experts. For example, faced in 1920s with the fact of elastic aftereffects, the academician A.F. Ioffe described the above phenomenon as follows: "... the result of the effect of this force on the body does not manifest itself entirely at once. For a long time,

exposed to a constant force, bending, twisting, tensile it continues becoming gradually weaker. The precision instruments can detect a slow progress even in a few months. When the force ceases to effect, the body does not immediately take its previous form. Each effect leaves a trace which can be noticed after a long period of time as the reasons of its emergence disappear. There is something similar to the body's memory experienced in the past" [1] (Fig. 1).

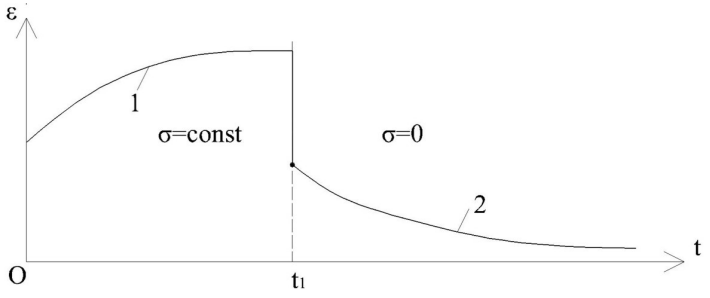

Fig. 1. The phenomenon of elastic aftereffects in viscoelastic and capillary-porous bodies: 1 - loaded; 2 - unloaded

Indeed, numerous experiments conducted with different materials show the deformation lag behind the stress. The deformation occurs in the process of force increasing and over a period of time after the termination of its increase. Such a phenomenon is called an elastic aftereffect after loading.

When the sample is unloaded, a similar pattern is observed: the elastic deformation decreases to zero during a certain period of time. The hereditary phenomena of the material include the phenomenon of stresses relaxation which consists in the fact that at constant deformation (and temperature), stresses decrease. In test machines, the load decrease and keeping the deformation unchangeable is made automatically with the help of special electronic equipment. The water is used as load in such tests. This allows a smooth stress decrease [2] (Fig. 2).

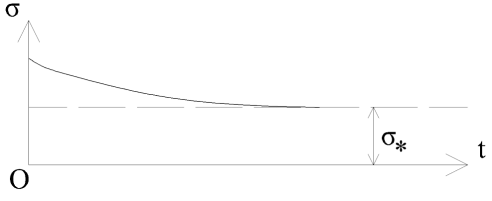

Fig. 2. The relaxation (on the plane) curve (σ, t), σ* – the boundary to which the stress is directed while "t" increases

All these phenomena occurring in viscoelastic/ capillary-porous bodies are accompanied by the energy accumulation and dissipation which should be taken into account in the oscillations theory considering the behavior of the material resulting from the force application.

It can be observed that the elastic aftereffect and stress relaxation produce opposite energy processes. In the first case, being constantly loaded, the deformation and work increase in course of time respectively. In the second case, being constantly exposed to the deformation, the load and work (energy) decrease.

Turning to researches for energy dissipation under load, i.e. for the theory of internal friction, it can be found that some theories are based on the dependence of oscillations friction on their velocity, other theories are based on the amplitude. Some research papers are based on the M.M. Davydenkov hypothesis. According to it, the energy in the conditions of oscillations application depends on the amplitude and does not depend on the velocity [3, 4].

E.S. Sorokin, author of one of these hypotheses [5], carries out a detailed analysis of research papers on these issues. On the basis of his table containing the comparative characteristics of various theories, he made an important remark that the theory of internal friction is poorly consistent with theories describing the hereditary properties of the materials. Moreover, the following tendency is observed: the better a theory reflects hereditary properties, the worse this theory is adapted to describe energy losses due to oscillations.

In this paper, an attempt has been made to harmonize both these theories and numerous experiments on the destruction of materials described in the academic literature, for example, in [7]. It turns out that in order to avoid contradictions, it is necessary to take into account the dependence of body deformation changing in the course of time specified below.

Modern technological processes control often requires the modeling of relaxation in reophysically complex media (viscoelastic medium (VEM), capillary-porous bodies (CPB)). Such media are encountered in the production of a wide variety of materials (rubbers, plastics, textiles, paints, lubricants, foods, etc.) [8-12, 15]. They are extremely important in processes related to oil extraction and transportation [13, 14, 16]. The interest of researchers to these materials is due to a huge variety of new effects which can occur in relaxing materials. Their rheology studying contributes to a better understanding and improvement of technological processes, rational development of new high-performance techologies and products.

Relaxation phenomena in rheophysically complex media are related to the slow development of processes regrouping structural units of different scales. (For example, in the case of polymers, these are flexible molecules, their individual segments, or bundles formed by these molecules). These processes result in deformation changes lag behind the stress change (hysteresis, elastic aftereffect, stress relaxation, etc.) and can be described using the model of elastic bodies with internal friction and viscous bodies with elasticity [8-12]. Mechanical models of viscoelastic, capillary-porous bodies are helpful for understanding the qualitative characteristics of relaxation phenomena, but their application to the quantitative description of real materials requires the construction of very complex systems consisting of a large number of different springs and viscous elements (due to the presence of structural units hierarchy of different scale leading to relaxation hierarchization in a wide range of time). It is clear that complex models can not prove effective: there are too many difficulties associated with the definition of numerous relaxation parameters with respect to experimental data, as well as solving problems of modeling the media motion in a wide range of relaxation time.

This research paper shows that the difficulties mentioned hereinabove can be overcome by specifying the structure of time hierarchies which determine the relaxation in rheophysically complex media. An analysis of experimental data shows that the distribution of relaxation time in these media may be scale-invariant, i.e. may have a fractal structure. It is shown that the time fractality allows to simplify the description of relaxation processes resulting in universal relaxation functions of a rather simple form in a wide range of relaxation time [17, 18]. It is also shown that in some cases, it is possible to use rheological models with derivatives of fractional order.

