DYNAMIC STABILITY OF A HEMISPHERICAL SHELL WITH SHAPE IMPERFECTIONS

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The nonlinear dynamic analysis of imperfect hemispherical shell under pressure was executed. The finite element model of hemisphere in the software NASTRAN was built. The shell wall in the form of the three-cornered and four-cornered finite element net was presented. Shape imperfection as a lower buckling form (*Buckling*) of perfect hemispherical shell under action of static pressure was modelled. Value of imperfection amplitude was set proportionally to a shell wall thickness. Two boundary conditions in the form rigid and hinged supports on the nodes of lower edge of the shell were considered. Excitation as external pressure, which linearly depended on time and uniform distributed on elements of hemispherical shell was presented. The modal analysis of perfect hemisphere with different wall thicknesses and shell with modelled shape imperfections by the Lanczos method using computational procedure of task on natural vibrations (*Normal Modes*) was executed. The nonlinear dynamic analysis (*Nonlinear Direct Transient*) of imperfect hemispherical shell under pressure by N'yumark method was executed. Influence of modelled shape imperfections amplitude on the critical values of dynamic loading and appropriate deformed forms of shell with different boundary conditions were investigated.

Keywords: hemispherical shell, shape imperfection, finite element method, modal analysis, nonlinear dynamic analysis, dynamic stability.

Introduction

It is known that initial imperfections play a significant role in the reliability problem of real shells [1-11]. A review of the literature has shown that the dynamic behavior of thin shells with shape imperfections has not been sufficiently studied to date. The unsolved problem in the field of dynamic mathematical modeling of elastic shell systems with shape imperfections is there. There is no comparative assessment of the influence of different models and geometric imperfections amplitudes on the dynamic behavior and stability of shells. The stability of shell structures with shape imperfections under dynamic loads using the theory and methods of nonlinear dynamics, which are implemented in modern software, have not been sufficiently investigated. Modern software of finite-element analysis [12] allows the shape imperfections to be entered directly into the geometric parameters of the

middle surface of the shell. Previous studies of elastic shells with shape imperfections under static and dynamic loads by the authors were executed using the software NASTRAN [7-11]. It is proven that the choice of the most dangerous model of shape imperfection depends on the geometric parameters of the shell and the type of load. But what model of geometric imperfections of the shell is the most dangerous in the dynamic stability problem remains unresolved. This issue is important for researching design reliability of the thin shells under dynamic loads.

The influence of boundary conditions and thickness of perfect hemisphere on its natural frequencies and normal modes

A hemispherical shell with radius 0,004 m and different thickness $h = [0,1–1]$ mm was considered. Material with the mechanical characteristics: elastic modulus – $2,06 \cdot 10^{11}$ Pa, shear modulus – $0,79 \cdot 10^{11}$ Pa, Poisson's ratio − 0,3 i density – 7800 kg/m^3 was set. The comp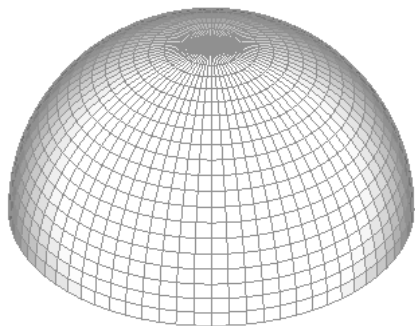uter model of shell was built in the software of finite element analysis NASTRAN [12]. The shell wall was presented as a set of plane quadrangular and triangular finite elements with six degrees of freedom in each node. The shell model consisted of 1441 nodes and 1400 elements (Fig. 1). Two types of boundary conditions of the hemispherical shell: rigid and hinged supports were examined.

Fig. 1. Finite element model of hemispherical shell with perfect surface

To research the dynamic stability of shells, a modal analysis must be performed. Using the solution of the eigenvalue problem (*Normal Modes*) and the Lanczos method, the frequencies and modes of the natural vibrations of the perfect shell with different thickness and boundary conditions were obtained (Tabl. 1).

