УДК 539.3; 534.1

INFLUENCE OF MATERIAL FUNCTIONAL HETEROGENEITY ON NON-STATIONAR OSCILLATIONS OF PIEZOCERAMIC BODIES

L.O. Hryhorieva[1],
Ph. D.

I.V. Yanchevskyi[2],
Dr. Sci.

[1] *Kyiv National University of Construction and Architecture, Povitroflotsky Ave., 31, Kyiv, 03680*

[2] *National Technical University of Ukraine «Igor Sikorsky Kyiv Polytechnic Institute» Peremogy ave., 37, Kyiv, 03056*

DOI: 10.32347/2410-2547.2022.109.359-368

The influence of functional heterogeneity of the material on mechanical oscillations of piezoelements atnon-stationary electrical loads is investigated. Within the assumption of the corresponding to body physicalproperties functional distribution of material characteristics by thickness of piezoelectric element, a unifiedsystem of solving equations was obtained to describe the thickness fluctuations of piezoelectric plates,thick-walled cylinders and balls. For controllingof accuracy, the calculation is performed with explicit andimplicit difference scheme. It was established that influence of functional heterogeneity within one material is reduced to decrease of wave propagation velocity and the amplitude of oscillations up to 2-3%. We do a conclusion that heterogeneity effect on the oscillations of the piezoelementis low, so the material characteristics can be averaged by thickness.

Key words: piezoceramics, electroelasticity, functional-gradient material, non-stationary oscillations.

Introduction. At the moment, the calculation of the dynamic behavior ofpiezoelectric bodies faces the problem of matching theoretical results with practicalones. During the operation of piezoelement, possible deviations of the characteristicsof the operating mode from the predicted ones can be up to 20% [12]. Such deviationsare possible as a result of differences in the actual material characteristics from thosedeclared by the manufacturer, non-homogeneity of the material, inconsistency of theoperating conditions with the predicted ones (the influence of the environment is not taken into account, the fastening and loading conditions are not correctly implemented or programmed, the influence of the body and protective shells is nottaken into account) etc.

Currently, the oscillations of piezoelectric bodies in linear assumption are widelypresented in the literature. In particular, modern methods and approaches tocalculating of dynamic behavior of electroelastic bodies, interaction with acoustic and thermal fields, working standards for determining material characteristics are given in [6]. The main types, geometry, operating modes and features of operation of the most common piezoelements are described in [1].

During calculations there are usually accepted determined experimentally averaged values of material characteristics. At the same time, at analyzing the manufacturing and polarization process of piezoelectric bodies [1], in particular thick-walled ones, the heterogeneity of material characteristics by thickness is obvious.

In the latest publications it is noted about the wide possibilities and prospects of practical application of constructive elements, in particular sensors, actuators, energy harvesters,made of so-called functional-gradient piezoceramics [2]. Such elements, thanks to the composition of piezoactive and piezopassive materials, allow to strengthen the advantages of piezoceramic elements. This is due to the increasing interest and relevance of dynamic behavior theoretical studies of functional-gradient piezoelectrics under various variants of their layout and electro-mechanical load. In [10], the latest technology for the manufacturing of a functional gradient piezoelectric film based on a polymer and barium titanate is described. In [2, 7, 9]the separating variables method is used for free vibrations and active control of rectangular plates [7, 9] and for the functional-gradient piezoceramics pherical rotating shell [2]. In [3], a cantilever nanobeam made of flexoelectric material is considered. The gradient dependence is given in the form of an exponent. There is also introduced the flexoelectriccoefficient μ, which allows to take into account large deflections. In [5] there is investigated a rectangular three-layer plate with outer piezoelectric layers and an inner passive functional-gradient layer.

This work presents a solution of the problem of the effect of material functional heterogeneity on non-stationary thickness fluctuations of thick-walled piezoelectric elements. At this it is considered that the specified heterogeneity corresponds to the geometric features and the procedure of manufacturing the bodies considered in the work. The problem is solved using the universal approach for studying the vibrations of flat layers, cylinders and spheres,proposed in publications [4, 8]. The calculationsare carried out using explicit and implicit difference schemesfor controlling of the results accuracy.