2. The stress and deformation in viscoelastic and capillary-porous bodies with respect to time

It is necessary to add to the known formula $\sigma_1 = E \cdot \varepsilon$ the component taking into account the temporal nature of the stress change:

$$\sigma_2 = E \cdot \varepsilon \cdot \exp(-t/\tau), \tag{1}$$

or

$$\sigma = E \cdot \varepsilon + E \cdot \varepsilon \cdot \exp(-t/\tau) = E \cdot \varepsilon \{1 + \exp(-t/\tau)\}, \tag{2}$$

where σ – body's general stress, E – stress module, ε – elongation ratio (deformation), t –time counted from the moment the load application, τ – relaxation time.

Elongation ratio (deformation) in the conditions of load application:

$$\varepsilon = \frac{\sigma}{E \cdot \{1 + \exp(-t/\tau)\}}, \tag{3}$$

After unloading, the maximum value of elongation ratio is as follows:

$$\varepsilon_0 = \frac{\sigma}{E \cdot \{1 + \exp(-t/\tau)\}} - \frac{\sigma}{2E} = \frac{\sigma \cdot [1 - \exp(-t/\tau)]}{2E \cdot [1 + \exp(-t/\tau)]} = \frac{\sigma}{2E} \cdot th\left[+\frac{t}{2\tau}\right]. \tag{4}$$

The value of ε_0 allows to determine the current value of its elongation ratio after unloading in time t, i.e. to take into account the unloading aftereffect and thus:

$$\varepsilon(t) = \varepsilon_0 \cdot \exp(-t/\tau) = \frac{\sigma}{2E} \cdot th[t/2\tau] \cdot \exp(-t/\tau), \tag{5}$$

where t – time running after unloading.

Example 1. The steel wire stretched and rigidly fixed. It is necessary to determine the stress relaxation. Sample data: elastic module E = 196 333 MPa, relaxation time τ = 168.2 s, elongation ratio ε = 0.001. The formula (2) allows to calculate the stresses occurring in the wire and depending on time, i.e. relaxation of stresses. The results are shown in Table 1.

Table 1

t, s	σ, MPa	t, s	σ, MPa	t, s	σ, MPa
0	392,627	200	256,12	1000	196,66
50	341,89	500	206,26	2000	196,13
100	304,60	900	197,08	3000	196,13

Example 2. The steel sample was stretched to have the stress σ = 300 MPa, module of elasticity E = 196 333 MPa, relaxation time τ = 168.2 s, the sample is

unloaded at the time t_p = 1000 s. It is necessary to determine the value of elongation ratio depending on time. The formula (3) allows to determine the value of elongation ratio in the conditions of loading, the formula (4) allows to calculate the unloading moment. Varying in time residual deformation can be found using the formula (5). The results of calculations are given in the Table 2.

Table 2

t, s	$\varepsilon_н$, dimensionless	t, s	$\varepsilon_н$, dimensionless
0	$7{,}6479\cdot10^{-4}$	1000	$7{,}6070\cdot10^{-4}$
200	$1{,}1713\cdot10^{-3}$	1200	$2{,}3265\cdot10^{-4}$
500	$1{,}4543\cdot10^{-3}$	1500	$3{,}9363\cdot10^{-5}$
1000	$1{,}5255\cdot10^{-3}$	2000	$2{,}0356\cdot10^{-6}$

Knowing ε_0, one can find the residual deformation $\varepsilon_{residual} = \varepsilon_r$ using the ratio (5). If the ratio $\varepsilon_r/\varepsilon_0 = \sigma$ is specified, the period of time (t^*) can be found, then the residual deformation will be $\varepsilon_r = \sigma\cdot\varepsilon_0$ (in periods τ). The results for viscoelastic and capillary-porous bodies are presented in Table 3.

Table 3

σ	t^*, s	σ	t^*, s	σ	t^*, s
0,1	2,303τ	0,0001	4,210τ	10^{-8}	18,421τ
0,01	4,605τ	0,00001	11,513τ	10^{-10}	23,026τ
0,001	6,908τ	0,000001	13,816τ	10^{-12}	27,631τ

3. The energy dissipation process when oscillations are applied

The equity (2) which explains the phenomena occurring in the material during its loading-unloading should be used to describe the process in viscoelastic and capillary-porous bodies. Thus, there is a relation between this dependence and the theory of internal friction given in [6].

In addition, there is no need to involve velocity dependent on external forces to explain energy losses, as was done in [6].

4. The effect of time on the results of experiments on the tensile of viscoelastic-type materials and capillary-porous bodies

It is known that time has a significant effect on the results of experiments on the materials destruction. Data from a number of scientists who carried out experiments involving various materials (Bach, Baumann, Volterra, Le Chatelier, Ludwig) were given in [7].

Curves can be constructed using three points and the equation of these curves can be described by applying these data (time is expressed in seconds, stress is expressed in pascals).

The changing of the destructive stress in time is expressed by the following general formula:

$$\sigma_t = \sigma_x + (\sigma_0 - \sigma_x)\cdot\exp(-t/\tau), \qquad (6)$$

where σ_t – ultimate breaking stress at the time t, where σ_x – ultimate breaking stress at the time $t = \infty$, t – load application time, τ – relaxation time.