It can see the first natural frequencies of the hemispherical shell with hinged support have lower values than in the case of rigid support; the natural frequency values increase a little with increasing the shell thickness.

As an example, in Fig. 3 the first ten modes of natural vibrations of a perfect shell with a thickness h=0,4 mm and h=1 mm were presented. All these normal modes didn't depend on the type of boundary conditions.

In further calculations, a hemispherical shell with a thickness 1 mm with different types of boundary conditions was considered and the effect of shape imperfections on the shell dynamic stability under the external surface pressure was evaluated.

Table 1

The first natural frequencies of the perfect hemisphere
with different wall thicknesses (Hz)

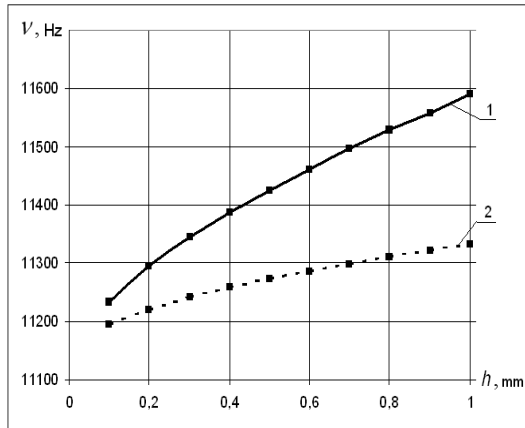| Thickness h, mm	Boundary conditions	
	Rigid support	Hinged support
0,1	11233,18	11195,31
0,2	11294,45	11220,79
0,3	11344,04	11240,61
0,4	11387,18	11257,44
0,5	11426,12	11272,38
0,6	11462,27	11286,01
0,7	11496,21	11298,67
0,8	11528,45	11310,58
0,9	11559,35	11321,89
1,0	11589,14	11332,71

Fig. 2. The influence of the thickness of perfect hemisphere and the type of boundary conditions
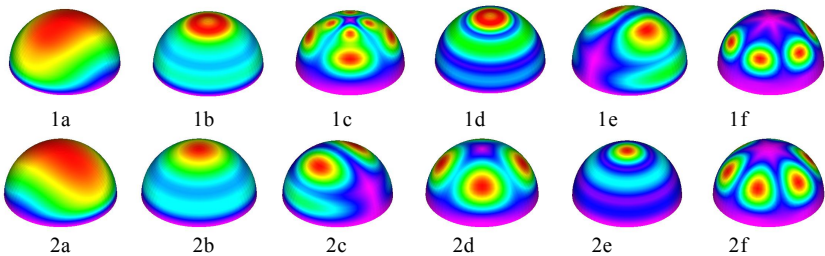on the natural frequencies: rigid (1) and hinged (2) supports

1a   1b   1c   1d   1e   1f

2a   2b   2c   2d   2e   2f

Fig. 3. The first ten normal modes of a perfect hemisphere with thickness h=0,4 mm (1) and
h=1 mm (2): 1and 2 (a); 3 (b), 4 and 5 (c), 6 and 7 (d), 8 (e), 9 and 10 (f)

Modal analysis of hemisphere with modelled shape imperfections

On the basis of previous studies of the shells dynamic stability [11], the authors took as the model of shape imperfection the first buckling forms of the hemisphere with the corresponding boundary conditions under static action of the surface pressure (Fig. 4 (a), (b)), which were obtained by the Lanczos method (*Buckling*).
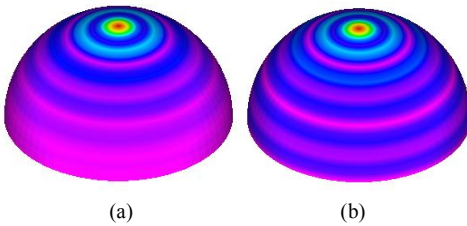

(a)   (b)

Fig. 4. The shape imperfection models
of the hemisphere: rigid (a) and hinged (b) supports

The amplitude of the shape imperfection of the hemispherical shell was set equal to $\delta=[0,4; 0,8; 1; 1,2; 1,6; 2]\,h$, where $h=1$ mm − the shell wall thickness. The shell imperfect surface for two types of boundary conditions was formed by adding the components of the buckling form with a given amplitude (Fig. 4 (a), (b)) to the corresponding coordinates of the middle surface of the perfect shell (Fig. 1). This procedure was implemented in a special program, which was adapted to the software NASTRAN [24] and was created by the authors.