Formulation of the problem. Thickness fluctuations of flat bodies (*N*=0), cylinders (*N*=*1*), and balls (*N*=*2*) are described by the equation of motion [4, 8]:

$$\frac{\partial \sigma_r}{\partial r}+\frac{N}{r}\left(\sigma_r-\sigma_\theta\right)=\rho\frac{\partial^2 u_z}{\partial t^2} \tag{1}$$

and Maxwell's quasi-static equation for electrical variables

$$\frac{\partial D_r}{\partial r}+\frac{N}{r}D_r=0\,, \tag{2}$$

where r is thickness coordinate.

Material relations with thickness polarization

$$\sigma_r=c_{33}^E\frac{\partial u}{\partial r}+N\frac{c_{13}^E}{r}u+e_{33}\frac{\partial \varphi}{\partial r},$$

$$\sigma_{\theta\theta}=c_{13}^E\frac{\partial u}{\partial r}+\frac{N}{r}\left(c_{11}^E-\frac{1}{2}(N-1)\left(c_{11}^E-c_{13}^E\right)\right)u+e_{13}\frac{\partial \varphi}{\partial r},$$

$$D_r=e_{33}\frac{\partial u}{\partial r}+\frac{Ne_{31}}{r}u-\varepsilon_{33}^S\frac{\partial \varphi}{\partial r}. \tag{3}$$

Further in the work, we will assume that all material characteristics are known functions of the thickness coordinate

$$c_*^E = c_*^E(r),\ e_* = e_*(r),\ \varepsilon_*^S = \varepsilon_*^S(r). \tag{4}$$

We introduce dimensionless in such a way that the original equations do not change their form:

$$\bar{r} = \frac{r}{h},\ \bar{t} = \frac{t}{t_h},\ \bar{u} = \frac{u}{h}, \bar{\sigma}_* = \frac{\sigma_*}{c_{00}}, \bar{\varphi} = \frac{\varphi}{h}\sqrt{\frac{\varepsilon_{33}^S}{c_{00}}}, \bar{D}_r = \frac{D_r}{\sqrt{c_{00}\varepsilon_{33}^S}},$$

$$\bar{c}_* = \frac{c_*^E}{c_{00}}, \bar{e}_* = \frac{e_*}{\sqrt{c_{00}\varepsilon_{33}^S}},\ \bar{\varepsilon}_{33} = 1,$$

where h is a layer thickness, $t_h = h\sqrt{\dfrac{\rho}{c_{00}}}$, $c_{00} = c_{33}^E + e_{33}^2 / \varepsilon_{33}^S$. In the future, we omit the dimensionless sign (macro) and only dimensionless variables are used.

We substitute (3) into (1), (2) taking into account the functional dependence of material constants from the coordinate r

$$\rho\frac{\partial^2 u}{\partial t^2} = c_{33}\frac{\partial^2 u}{\partial r^2} + \left(c_{33}' + N\frac{c_{33}}{r}\right)\frac{\partial u}{\partial r} + \left(N\frac{c_{13}'}{r} - N\frac{c_{13}}{r^2} + N^2\frac{c_{13}-a}{r^2}\right)u +$$
$$+e_{33}\frac{\partial^2 \varphi}{\partial r^2} + \left(e_{33}' + N\frac{e_{33}-e_{31}}{r}\right)\frac{\partial \varphi}{\partial r}, \tag{5}$$

$$e_{33}\frac{\partial^2 u}{\partial r^2} + \left(e_{33}' + N\frac{e_{13}+e_{33}}{r}\right)\frac{\partial u}{\partial r} + \left(N\frac{e_{31}'}{r} + N(N-1)\frac{e_{13}}{r^2}\right)u -$$
$$-\varepsilon_{33}\frac{\partial^2 \varphi}{\partial r^2} - \left(\varepsilon_{33}' + N\frac{\varepsilon_{33}}{r}\right)\frac{\partial \varphi}{\partial r} = 0, \tag{6}$$

where $a = c_{11} - (N-1)(c_{11} - c_{23})/2$. The dash denotes the functionderivative along the thickness coordinate.