The three points $t_1 \rightarrow \sigma_1$, $t_2 \rightarrow \sigma_2$, $t_3 \rightarrow \sigma_3$ available from the experiment allow to derive the equation:

$$\begin{cases} \sigma_2 = \sigma_x + (\sigma_1 - \sigma_x) \cdot \exp\left(-(t_2 - t_1)/\tau\right); \\ \sigma_3 = \sigma_x + (\sigma_1 - \sigma_x) \cdot \exp\left(-(t_3 - t_1)/\tau\right), \end{cases} \tag{7}$$

it is resulting in

$$\sigma_x = \frac{\sigma_3 - \sigma_1 \cdot \exp\{-(t_3 - t_1)/\tau\}}{1 - \exp\{(t_3 - t_1)/\tau\}}. \tag{8}$$

The relaxation time τ can be found by means of the iteration method using the following formula derived from (7):

$$\sigma_2 - (\sigma_1 - \sigma_3) \cdot \exp\{-(t_2 - t_1)/\tau\} = \sigma_3 - (\sigma_1 - \sigma_2) \cdot \exp\{-(t_3 - t_1)/\tau\}. \tag{9}$$

The calculations using the formulas (6) - (9) are easy to perform on a PC. The general view of curves showing the ultimate breaking stress changing in time is given in Fig. 3. Experimental data processing results for [3] are presented in Table 4.

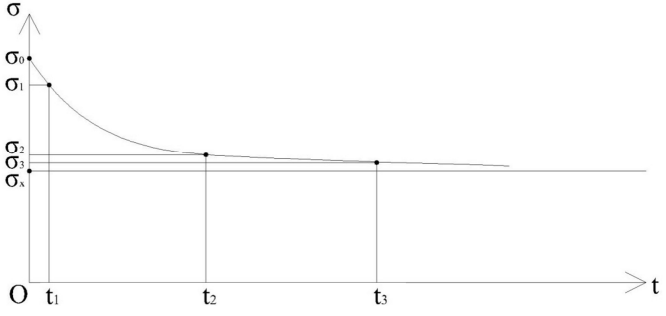

Fig. 3. General view of curves showing the ultimate breaking stress changing in time

The ultimate breaking stress changing in time is characterized by the same exponential function dependent on time. The ultimate breaking stress decreases in course of time.

Consequently, the material (viscoelastic/capillary-porous) offering the property of elastic aftereffect has also the property of stresses relaxation. The stress in the material depends on elongation ratio decreasing in time and showing the exponential dependence. If the material is loaded, the main energy is spent to overcome the forces of elasticity and the additional energy (accumulated in the material) dissipates within a certain time period. To solve the problems of energy dissipation when oscillations are applied, it should be taken into account that the additional energy varies in time according to the exponential law. The ultimate breaking stress decreases in time according to the exponential law. The results of experiments on certain materials breaking dependent on time are consistent with the given formulas.

Table 4

Material of sample	Experimental data		Calculation results		
	Time t_1, t_2, t_3, s	Ultimate breaking stress, MPa σ_1, σ_2, σ_3	Relaxation time τ, s	Ultimate breaking stress, MPa if $t = 0$ σ_0	Ultimate breaking stress, MPa if $t = \infty$ σ_x
1. Foundry steel Experiments of Bach and Baumann	19 150 1320	392,66 385,30 379,03	168,8 212	394,284	379,024
2. Steel Experiments of Bach and Baumann	17 150 1560	550,94 540,15 522,20	284,1 459	552,720	522,073
3. Zinc Experiments of Le Chatelier	60 3600 86400	235,34 112,78 49,875	330,8 75	238,757	49,03
4. Copper Experiments of Ludwig	0 300 37584000	248,20 248,11 198,09	1668 70,6	248,20	198,09
5. Brass Experiments of Volterra	0 2160000 9504000	505,04 294,20 156,90	2395 187	505,04	150,19

5. Relaxation of stresses in viscoelastic and capillary-porous bodies. Generalized Maxwell model

Considering the generalized Maxwell model representing a set of parallel connected chains composed of a series of sequentially connected springs and a viscous element. The rheology of such a body is determined by the known relations: $\sigma = \sum_{n=1}^{\infty} \sigma_n$, $\varepsilon_n = \varepsilon_n^{(1)} + \varepsilon_n^{(2)}$, where ε – body deformation, σ – stress, $\sigma_n = E_n \cdot \varepsilon_n^{(1)} = \eta_n \cdot D \cdot \varepsilon_n^{(2)}$ – stress, E_n, η_n – spring stiffness and viscous resistance coefficient of element n, $\varepsilon_n^{(1)}, \varepsilon_n^{(2)}$ – elongation of the spring n and displacement of viscous element n, $D = d/dt$ – operator of differentiation.

It is given that a body is deformed $\varepsilon = h(t)$ at the time $t = 0$ where $h(t)$ is Heaviside function. The stress relaxation is determined by the function:

$$\Phi(t) = \sum_{n=1}^{\infty} E_n \cdot \exp(-t/\tau_n), \tag{10}$$

where $\tau_n = \eta_n / E_n$ – relaxation time of the element n. The value E_n (n = 1, 2, …) determines the contribution of the element n in the total stress $\sigma(t)$.

Authors of many studies have found that multilevel relaxation processes in the majority of the most varied systems are characterized by scale-invariant

(fractal) distributions of characteristic time [18, 19]. Based on this, it is assumed that the values E_n and τ_n are determined by the scaling laws having the following form:

$$E_n = E_0 / \lambda_1^n = E_0 \cdot \exp(-n\lambda); \lambda = \ln \lambda_1 \quad (11)$$

and

$$\tau_n = \tau_0 \cdot \mu_1^n = \tau_0 \cdot \exp(n\mu); \mu = \ln \mu_1, \quad (12)$$

or

$$\tau_n = \tau_0 \cdot n^{\nu}. \quad (13)$$

After taking the logarithm (11), the following formula can be obtained: $\ln E_n = \ln E_0 - n\lambda$.