Modal analysis of the hemisphere with shape imperfections of different amplitudes was performed by the Lanczos method. The values of the first ten natural frequencies of the shell in the Tabl. 2 were shown.

Table 2

Natural frequencies of the hemisphere with shape imperfections (Hz)

Number frequency	The amplitude of the shape imperfection δ/h ($h=1$ mm − shell wall thickness)						
	0	0,4	0,8	1,0	1,2	1,6	2,0
1	11589	11585	11577	11572	11565	11549	11413
2	11332	11364	11382	11386	11387	11379	11242
3	15579	15479	15217	15025	14641	12842	11528
	15185	15125	14866	14662	14408	12646	11356
4	18216	17903	16564	15614	14792		
5	17940	17852	16436	15457	14462	14216	13528
						13772	13012
6	18217	18188	18085	17993	17583	17472	16914
7	18216	17920	17821	17729	17593	16735	15824
8	19120	18879	18417	18183	17863	17590	
	18897	18634	18118	17845	17594	16857	17101
9						17658	15888
						16860	
	19351	19178	18771	18535	18257		
10						17658	17295
	19342	19040	18398	18008	17597	17112	16342
Rigid support Hinged support							

We can see in Tab. 3, the modelled imperfections increased the values of the shell natural frequencies by a maximum 25,7 % for the shell with rigid support and 27,4 % − hinged one.

Research showed that the first ten of hemisphere normal modes with an imperfection amplitude δ/h=0,1 and the corresponding ones of the perfect shell (Fig. 4) coincided. But with increasing in imperfection amplitude the differences in the hemisphere normal modes were observed. The first ten normal modes of the hemisphere with the amplitudes of the shape imperfection δ/h=0,4 and δ/h=1 were presented in Fig. 5. The normal modes of shell with the corresponding amplitude of the imperfection coincided for different boundary conditions.
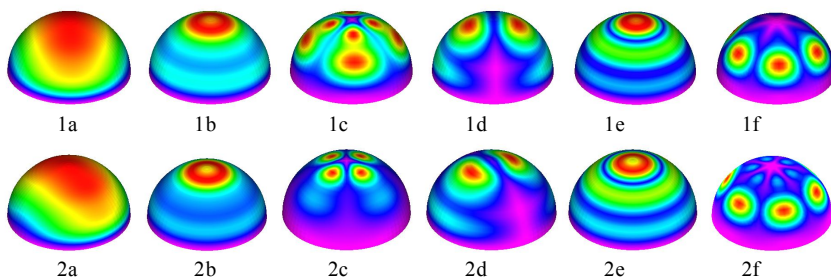

| 1a | 1b | 1c | 1d | 1e | 1f |

| 2a | 2b | 2c | 2d | 2e | 2f |

Fig. 5. The first ten normal modes of the hemisphere with the shape imperfection amplitudes δ/h=0,4 (1) and δ/h=1 (2): 1 i 2 (a); 3 (b), 4 i 5 (c), 6 i 7 (d), 8 (e), 9 i 10 (f).

We can see (Fig. 5) that the hemispherical shell with different boundary conditions from the fourth to the seventh normal modes has a discrepancy with the corresponding modes of the perfect shell (Fig. 3, b). But at the same time, the maximum number (six) of half-waves in the circular direction of the shell remains unchanged.

Modal analysis also showed that the first ten modes of natural vibrations of the hemisphere with an imperfection amplitude δ/h=1,2 depended on the type of boundary conditions and differed from the corresponding modes of the perfect shell (Fig. 4). At the same time, the maximum number of half-waves (six) in the circular direction of the hemisphere also remains unchanged.