Equations (5)(6) are supplemented with mechanical boundary conditions

$$u(R_0,t) = 0, \sigma_{rr}(R_1,t) = P_1(t) \tag{7}$$

and electrical boundary conditions

$$\varphi(R_0) = 0,\ \ \varphi(R_1) = V(t), \tag{8}$$

where $r_1 = R_0$ is a coordinate of bottom surface of a layer $r_m = R_1$ is a coordinate of top surface. Relations (7) (9) model the rigid fixation of the lower boundary surface of the piezoelement and zero electric potentialonit.

We write down the initial conditions as follows

$$u\big|_{t=0} = U_0(r),\ \left.\frac{du}{dt}\right|_{t=0} = W_0(r). \tag{9}$$

Solution method. The problem is solved using the finite difference method. To implement the method, we introduce a division by spatial coordinate

$$r_i = R_0 + (i-1)\Delta\,,\ \Delta = \frac{h}{m}\ (\,i = 1,...,m+1\,). \tag{10}$$

We write equation (5), (6) in differential form at the internal points of division (10) ($i = 2,...,m$):

$$\rho\frac{\partial^2 u}{\partial t^2} = \left(\frac{c_{33}}{\Delta^2} + \frac{1}{2\Delta}\left(c_{33}' + N\frac{c_{33}}{r_i}\right)\right)u_{i+1} + \left(N\frac{c_{13}{}'}{r_i} - N\frac{c_{13}}{r_i^2} + N^2\frac{c_{13}-a}{r_i^2} - 2\frac{c_{33}}{\Delta^2}\right)u_i +$$

$$+\left(\frac{c_{33}}{\Delta^2} - \frac{1}{2\Delta}\left(c_{33}' + \frac{N}{r_i}c_{33}\right)\right)u_{i-1} + \left(\frac{e_{33}}{\Delta^2} + \frac{1}{2\Delta}\left(e_{33}{}' + N\frac{e_{33}-e_{31}}{r_i}\right)\right)\varphi_{i+1} -$$

$$-2\frac{e_{33}}{\Delta^2}\varphi_i + \left(\frac{e_{33}}{\Delta^2} - \frac{1}{2\Delta}\left(e_{33}{}' + N\frac{e_{33}-e_{31}}{r_i}\right)\right)\varphi_{i-1},$$

$$\left(\varepsilon_{33} + \frac{\Delta}{2}\left(\varepsilon_{33}' + N\frac{\varepsilon_{33}}{r_i}\right)\right)\varphi_{i+1} - 2\varepsilon_{33}\varphi_i + \left(\varepsilon_{33} - \frac{\Delta}{2}\left(\varepsilon_{33}' + N\frac{\varepsilon_{33}}{r_i}\right)\right)\varphi_{i-1} =$$

$$= \left(e_{33} + \frac{\Delta}{2}\left(e_{33}' + N\frac{e_{31}+e_{33}}{r_i}\right)\right)u_{i+1} + \left(\Delta^2\left(N\frac{e_{31}'}{r_i} + N(N-1)\frac{e_{31}}{r_i^2}\right) - 2e_{33}\right)u_i +$$

$$+\left(e_{33} - \frac{\Delta}{2}\left(e_{33}' + N\frac{e_{31}+e_{33}}{r_i}\right)\right)u_{i-1}.$$

It is necessary to exclude contour values of displacements from this system using one-sided difference expressions of the second order of accuracy:

$$\left.\frac{du}{dr}\right|_i = \frac{3u_i - 4u_{i-1} + u_{i-2}}{2\Delta}\,,\ \left.\frac{du}{dr}\right|_i = \frac{-3u_i + 4u_{i+1} - u_{i+2}}{2\Delta}\,.$$