Thus, the time-scale invariance should linearly decrease at the same time as n increases.

It contains logarithms of E_n and τ_n values corresponding to several hierarchical levels of mono- and polydisperse polystyrene sample. The exact number of selected levels is not defined, so they have numbers $n = m+k$ where m is an unknown level number with the smallest (of the given) relaxation time, $0 < k \le 8$, $k/m << 1$.

Table 5

Relaxation characteristics of polystyrene

K	E – in 10^{-1} Pa, τ in seconds			
	monodisperse polystyrene		polydisperse polystyrene	
	$\ln E_{m+k}$	$\ln \tau_{m+k}$	$\ln E_{m+k}$	$\ln \tau_{m+k}$
1	5,46	5,04	5,22	5,44
2	6,08	4,34	5,55	4,76
3	5,92	3,67	5,95	4,09
4	6,20	3,01	6,19	3,54
5	6,34	2,20	6,39	2,91
6	6,50	1,14	6,68	2,33
7	6,95	0,40	7,03	1,51
8	7,03	-0,30	-	-

It is easy to see that ln$E$ is really linearly dependent on the level number. The linear dependence of the relaxation time logarithm on the level number which may be the scaling law effect (12). However, it should be noted immediately that using only the given data, any of the above possible dependencies for relaxation times can not be preferable (laws (12) and (13)). Indeed, $k/m << 1$ together with (13) allow to obtain the following formula: $\ln \tau_{m+k} = \ln \tau_0 + \nu \cdot \ln(m+k) \approx (\ln \tau_0 + \nu \cdot \ln m) + (\nu/m) \cdot k$ which also gives a linear dependence between the level number and relaxation time logarithm.

By choosing the scaling laws (11) and (12) and turning the sum (10) into an integral, it is possible to obtain:

$$\Phi(t) = E_0 \cdot \int_0^{\infty} \exp(-x\lambda) \cdot \exp(-t \cdot \exp(-x\mu)/\tau_0) dx .$$

To determine the asymptotic behavior of this integral at large values of relaxation time, the variable $z = \exp(-x\mu)$ should be replaced: $\Phi(t) = \frac{E_0}{\mu} \cdot \int_0^1 z^{\lambda/\mu - 1} \cdot \exp(-t \cdot z/\tau_0) dz$, hence, using Laplace method, it is possible to obtain the following formula:

$$\Phi(t) \approx (E_0/\mu) \cdot \Gamma(\lambda/\mu) \cdot (t/\tau_0)^{-\lambda/\mu}, \quad (14)$$

where $\Gamma(x)$ – gamma function.

If the relaxation time is given by the law (13), then the following integral can be obtained: $\Phi(t) = E_0 \cdot \int_0^{\infty} \exp(-x\lambda - \frac{t}{\tau_0} \cdot x^{-\nu}) dx$. It has to be noted that the function $x\lambda - \frac{t}{\tau_0} \cdot x^{-\nu}$ reaches its minimum if: $x = x_* = (t\nu/(\tau_0\lambda))^{1/(\nu+1)}$. By replacing the variable, it is possible to obtain the following formula:

$$\Phi(t) = E_0 \cdot x \int_0^{\infty} \exp\left[-A(z + z^{-\nu}/\nu)\right] dz ,$$

where $A = \left(\frac{t \cdot \lambda^{\nu} \cdot \nu}{\tau_0}\right)^{1/(\nu+1)}$.

If $t/\tau_0 \to \infty (A \to \infty)$ then asymptotic behavior of this integral is easily determined using Laplace method that leads to an expanded exponential law (Kohlrausch law [18, 19]):

$$\Phi(t) \approx \exp\left[-(t/\tau)^{1/(\nu+1)}\right], \quad (15)$$

where $\tau = \left(\tau_0 \cdot \lambda^{-\nu}/\nu\right) \cdot \left(1 + \nu^{-1}\right)^{1/(\nu+1)}$.

Thus, the scale invariance of relaxation processes substantially simplifies their description and allows to use a rather simple universal relaxation functions having the form (14) and (15).

It should be noted that the relaxation function (14) with an exponent equal to (-1/2) can be obtained using Gauss and Bueche molecular theory of viscoelasticity [12]. However, this theory can explain neither the exponent value deviation (which is often observed in practice) nor the origin of the relaxation functions (15).

The scale invariance of the relaxation parameters distribution can serve to explain the principle of temperature-time superposition [12] which can be expressed using the following dependence:

$$\Phi\left[k(T)t\right] = k_1(T) \cdot \Phi_0(t), \quad (16)$$

where T_0 – some characteristic temperature, $\Phi(t)$ and $\Phi_0(t)$ – relaxation functions at temperatures T and T_0, k, k_1 – coefficients depending on temperature $\{k(T_0) = k_1(T_0) = 1\}$.

Indeed, if it is assumed that the scaling parameters λ, μ are not dependent on temperature then using (14), it is possible to obtain:

$$\Phi_0(t) = \frac{E_0(T_0)}{E_0(T)} \cdot \Phi\left\{\frac{\tau_0(T)}{\tau_0(T_0)} \cdot t\right\}$$

whence it follows (16) with $k = \frac{\tau_0(T)}{\tau_0(T_0)}$; $k_1 = \frac{E_0(T_0)}{E_0(T)}$.

As an example, consider the stress relaxation curve in a monodisperse polystyrene sample given in [12] (see Table 6 in which the value of depending on time dimensionless values are specified).