The first five normal modes of the shell with imperfection amplitude δ/h=1,6 with rigid and hinged supports coincided, other modes depended on the type of boundary conditions. The first ten modes of natural vibrations of the hemisphere with a rigid support coincided with the corresponding modes of the shell with an imperfection δ/h=1,2. But for the shell with hinged support an increase in the maximum number of half-waves in the circular direction to eight was observed.

In further research of the hemisphere showed that with increasing in imperfection amplitude to δ/h=2 (Fig. 6), not only the maximum number of half-waves in the circular direction increased to ten but also the first and second normal modes changed. At the same time, the type of support didn't affect the first five modes of the hemisphere.
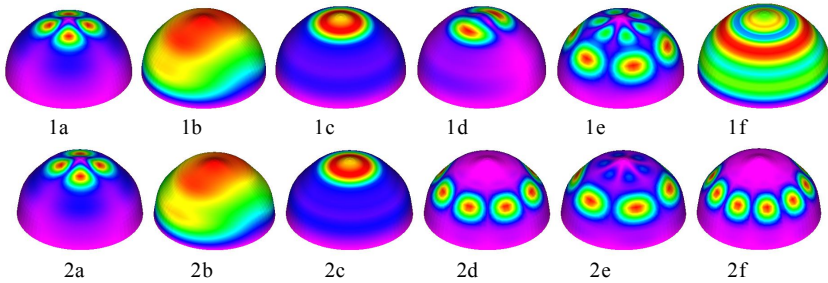
Fig. 6. The first ten normal modes of the hemisphere with the imperfection amplitudes δ/h=2 with the rigid (1) and hinged (2) supports: 1 i 2 (a); 3 i 4 (b), 5 (c), 6 i 7 (d), 8 i 9 (e), 10 (f)

Research showed that the shape imperfection model of the hemisphere as the first buckling form was effective in modal analysis of this shell.

The influence of shape imperfections on the dynamic stability of the hemispherical shell

Nonlinear dynamic analysis of a hemispherical shell with modelled shape imperfections under pressure was performed by the N'yumark method (*Nonlinear Direct Transient*), which was implemented in the NASTRAN software [12]. The excitation was given in the form of external pressure $q(t) = q_0 t$, where the amplitude of the excitation was $q_0 = 150$ MPa, the duration of the excitation from $t = 0$ to $t = 2$ s, the integration step $\Delta t = 0{,}002$ s and the structural damping coefficient $G = 0{,}01$ were set the same for all statements of the nonlinear dynamics analysis.

The influence of the excitation on the total maximum nodal displacements of the shell model (*Total Translation*) with different imperfection amplitude δ=[0,4; 0,8; 1; 1,2; 1,6; 2] h, where h=1 mm − the shall wall thickness, was investigated.

Table 3

Influence of imperfection amplitude of the hemisphere
on the critical value of pressure

Amplitude of imperfection δ/h (h=1mm)	Critical value of pressure q_{cr}^{dyn} (MPa)	
	Rigid support	Hinged support
0	133,51 / 124,64	129,22 / 105,00
0,4	103,87 / 116,42	102,72 / 94,61
0,8	84,70 / 93,32	83,53 / 89,06
1	76,81 / 82,50	75,30 / 81,35
1,2	69,38 / 75,04	68,49 / 72,08
1,6	61,95 / 60,12	58,81 53,61
2	60,15 / 52,53	57,68 / 40,22
Dynamic stability / Static stability [9]		

We can see (Tab. 3), the increasing of imperfection amplitude the critical value of the excitation were reduced by a maximum 54,9 % for the shell with rigid support and 55,4 % − hinged one. If we compare the obtained results with ones of the hemisphere static stability which was researched in the article [9], we can see that the trend of influence of the imperfection amplitude on the critical static load was preserved.