We also write conditions (7), (8) in difference form and find the values of unknown quantities at contour points

$$u_1 = 0\,,$$

$$u_{m+1} = -R_0\,\frac{c_{33}(R_0)(-4u_m + u_{m-1}) + e_{33}(R_0)(3\varphi_{m+1} - 4\varphi_m + \varphi_{m-1}) - 2\Delta P_1}{3R_0 c_{33}(R_0) + 2\Delta N c_{13}(R_0)}\,,$$

$$\varphi_1 = 0\,,\quad \varphi_{m+1} = V(t)\,. \tag{11}$$

After simple mathematical operations, a system of equations can be obtained and written in matrix form as follows

$$\frac{\partial^2\mathbf{u}}{\partial t^2} = \mathbf{Au} + \mathbf{B\varphi}\,,$$

$$\mathbf{M\varphi} = \mathbf{Cu}\,, \tag{12}$$

where $\mathbf{u}, \boldsymbol{\varphi}$ are the vectors of displacements and electrical potential in the internal points $i = 2,...,m$ of partitioning (10).

Integration by time. Explicit numerical scheme. We will look for the solution of the problem at times t^k with a timestep Δ_t. From the initial conditions (9) we obtain a discretized distribution of displacements at zero ($t^0 = 0$) and first points of time ($t^1 = \Delta_t$):

$$u_i^0 = U_0(r_i), \quad u_i^1 = \Delta_t W_0(r_i) + U_0(r_i).$$

The distribution of the electrical potential at $t = t^1$ is determined from the second equation of the system (12):

$$\boldsymbol{\varphi}^1 = \mathbf{M}^{-1}\mathbf{C}\,\mathbf{u}^1 .$$

At the following points of time ($t^k = k\Delta_t$, $k = 2,3,\dots$) the displacement at the internal points is found through displacement and electrical potential at previous moments of time:

$$\mathbf{u}^{k+1} = 2\mathbf{u}^k - \mathbf{u}^{k-1} + \frac{\Delta_t^2}{\rho}\left(\mathbf{A}\mathbf{u}^k + \mathbf{B}\boldsymbol{\varphi}^k\right),$$

$$\mathbf{D}\boldsymbol{\varphi}^{k+1} = \mathbf{C}\mathbf{u}^{k+1}. \tag{13}$$

For convergence of the implicit scheme, it is necessary fortimestep be much smaller than a step in spatial coordinate. Therefore, we accept $\Delta_t = 0{,}1\Delta_r$.

Implicit scheme. We use one-parameter Newmark scheme with parameter ξ ($0{,}5 < \xi < 1$):

$$\dot{u}_i^{k+1} = \frac{u_i^{k+1} - u_i^k}{\xi\Delta_t} - \frac{1-\xi}{\xi}\dot{u}_i^k, \quad \ddot{u}^{k+1} = \frac{u^{k+1} - u^k}{\xi^2\Delta_t^2} - \frac{1}{\xi^2\Delta_t}\dot{u}^k - \frac{1-\xi}{\xi}\ddot{u}^k .$$

Equations (12) are written as

$$\frac{\rho}{\xi^2\Delta_t^2}\mathbf{u}^{k+1} - \left(\mathbf{A}\mathbf{u}^{k+1} + \mathbf{B}\boldsymbol{\varphi}^{k+1}\right) = \frac{\rho}{\xi^2\Delta_t}\dot{\mathbf{u}}^k + \rho\frac{1-\xi}{\xi}\ddot{\mathbf{u}}^k + \frac{\rho}{\xi^2\Delta_t^2}\mathbf{u}^k,$$

$$\mathbf{M}\mathbf{u}^{k+1} - \mathbf{D}\boldsymbol{\varphi}^{k+1} = 0. \tag{14}$$