Table 6

Stress relaxation curve in a monodisperse polystyrene sample

t, 10^5 s	$\bar{\sigma}$	t, 10^5 s	$\bar{\sigma}$
0,000	1,00	1,015	0,24
0,062	0,78	1,169	0,22
0,092	0,63	1,415	0,20
0,215	0,53	1,600	0,18
0,277	0,45	1,785	0,16
0,400	0,40	2,092	0,15
0,492	0,36	2,800	0,12
0,646	0,31	3,569	0,09
0,800	0,28		

The Fig. 4 presents approximation results of this curve by means of functions $\bar{\sigma} = \exp(-t/\tau)$ and $\bar{\sigma} = \exp\left[(-t/\tau)^{\beta}\right]$ parameters of which τ and β were determined using methods of the sensitivity theory [20, 21]. Apparently, the stress relaxation curve is quite well described by the law of Kohlrausch. For this curve, the parameter β is equal to 0.5.

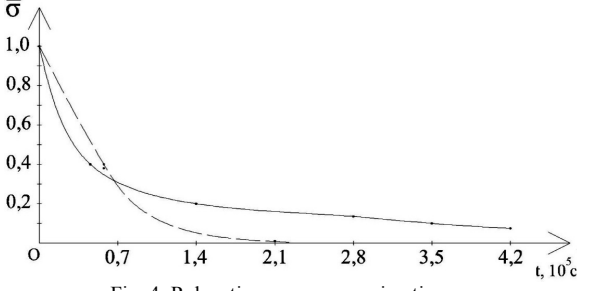

Fig. 4. Relaxation curve approximation:

1 – relaxation function $\bar{\sigma} = \exp[(-t/\tau)^{\beta}]$, 2 – function $\bar{\sigma} = \exp(-t/\tau)$

6. Rheological models of viscoelastic and capillary-porous bodies in fractional derivatives

A viscoelastic/capillary-porous body which can be given using a set of sequentially connected Voigt bodies (chains which consist of parallel connected springs and a viscous element) should be considered now. The stress $\sigma = \sigma_0 \cdot h(t)$ applied to a body at the time $t = 0$. Then the deformation rate is determined using the expression:

$$D\varepsilon(t) = \frac{\sigma_0}{\eta} + \sigma_0 \sum_{n=1}^{\infty} \frac{1}{\eta_n} \cdot \exp(-t/\tau_n).$$

By determining the relaxation function $\Psi(t)$ as $\Psi = \{D\varepsilon - \sigma_0/\eta\}/\sigma_0$, the following result is obtained $\Psi(t) = \sum_{n=1}^{\infty} \left(\frac{1}{\eta_n}\right) \cdot \exp(-t/\tau_n)$. The deformation velocity with the arbitrary stress changing is determined using this function [8, 10]:

$$D\varepsilon(t) = \frac{\sigma(t)}{\eta} + \int_0^t \Psi(t-\xi) d\sigma(\xi). \qquad (17)$$

As it was shown above, it is assumed that there is the scale invariant distribution of relaxation parameters.

Then (see (14)):

$$\Psi(t) = L \cdot t^{-\chi_1}, \qquad (18)$$

and (17) can be rearranged in the following form:

$$D\varepsilon(t) = (1/\eta) \cdot \sigma(t) + \alpha \cdot D^{-\chi} D\sigma(t), \qquad (19)$$

where $\chi = 1 - \chi_1$; $\chi_1 = \lambda'/\mu$; $\alpha = L \cdot \Gamma(\chi)$; $L = \Gamma(\chi_1) \cdot \tau_0^{\chi_1} / (\eta_0 \cdot \mu)$;

$D^{-\chi} f(t) = \frac{1}{\Gamma(\chi)} \cdot \int_0^t (t-\xi)^{\chi-1} f(\xi) d\xi$ – fractional derivative of order $(-\chi)$.

Accepting that $E_n = E_0 \cdot \exp(-\lambda n)$, the following formula is obtained: $\tau_n = (\eta_0 / E_0) \cdot \exp[(\lambda' + \lambda) n]$ whence it follows that $0 < \chi_1 < 1, 0 < \chi < 1$.

Thus, the time-scale invariance leads to the need to use rheological models in fractional derivatives. It should be noted that such models are entered (on other grounds) in [10, 11, 21].

The obtained result is also related to the research paper [22] establishing that relaxation processes in dielectrics with fractal distribution of clusters are described by means of fractional derivative equations. It is necessary to emphasize that the rheological law with fractional derivatives is obtained here as model including only different springs and viscous elements (unlike in the research paper [11]) where the existence of an independent type of deformation is postulated which can not be reduced to the sum of elasticity and viscous friction.

7. Relaxation processes in viscoelastic and capillary-porous bodies in conditions of bulk deformation

During some experiments [13, 23] performance, it was observed that if the cavity was filled with a structured liquid (for example, with oil containing asphalt-resinous admixtures) and if a pressure excess was created in it and it was sealed hermetically then the pressure in the cavity gradually and slowly decreases to some stationary value. Relaxation processes of this kind are associated with the rearrangement of macromolecules and clusters formed by them. If a rapid compression is applied then such a system is influenced by instantaneous elastic deformation whose value is determined using the bulk elasticity coefficient of the medium being in the initial state. Then a slow structural units rearrangement of varying complexity which (due to the sealing of the medium) leads to decrease of its volume and, consequently, to the pressure decrease. Considering structural units as viscoelastic elements, the mechanical model given in Fig. 5 is proposed to describe the bulk relaxation processes. For this model, the value β_0 characterizes the instantaneous volume compressibility of the medium and the values $E_n, \eta_n\ (n=1, 2, ...)$ describe elasticity of structural units and viscosity forces that counteract their movement.