As an example, the dependence of the total maximum nodal displacements of the hemisphere with imperfection amplitude δ/h=1 and δ/h=2 on the excitation $q(t)$ was shown in Fig. 7.

1a

1b

2a

2b
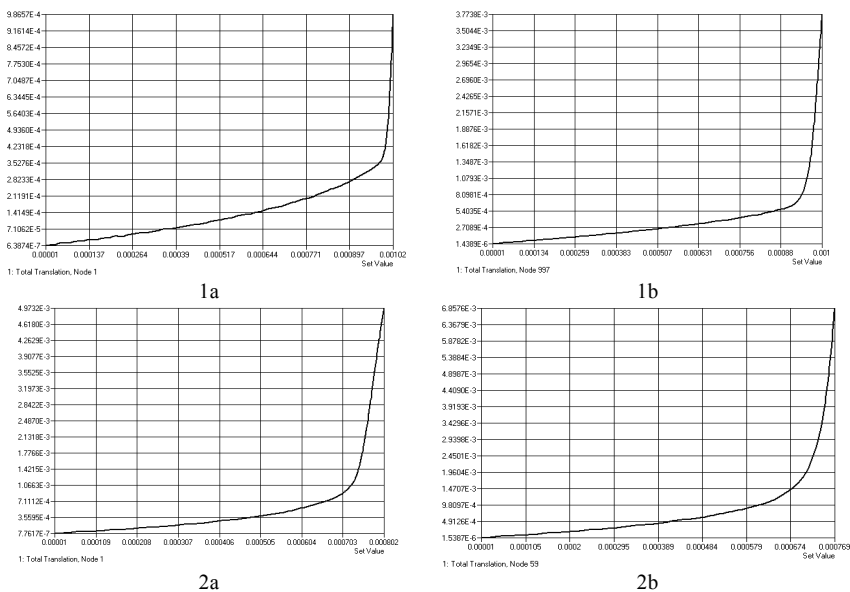
Fig. 7. Dependence of the total maximum nodal displacements of the hemisphere with imperfection δ/h=1 (1) and δ/h=2 (2) on the excitation $q(t)$: rigid (a) and hinged (b) supports

We observed (Fig. 7) that the curves of dependence of the total maximum nodal displacements on the excitation were the similar appearance, however the critical values of the pressure decrease with increasing of imperfection amplitude.

The deformed forms of the hemisphere with imperfection amplitude δ/h=1 and δ/h=2 at different stages of dynamic loading were shown in Fig. 8.

At the moment when shell dynamic stability was lost (the third deformed form in each statement of the problem in Fig. 8) the deformed forms of the hemisphere with given imperfection amplitude were different. Other deformed forms of the hemisphere at different stages of dynamic loading were the similar appearance.

The influence of the hemisphere imperfection amplitude on the critical value of the dynamic load was presented in Fig. 9.
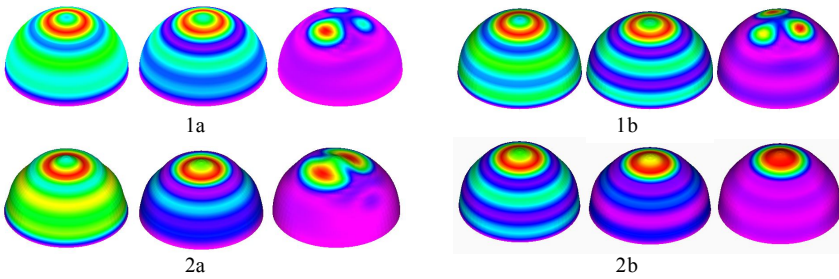
1a   1b

2a   2b

Fig. 8. The deformed forms of the hemisphere with imperfection amplitude *δ/h* =1 (1) and *δ/h* =2
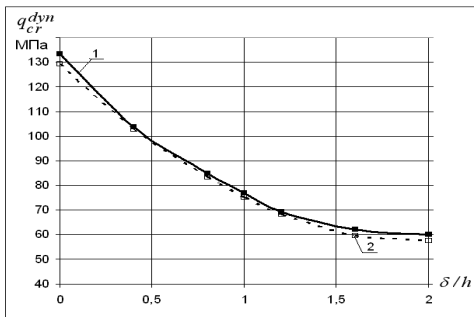(2) at different stages of dynamic loading: rigid (a) and hinged (b) supports