We introduce a vector $\mathbf{y} = (\mathbf{u}, \boldsymbol{\varphi})$ and, taking into account (14), form a system of algebraic equations like $\mathbf{C}\mathbf{y} = \mathbf{F}$. For calculations start we need displacements, speed, potential and acceleration at zero point of time:

$$u_i^0 = U_0(r_i),\ v_i^0 = W_0(r_i),\ \boldsymbol{\varphi}^0 = \mathbf{M}^{-1}\mathbf{C}\,\mathbf{u}^0,\ \mathbf{w}^0 = \left(\mathbf{A}\mathbf{u}^0 + \mathbf{B}\boldsymbol{\varphi}^0\right)/\rho .$$

The implicit scheme is absolutely stable, allowing us to take step by time and by the spatial coordinate of the same order.

Numerical results. We consider the piezoceramic flat layer, thick-walled cylinder and ball of piezoceramic PZT-4, the averaged material characteristics values of which are given in [8]. It is assumed that there is no mechanical load ($P_1 = 0$), and to the conductive coatings of the piezoelements a stepped single electric signal ($V(t) = H(t)$, $H(t)$ is the Heaviside function is applied. The geometry of piezoelements was determined by dimensionless parameters $R_0 = 1$ and $R_1 = 2$ ($h = R_1 - R_0 = 1$).

When modeling the functional heterogeneity of material characteristics (4), it was assumed that their distribution by thickness occurs according to a parabolic law with a multiplier

$$f(r) = ar^2 + br + c, \tag{15}$$

relative to the table value.

The coefficients a, b and c of the law (15) were determined on the basis of the assumption that the average value of the function (4) is equal to one

$$\frac{1}{R_1 - R_0} \int_{R_0}^{R_1} f(r)dr = 1, \tag{16}$$

on the middle surface of the layer the material characteristics are smaller by a parameter y relative to the average value

$$f\left(\frac{R_0 + R_1}{2}\right) = 1 - y, \tag{17}$$

and on the outer surfaces, they are inversely proportional to the area of electric rods:

$$R_1^N f(R_1) = R_0^N f(R_0),$$

what corresponds to the technological features of the piezoceramic manufacturing and polarization process.

Normalized relative to the tabular (averaged) material characteristics values, the thickness distribution laws (15) for flat ($N = 0$), cylindrical ($N = 1$) and spherical ($N = 2$) bodies at $y = 10\%$ are shown in Fig. 1.

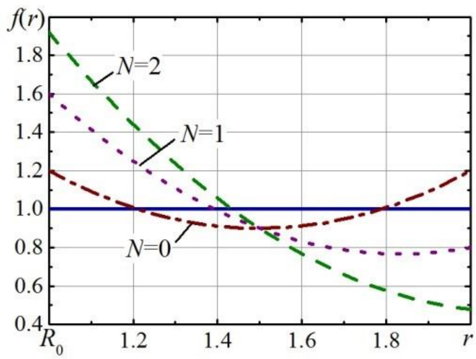

Fig. 1. Normalized distribution of material characteristicsby thickness at $y = 10\%$

Fig. 2(a) presents the displacement curves of the moving surface of the flat layer ($N = 0$), got from the system of equations (13) or (14) at different values of parameter y (17). Dashed-dashed lines in this figure represent displacements at 20%, obtained by explicit (13) and implicit (14) methods. In the implicit scheme the parameter ξ was assumed equal to 0.6. From the comparison of these lines, it follows that the difference between them does not exceed 1%. From the analysis of fig. 2(a), it can be seen that at the initial stage of oscillations, the effect of heterogeneity is not very noticeable, and with increasing time and at larger y there is a slight change in the amplitude of oscillations and a decrease in the speed of wave propagation (for 20%, this difference is approximately 2%).