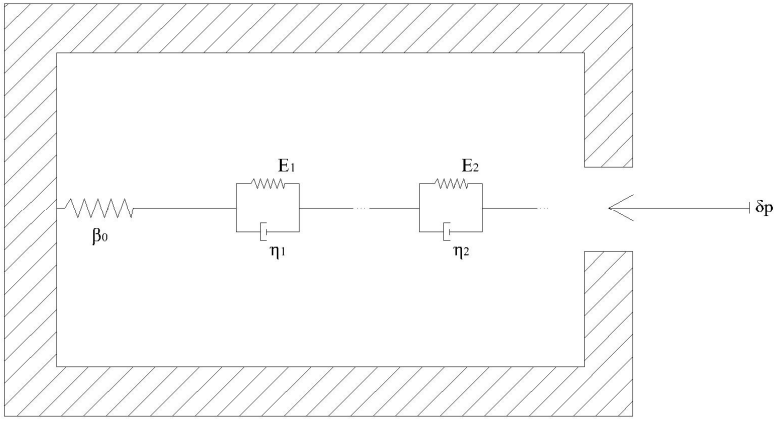

Fig. 5. Mechanical model of volume relaxing body

Similar to the previous cases, it is easy to obtain:

$$-\delta V(t)/V_0 = \beta_0 \cdot \delta p(t) + \int_0^t \Psi_1(t-\xi) D \delta p(\xi) d\xi, \qquad (20)$$

where δV – decrease of medium volume when the pressure is increased by value δp, V_0 – initial volume, $\Psi_1(t) = \beta' \cdot \sum_{n=1}^{\infty} \frac{1}{E_n} \cdot \left[1 - \exp(-t/\tau_n)\right]$ – relaxation function, $\tau_n = \eta_n / E_n$, β' – quantity determining the volume changing due to the displacement of the structural elements. Having differentiated (20) with respect to time, the equation the viscoelastic medium (capillary-porous body)

state is obtained: $\frac{1}{\rho_0} Dp = \beta_0 \cdot Dp + \int_0^t \Psi(t-\xi) Dp(\xi) d\xi$, where ρ – medium density. $\left(\frac{1}{\rho_0} Dp = -\frac{1}{V_0} D\delta V\right)$, $\Psi(t) = \beta' \cdot \sum_{n=1}^{\infty} \left(\frac{1}{\eta_n}\right) \cdot \exp(-t/\tau_n)$.

Again, taking the scaling laws (11), (12) and retaining previous notations, it can be obtained in the same way as (19):

$$\frac{1}{\rho_0} Dp(t) = \beta_0 \left(Dp + \beta_1 \cdot D^{-\chi} Dp\right), \tag{21}$$

where $\beta_1 = \beta' \cdot \Gamma(\chi_1) \cdot \tau_0^{\chi_1} / (\mu \cdot \beta_0 \cdot \eta_0)$.

Thus, the equation for viscoelastic/capillary-porous bodies (media) state may have fractional derivatives (it has to be noted that powers of derivatives in (19) and (21) may differ, although the same notations is still held for them).

It is supposed that the pressure relaxation in a particular container is described by the equation (21). The operation method [24, 25] is used to identify this model. It is considered that during the process of pressure relaxation, the density of the structured liquid in the container does not change, i.e. $\delta p(t) = \rho_0 \cdot \beta_0 \cdot \delta p(0) \cdot h(t)$. By taking $\delta p(t) = \delta p(0) \cdot [\ln t - 1] + \delta p_1(t)$ and applying Laplace conversion (21), the following formula is obtained:

$$\frac{1}{S} U = 1 + \beta_1 \cdot S^{-\chi}, \tag{22}$$

where $\delta p_1(t)$ – pressure measured during the experiment, $U = \frac{1}{\delta p(0)} \cdot \int_0^{\infty} e^{-St} \cdot \delta p_1(t) dt e^{i\theta}$. After converting (22), the following result is obtained: $\ln\left(\frac{1}{S} U - 1\right) = \ln \beta_1 - \chi \ln S$.

Thus, if the bulk relaxation is in fact described by the model (21) then the pressure curve should be straightened in coordinates $Y(S) = \ln\left\{\frac{1}{S} U - 1\right\}, \ln S$. The inclination of line can be found with respect to χ.

8. Equation for relaxing liquid motion

If the motion of a relaxing medium in a pipe (capillary) of radius R is considered then the rheological equation of the medium is presented as follows:

$$-\frac{\partial v}{\partial r} = \frac{\sigma}{\eta} + \alpha \cdot D^{-\chi} \cdot \frac{\partial \sigma}{\partial t}, \tag{23}$$

where $v(r, t)$ – component of velocity along the pipe axis, σ – shear stress, η – viscosity of medium. By averaging (23) over the section of the pipe, the following motion equation can be obtained within the frame of the quasi-stationary approximation [26]:

$$\rho_0 \cdot \left\{\frac{\partial w}{\partial t} + 2aw\right\} = -\left\{\frac{\partial p}{\partial x} + \alpha \cdot D^{-\chi} \cdot \frac{\partial^2 p}{\partial x \partial t}\right\}, \tag{24}$$

where w – average cross-sectional velocity, $2a = 8\eta / (\rho_0 R^2)$, $\partial p/\partial x$ – pressure gradient along the axis of the pipe.