Fig. 9. The influence of imperfection amplitude of the hemisphere on the critical value of the
dynamic load: rigid (1) and hinged (2) supports

In Fig. 9 the values of the critical value of the pressure were significantly decreased with increasing the hemisphere imperfection amplitude. The critical values of dynamic load for the imperfect hemispherical shell with the rigid and hinged supports were little different.

Conclusion. Research has shown that the shape imperfection model of the hemispherical shell as the lower buckling form of shell under action of static pressure is effective in the modal and nonlinear dynamic analysis. A significant influence of the shape imperfection amplitude on the critical values of the dynamic load (55 %) was observed. Also, in our opinion, the presented model of hemisphere shape imperfection can be used to assess design reliability of the hemisphere under the action of dynamic loads using Bolotin probabilistic approach [7].

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ДИНАМІЧНА СТІЙКІСТЬ НАПІВСФЕРИЧНОЇ ОБОЛОНКИ З НЕДОСКОНАЛОСТЯМИ ФОРМИ

Виконано нелінійний динамічний аналіз стійкості пружної тонкої напівсферичної оболонки з недосконалістю форми від дії поверхневого тиску. Побудована скінченно-елементна модель півсфери в програмному комплексі NASTRAN. Стінка оболонки представлена у вигляді трикутної та чотирикутної скінченно-елементної сітки. Недосконалість змодельовано у вигляді нижчої форми втрати стійкості (*Buckling*) ідеальної напівсферичної оболонки під дією статичного тиску. Значення амплітуди недосконалості встановлювалася пропорційно товщині стінки оболонки. Розглянуто дві граничні умови у вигляді жорсткої і шарнірної опор на вузлах нижньої кромки оболонки. Збурення представлено у вигляді зовнішнішнього тиску, який рівномірно розподілений по елементах напівсферичної оболонки і лінійно залежить від часу. Виконано модальний аналіз ідеальної півсфери з різною товщиною стінки та оболонки зі змодельованими недосконалостями форми методом Ланцоша з використанням обчислювальної процедури розв'язання задачі на власні коливання (*Normal Modes*). Виконано нелінійний динамічний аналіз (*Nonlinear Direct Transient*) неідеальної напівсферичної оболонки від дії поверхневого тиску методом Н'юмарка. Досліджено вплив амплітуди змодельованих недосконалостей форми оболонки і різних граничних умов на критичні значення динамічного навантаження та відповідні форми деформації. Виявлено, що змодельована форма напівсферичної оболонки у вигляді нижчої форми втрати стійкості ідеальної оболонки від статичної дії тиску була ефективною у модальному та динамічному аналізі оболонки від такого ж виду навантаження. Спостерігався значний вплив амплітуди недосконалості на критичні значення динамічного навантаження (55 %). Також, на думку авторів, представлена модель недосконалості може бути використана для оцінки проектної надійності півсфери при дії динамічних

навантажень із застосуванням імовірнісного підходу Болотіна.

Ключові слова: напівсферична оболонка, недосконалість форми, метод скінченних елементів, модальний аналіз, нелінійний динамічний аналіз, динамічна стійкість.

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It was discovered that a modelled imperfection in the form of a lower buckling form of perfect shell under static pressure in the modal and the dynamic analysis of hemisphere under the same type of the loading was effective. A significant influence of the amplitude of the shape imperfection on the critical values of the dynamic load (55 %) was observed. Also, in our opinion, the presented model of shape imperfection can be used to assess design reliability of the hemisphere under the action of dynamic loads using Bolotin probabilistic approach.

Keywords: hemispherical shell, shape imperfection, finite element method, modal analysis, nonlinear dynamic analysis, dynamic stability.

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