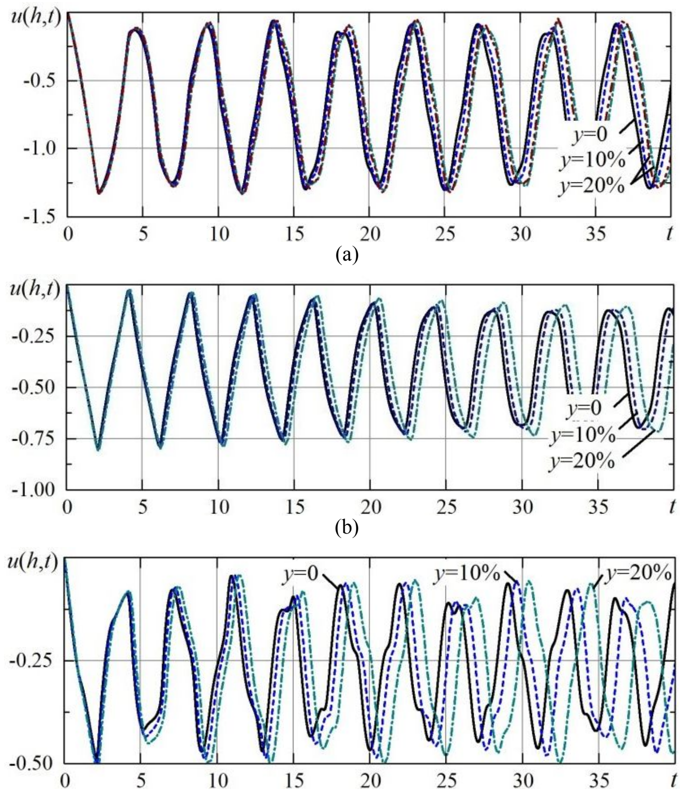

Fig. 2. Moving the external surface of the piezoelement:
(a) - flat layer, (b) – cylinder, (c) - ball

Fig. 2(b) illustrates the displacement of the moving surface of a hollow cylinder ($N=1$) with different parameters of the parabolic distribution (15) for all material characteristics. The parabolic distribution considered in this paper leads to a decrease in the speed of propagation and a slight increase in the amplitude of oscillations with an increase in the parameter y . It should be noted that a cylinder with a rather large curvature is considered here, since for cylinders with a smaller curvature ($R_1/R_0<2$) the influence of the inhomogeneity of the material characteristics along the thickness becomes less noticeable.

Similar results and conclusions were obtained for the spherical layer (Fig. 2(c)). However, in this case, the increase in the amplitude of oscillations reaches 3% at $y=20\%$.

It can be noted that with the considered version of the electromechanical load, oscillations occur in the compressed zone without entering the undeformed state ($u(h,t)<0$).

An additional numerical analysis was also conducted to assess the influence of the law of the distribution of material characteristics by thickness $f(r)$ on the dynamic behavior of piezoelements. It was found that, under the condition of equality (16), this law has little effect on the graphs of the function $u(h,t)$. However, due to the reduction of the scope of this work, these numerical results are not presented here.

Conclusions. The analysis of the numerical results shows that the heterogeneity of the material characteristics distribution by thickness can be neglectedrelative to their average values, since the deviation between the obtained results lies within acceptable limits for engineering calculations. Also, an important result is the confirmation of the assumption that for curved bodies such as hollow cylinders and spheres, the material characteristics can be considered constant in thickness, regardless of the curvature of the body.

The proposed technique can be applied forsimulating of vibrations of different geometries bodies with significantly heterogeneous functional material or combined from several materials with a gradient transition between them.