It is possible to write down the continuity equation $\frac{\partial \rho}{\partial t} = -\rho_\partial \cdot \frac{\partial w}{\partial x}$ with respect to (21) in the following form:

$$\frac{\partial p}{\partial t} + \beta_1 \cdot D^{-\chi'} \cdot \left(\frac{\partial p}{\partial t}\right) = -\rho_0 \cdot C_0^2 \cdot \left(\frac{\partial w}{\partial x}\right), \tag{25}$$

where $C_0 = (\beta_0 \rho_0)^{-1/2}$ – "instantaneous" acoustic velocity in a medium. Excluding the velocity from (24) and (25), the equation for the relaxing liquid motion can be obtained in the following:

$$(D + 2a)(Dp + \beta_1 \cdot D^{-\chi'} Dp) = C_0^2 \cdot (1 + \alpha \cdot D^{-\chi} D) \cdot \left(\frac{\partial^2 p}{\partial x^2}\right). \tag{26}$$

As far as is known, the filtration equation can be obtained by throwing away the inertia term in (24) and taking $1/2a = k/\eta$ where now w – filtration rate, k – permeability of the porous (capillary-porous) medium instead of (26); then it is easy to obtain [27]:

$$Dp + \beta_1 \cdot D^{-\chi'} Dp = \kappa(D^{-\chi} \cdot D + 1) \cdot \left(\frac{\partial^2 p}{\partial x^2}\right), \tag{27}$$

where $\kappa = k / \eta^m \cdot \beta_o$ - thermal conduction coefficient, m – porosity.

9. Conclusions

1. Thus, the distribution of relaxation time in viscoelastic and capillary-porous media may have a scale-invariant (fractal) structure. To confirm this, the spectra of relaxation parameters obtained during experiments carrying out the stretching of polystyrene samples are given in the present paper. It is shown that the indirect confirmation of the scale invariance of relaxation time hierarchy can be the principle of temperature-time superposition according to which the experimental relaxation functions obtained for different temperatures can be combined with each other using the appropriate coordinate axes stretching.

2. Taking into account the scale invariance of relaxation parameters distribution substantially eases the modeling of multilevel relaxation processes and allows to use a rather simple universal relaxation functions having the form (14) and (15).

The present paper shows that in some cases, time fractality can lead to an algebraic relaxation law and, thus, to the need to use rheological models and state equations having fractional derivatives. It is precisely fractional derivatives that can be used for modeling, in particular, bulk relaxation processes.

3. The derived equations of motion of relaxation media in tubes, capillaries, porous media which take into account the time scale invariance of shear and bulk deformation processes are given.

REFERENCES

1. *Ioffe A.F.* O fizike i fizikakh [About physics and physicists]. Leningrad: Nauka, 1977. – P. 244.
2. *Filin A.P.* Prikladnaia mekhanika tverdogo deformiruemogo tela [Applied mechanics of solid deformable body]. – Moskva: Nauka, 1975. – V.1. – 832 p.
3. *Davidenkov N.N.* Obzor o rasseianii energii pri vibratsiiakh [Energy dissipation during oscillations overview] // Zhurnal technicheskoy fiziki. – 1938. – V. 8. – Iss. 6. – P.11.
4. *Pisarenko G.S., Yakovlev A.P., Matveev V.V.* Vibropogloshchaiushchie svoistva konstruktsionnykh materialov: spravochnik [Vibration absorbing properties of construction materials: handbook]. – Kiev: Naukova думка, 1971. – 376 p.
5. *Sorokin Ye. S.* K teorii vnutrennego treniia pri kolebaniiakh uprugikh sistem [On the theory of internal friction during oscillations of elastic systems]. – Moskva: Gosudarstvennoe izdatelstvo literatury po stroitelstvu arkhitekture i stroitelnym materialam, 1960. – P. 132.
6. *Kittel Ch., Nait U., Ruderman M.* Mekhanika [Mechanics]. - Moskva: Nauka, 1977. – V.1. – 480 p.
7. *Ivanov N.I.* Soprotivlenie materialov [Strength of materials]. – Moskva-Leningrad: GNTTL, 1942. – P. 27, 73.
8. *Lodg A.* Elastichnye zhidkosti [Elastic liquids]. – M.: Nauka, 1984.
9. *Uilkinson U.L.* Neniutonovskie zhidkosti [Non-newtonian liquids]. - M.: Mir, 1984.
10. *Blend D.* Teoriia lineinoi viazkouprugosti [Theory of linear viscoelasticity]. M.: Mir, 1965.
11. *Slonimskii G.L.* // DAN SSSR. – 1961. – V. 140. - № 2. – P. 343.
12. *Tobolskii A.* Svoistva i struktura polimerov [Properties and structure of polymers]. – M.: Himiia, 1964.
13. *Oshbalov P.M.,* Mirzadzhanzade A.Kh. Mekhanika fizicheskikh protsessov [Mechanics of physical processes]. – M.: MGU, 1976.
14. *Mirzadzhanzade A.Kh.,* Kovalev A.G., Zaitsev U.V. Osobennosti ekspluatatsii mestorozhdenii anomalnykh neftei [Operational features of anomalous oil fields]. - M.: Nedra, 1972.
15. *Shulman Z.P., Khusid B.M.* Nestatsionarnye protsessy konvektivnogo perenosa v nasledstvennykh sredakh [Non-stationary processes of convective transfer in hereditary media]. – Minsk: Nauka i tekhnika, 1983.
16. Gidrodinamika truboprovodnogo transporta nefti i nefteproduktov [Hydrodynamics of pipeline transportation of oil and oil products]. // *A.Kh. Mirzadzhanzade, A.K. Galiamov, V.I. Maron i dr.* – M.: Nedra, 1984.
17. *Shlezinger M., Klafter Dzh.* // V kn.: Fraktaly v fizike [Fractals in physics]. - M.: Mir, 1988. – P. 552.
18. *Bliumen A., Klafter Dzh., Tsumofen G.*// V kn.: Fraktaly v fizike [Fractals in physics]. - M.: Mir, 1988. – P. 561.
19. *Petrov B.N., Krutko P.L. Izvestiia AN SSSR.* Tekhnicheskaia kibernetika [Technique cybernetics]. – 1970. - №2. – P. 128.
20. *Rozenvasser E.N., Iusupov R.M.* Chuvstvitelnost sistem upravleniia [Sensitivity of guidance systems]. - M.: Nauka, 1981.
21. *Rabotnov U.N.* Elementy nasledstvennoi mekhaniki tverdykh tel [Elements of hereditary mechanics of solids]. - M.: Nauka, 1977.
22. *Nigmatullin R.R.*// Fizika tverdogo tela [Solid state physics]. - 1985. – V.27. – №5. – P. 1983.
23. *Mirzadzhanzade A.Kh., Ametov I.M.* Prognozirovanie promyslovoi effektivnosti metodov teplovogo vozdeistviia na neftianye plasty [Forecasting of field efficiency regarding methods of thermal effect on oil reservoirs]. - M.: Nedra, 1983.
24. *Barenblatt G.I. i dr.*// Izvestiia AN SSSR. OTN. – 1957. - №11. – P. 84.
25. *Khimmelblau D.* Analiz protsessov statisticheskimi metodami [Analysis of processes using statistical methods]. - M.: Mir, 1973.
26. *Charnyi I.A.* Neustanovivsheesia dvizhenie realnoi zhidkosti v trubakh [Unsteady motion of real liquid in pipes]. - M.: Nedra, 1975.
27. *Barenblatt G.I., Enotov V.M., Ryzhik V.M.* Teoriia nestatsionarnoi filtratsii zhidkosti i gaza [Theory of non-stationary filtration of liquid and gas]. - M.: Nedra, 1972.

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Човнюк Ю.В., Чередніченко П.П., Москвітіна А.С., Шишина М.О.

ФРАКТАЛЬНА МАСШТАБНО-ІНВАРІАНТНА СТРУКТУРА ЧАСОВИХ ІЄРАРХІЙ У ПРОЦЕСАХ РЕЛАКСАЦІЇ ТА РОЗСІЮВАННЯ ЕНЕРГІЇ В'ЯЗКО-ПРУЖНИХ/ КАПІЛЯРНО-ПОРИСТИХ СЕРЕДОВИЩ

Явища пружної післядії при навантаженнях/ розвантаженнях в'язко-пружних та капілярно-пористих тіл, релаксація напружень у них супроводжується накопиченням і розсіюванням енергії, що слід враховувати у теорії коливань, де також розглядається поведінка матеріалів під дією сили. Пружна післядія й релаксація напружень утворюють немовби протилежні енергетичні процеси. У першому випадку, при постійному навантаженні деформація й робота з плином часу збільшуються, а у другому – при постійній деформації навантаження, а разом з нею й робота (енергія) зменшуються. При дослідженнях щодо розсіювання енергії під час коливань, тобто у межах теорій внутрішнього тертя можна виявити, що одні теорії засновані на залежності тертя від швидкості коливань, інші – від амплітуди. В основу багатьох робіт покладена гіпотеза М.М. Давиденкова, згідно з якою енергія при коливаннях залежить від амплітуди і не залежить від швидкості. Згідно з думкою Є.С. Сорокіна, теорія внутрішнього тертя погано узгоджується з теоріями, котрі описують спадкові властивості матеріалів (в'язко-пружних та капілярно-пористих). Помічена тенденція: чим краще теорія відображає спадкові властивості, тим гірше вона прилаштована для опису енергетичних втрат при коливаннях. У даній роботі зроблена спроба узгодження як безпосередньо цих теорій, так і численних дослідів по руйнуванню матеріалів, які описуються у літературі. Виявляється, що для усунення протиріч слід враховувати наведену у даній роботі залежність зміни деформації тіла з плином часу. Показано, що ієрархія часів, котрі визначають зсувну й об'ємну релаксацію у в'язко-пружних/ капілярно-пористих середовищах, має фрактальну (масштабно-інваріантну) структуру. Помічено, що наявність часової фрактальності полегшує моделювання в'язко-пружних/ капілярно-пористих тіл, приводячи до універсальних релаксаційних функцій досить простого виду. Зокрема, у середовищах з масштабно-інваріантним розподілом релаксаційних характеристик можливий прояв алгебраїчного закону релаксації для в'язко-пружних/ капілярно-пористих матеріалів, що призводить до реологічних моделей та рівнянь стану, які мають похідні дробового порядку.

Ключові слова: фрактальність, масштабно-інваріантна структура, часові ієрархії, процеси, релаксація, післядія, розсіювання енергії, внутрішнє тертя, в'язкопружність, капілярно-пористі середовища.

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Човнюк Ю.В., Чередніченко П.П., Москвітіна А.С., Шишина М.А. **Фрактальна масштабно-інваріантна структура часових ієрархій у процесах релаксації та розсіювання енергії в'язко-пружних/капілярно-пористих середовищ. Аналіз ризиків при забезпеченні інформаційної безпеки** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2022. – Вип. 110. – С. 277 – 293. – Англ.
Зроблена спроба узгодження як безпосередньо теорій М.М. *Давиденкова і Є.С. Сорокіна, так і численних дослідів по руйнуванню матеріалів, які описуються у літературі. Виявляється, що для усунення протиріч слід враховувати наведену у даній роботі залежність зміни деформації тіла з плином часу.*
Табл. 6. Іл. 5. Бібліогр. 27 назв.

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Chovniuk Y.V., Cherednichenko P.P., Moskvitina A.S., Shyshyna M.O. **The fractal scale-invariant structure of a temporal hierarchy in the relaxation and energy dissipation processes in a visco-elastic/capillary-porous medium**// Strength of Materials and Theory of Structures: Scientific-and-technical collected articles.– K.: KNUBA. 2022. – Issue 110. – P. 277 – 293.
The article coordinates the theories of M.M. Davydenkov, E.S. Sorokin and numerous experiments on the destruction of materials described in the scientific literature. It turns out that in order to remove contradictions, it is necessary to take into account the dependence of the change in the deformation of the body with time.
Table 6. Fig. 5. Ref. 27

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