REFERENCES

1. *Sharapov V.M., Sotula Zh.V., Kunitskaya L.G.* Elektroakusticheskie preobrazovateli (Electroacoustic converters*) //* Ed. V.M. Sharapov. - Moscow: Technosphere, 2013. - 296 p.
2. *W.Q. Chen, H.J. Ding, J. Liang*. The exact elasto-electric field of a rotating piezoceramic spherical shell with a functionally graded property // International Journal of Solids and Structures, September 2001. 38(38-39):7015-7027. DOI:10.1016/S0020-7683(00)00394-2
3. *Chen, Y.; Zhang, M.; Su, Y.; Zhou, Z.* Coupling Analysis of Flexoelectric Effect on Functionally Graded Piezoelectric Cantilever Nanobeams. Micromachines 2021, 12, 595. https://doi.org/10.3390/ mi12060595
4. *Grigoryeva, L.O.* Transient responses in Piezoceramic Multilayer Actuators Taking into Account External Viscoelastic Layers // Strength of Materials and Theory of Structures. - № 105 (2020). – P. 255-266. DOI: 10.32347/2410-2547.2020.105.255-266
5. *H. Nguyen Thi*. Free Vibration and Static Bending Analysis of Piezoelectric Functionally Graded Material Plates Resting on One Area of Two-Parameter Elastic Foundation /Hindawi Mathematical Problems in Engineering. V. 2020, Article ID 9236538, 18 p. DOI:10.1155/2020/9236538
6. *S. J. Rupitsch*. Piezoelectric Sensors and Actuators: Fundamentals and Applications. Springer: 2019. 559 p. ISBN: 978-3-662-57534-5
7. *Li, J., Xue, Y., Li, F., Narita, Y.* Active Vibration Control of Functionally Graded Piezoelectric Material Plate, Composite Structures (2018). DOI: 10.1016/j.compstruct.2018.09.053.
8. *Shulga M.O., Grigoryeva L.O.* Electromechanical unstationary thickness vibrations of piezoceramic transformers at electric excitation // Mechanical Vibrations: Types, Testing and Analysis. − Nova Science Publishers, New York. − 2011. − P.179 – 204.
9. *Zhu Su, G. Jin, T. Ye.* Electro-mechanical vibration characteristics of functionally graded piezoelectric plates with general boundary conditions, International Journal of Mechanical Sciences (2018), doi: 10.1016/j.ijmecsci.2018.01.040.
10. *Zh. Wang, K. Maruyama, F. Narita.* A novel manufacturing method and structural design of functionally graded piezoelectric composites for energy-harvesting. Materials & Design 214 (2022) 110371. DOI: 10.1016/j.matdes.2021.110371.

Стаття надійшла 29.08.2022

Григор'єва Л.О., Янчевський І.В.

ВПЛИВ ФУНКЦІОНАЛЬНОЇ НЕОДНОРІДНОСТІ МАТЕРІАЛУ НА НЕСТАЦІОНАРНІ КОЛИВАННЯ П'ЄЗОКЕРАМІЧНИХ ТІЛ

Досліджується вплив функціональної неоднорідності матеріалу на механічні коливання п'єзоелемента при нестаціонарних електричних збуреннях. В межах припущення про функціональний розподіл матеріальних характеристик по товщині п'єзоелемента отримано уніфіковану систему розв'язуючих рівнянь для опису товщинних коливань п'єзоелектричних пластин, циліндрів і куль. Для контролю точності розрахунок проводиться за допомогою явно та неявно різницевої схеми.

Досліджуються нестаціонарні коливання плоского п'єзокерамічного шару, циліндра та кулі при параболічному розподілі всіх матеріальних характеристик за товщиною елемента.

Встановлено, що функціональна неоднорідність у межах одного матеріалу призводить до зміни швидкості поширення хвилі та зміни амплітуди коливань у межах 2-3%, тобто, при розрахунку дійсно можна усереднювати матеріальні характеристики по товщині. Також важливим результатом є підтвердження припущення, що для криволінійних тіл типу циліндру та кулі матеріальні характеристики можна вважати сталими за товщиною незалежно від кривизни тіла.

Запропонована методика може бути застосована для дослідження коливань тіл різної геометрії з суттєво неоднорідним функціональним матеріалом або що скомбіновані з кількох матеріалів з градієнтним переходом між ними.

Ключові слова: п'єзокераміка, електропружність, функціонально-градієнтний матеріал, нестаціонарні коливання.

Hryhorieva L.O., Yanchevsky I.V.

INFLUENCE OF MATERIALFUNCTIONAL HETEROGENEITYON NON-STATIONAR OSCILLATIONS OF PIEZOCERAMIC BODIES

The influence of material functional heterogeneity on mechanical oscillations of piezoelement under non-stationary electrical loading is investigated. Within the assumption of functional distribution of material characteristics by thickness of the piezoelectric element, which corresponds to the physical properties of the body, a unified system of solving equations was obtained to describe the thickness fluctuations of piezoelectric plates, cylinders, and balls. For controllingof accuracy, the calculation is carried out using an explicit and implicit difference scheme.

Unsteady oscillations of a flat piezoceramic layer, cylinder, and sphere are investigated with a parabolic distribution of all material characteristics along the thickness of the element. It is assumed that the average value of the function along the thickness is equal to the tabular value of the material characteristic, and the value on electrodes is proportional to the area of electrodes. At such conditions, we obtained a decrease in the speed of disturbances propagation and a slight change in the amplitude associated with the curvature of the element. The increase in amplitude reaches 3% for balls. It should be noted that at given load oscillations occur in the compressed zone without entering the undeformed state. The considered cylinder and ball have a rather large curvature, for bodies with a smaller curvature the influence of the described effect will be smaller. The additional analysis indicates that the shape of the distribution curve under described above conditions also has little effect on the results.

It was established that the effect of functional heterogeneity within the same material has little effect on the oscillations of the piezoelement, that is, it is really possible to average the material characteristics by thickness at calculating, since the deviation between the results is within acceptable limits (up to 2.5%). Also, an important result is the confirmation of the assumption that for curved bodies such as cylinder and sphere, the material characteristics can be considered constant on thickness, regardless of the curvature of the body.

The proposed technique can be applied for studyingof the vibrations of different geometries bodies with significantly heterogeneous functional material or what are combined from several materials with a gradient transition between them.

Key words: piezoceramics, electroelasticity, functional-gradient material, non-stationary oscillations.

УДК 539.3; 534.1
Григор'єва Л.О., Янчевський І.В. **Вплив функціональної неоднорідності матеріалу на нестаціонарні коливання п'єзокерамічних тіл** // Опірматеріалів і теоріяспоруд: наук.-тех. збірн. – К.: КНУБА, 2022. – Вип. 109. – С. 359-368. – Англ.
Досліджуються коливання п'єзокерамічних плоских, циліндричних та сферичних тіл з урахуванням функціональної неоднорідності матеріалу.
Іл. 2. Бібліогр. 10 назв.

UDC 539.3; 534.1
Hryhorieva L.O., Yanchevsky I.V. **Influence of material functional heterogeneityon non-stationar oscillations of piezoceramic bodies** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – K.: KNUCA, 2022. – Issue 109. – P. 359-368.
The oscillations of piezoceramic flat, cylindrical and spherical bodies are investigated taking into account the functional heterogeneity of material.
Fig. 2. Ref. 10.

Автор (вчена ступень, вчене звання, посада): кандидат фізико-математичних наук, доцент кафедри опору матеріалів ГРИГОР'ЄВА Людмила Олександрівна.
Адреса робоча: 03680,Україна, м. Київ, Повітрофлотський проспект, 31, Київський національний університет будівництва і архітектури, кафедра опору матеріалів, Григор'євій Людмилі Олександрівні.
Роб.тел.: +38(044) 241-54-21
Моб.тел.: +38(067) 304-34-32
E-mail: grygorieva.lo@knuba.edu.ua
ORCID ID: https://orcid.org/0000-0001-7013-0327

Автор (вчена ступень, вчене звання, посада): доктор фізико-математичних наук, професор, професор кафедри динаміки і міцності машин та опору матеріалів ЯНЧЕВСЬКИЙ Ігор Владиславович.
Адреса робоча: 03056, Україна, м. Київ, пр. Перемоги, 37, Національний технічний університет України «Київський політехнічний інститут імені Ігоря Сікорського», кафедра динаміки і міцності машин та опору матеріалів
Роб.тел.: +38 (044) 204-95-35
E-mail: i.yanchevskyi@kpi.ua
ORCID ID: https://orcid.org/0000-0002-7113-2276