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SPECIFIED MODELS IN THE PROBLEMS OF THE DEFORMATION OF MULTILAYER PLATES ON A RIGID FOUNDATIONS

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In order to research the stress-strain state (SSS) of multilayer thick plates on a rigid foundation, an investigation has been carried out on the use of constructed refined models of unflexural SSS. The double-thickness plate bilaterally symmetrically loaded about its mid-surface is considered. The plate is formed by symmetric supplementing it with regard to the contact surface and the foundation. Calculations confirm the efficiency and accuracy of such an approach, which allows finding solutions that are close to three-dimensional ones.

Keywords: refined model, multilayered plate, rigid foundation, transverse shear, transverse compression.

Introduction. The evaluation of strength and deformation likelihood of various kinds of homogeneous and inhomogeneous composite coatings, especially the multilayered ones, is ultimately based on assessing their stress-strain state as of the plates contacting the foundation. This also applies when calculating the multilayered road clothes on a quite rigid bridge, tunnel, and other parts of transport constructions, multilayer coatings of flat construction elements or parts, functional coatings of working surfaces of various equipment, in particular, enamel coatings of the shell of the chemical apparatus, etc. Thus, correctly determining the SSS of the multilayer plate on rigid foundations under the force of the fixed transverse loading is an urgent problem.

Combining materials with isotropic and transformation-isotropic physical characteristics in a multilayer package allows creating multifunctional structures. The SSS of such structures, given their structural inhomogeneity and relatively low transverse stiffness of the individual layers, is mostly applied under the influence of the deformation of the transverse shear and compression. Therefore, the problem of refined modeling of the SSS plates, that takes into account these types of deformations, is urgent. Requirements for the accuracy of approximate modeling depend on the purpose of the structure and many other factors. It is also necessary to assess the accuracy of the SSS obtained according to the refined model. It should be noted that the application of classical plate theory or refined transverse shear models leads to a trivial-zero solution for the SSS of transversely loaded plates on rigid foundation.

There are refined SSS models [1, 2], that take into account transverse shear and transverse compression, oriented mainly when describing the flexural SSS, whereas in the plates on a rigid basis it is more common to consider the unflexural component of the SSS. As a result, it is necessary to develop a refined model that would accurately describe the unflexural component of the SSS plate on a rigid foundation.

This research [3] suggests hypotheses and a special model of unflexural deformation of a homogeneous and layered plate, which in combination with the flexural deformation model gives results that are close to the exact three-dimensional solution. The models [4, 5] consider structural-continuous models of the SSS of transversely loaded plates, in which the idea of iterative modeling [3] is applied to plates on a rigid foundation.

The purpose of this work is to assess the accuracy and feasibility of the approaches [3-5] to the modeling of SSS plates on a rigid foundation, depending on the physical and geometric characteristics of the plates.

Materials and methods of research. The deformation of a rectangular multilayer plate, which rests on a rigid foundation in the linearly elastic statement is considered.

Layers of plate are isotropic and transversely isotropic of arbitrary but constant thickness. Instead of the actual design of the multilayer plate (Fig. 1(a)), it is suggested to consider the design diagram of the plate, which is formed by supplementing it with a symmetric one about the contact surface of the foundation. In this case, the plate will be bilaterally symmetrically loaded against the middle surface of the plate, and the thickness of the plate will double $H = 2b_n$ (Fig. 1(b)).The contact of the plate with the foundation corresponding to the conditions of sliding without friction (Fig. 1(b)). The rigid contact of the plate with the foundation is modelled by introducing an additional absolutely rigid thin interlayer of thickness h_0 (Fig. 1(c)).

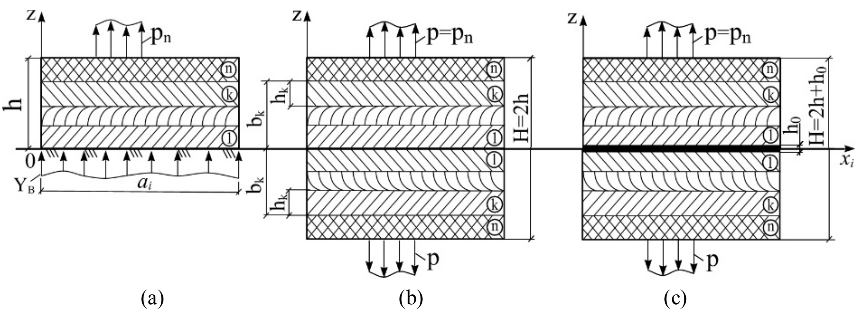

Fig. 1. Options of optimizing the design scheme of the plate:
(a) – multilayer plate on a rigid foundation; (b) – symmetrical plate with sliding contact with the foundation; (c) – symmetrical plate with absolutely rigid contact with the foundation

Conditions of rigid contact without slipping are fulfilled between layers of a plate, however, when introducing thin layers of small relative rigidity, it is possible without any modifications of the statement of a problem to consider other conditions of interlayer contact.

The approach offered allows optimizing the refined model of the SSS plate considered in [4], which consisted of two qualitatively different types of SSS the flexural and unflexural ones. The flexural component of the SSS disappears because the SSS in a symmetrical structure of the plate under bilateral symmetrical loading is completely defined by the unflexural SSS. As a result, the number of unknown functions and thus, the order of differentiation of the resolving system of equations in the problem is significantly reduced.

In a continuous model [5], the components of the vector of normal $u_3^{(k)}$ and tangential $u_i^{(k)}$ displacements to the coordinate surface (see Fig. 1) are represented by the sums of the products of hypothetically given specifiedpower functions $\psi_t^{(k)}, \psi_{ir}^{(k)}$ of the transverse coordinate z and the required functions γ_t, β_{ir} and v_i ofthe coordinate surface $x_1 O x_2$:

$$u_3^{(k)} = \psi_{3t,3}^{(k)}(z)\gamma_t + \psi_{33,3}^{(k)}(z)p\,;\, t = \overline{1,\,2}\,;\quad i = \overline{1,\,2}\,;$$

$$u_i^{(k)} = v_i - \psi_{3t}^{(k)}(z)\gamma_{t,i} - \psi_{33}^{(k)}(z)p_{,i} - \psi_{ir}^{(k)}(z)\beta_{ir}\,;\quad r = \overline{1,\,4}\,, \tag{1}$$

where two functions γ_t model the influence of transverse compression, and eight functions β_{ir} shows the influence of transverse shear in the fourth approximation for each variable x_i; p is the function of the given transverse load. Hereinafter, the differentiation with respect to x_α is designated bysubscripts after the comma, as well as the summation with respect to repeating subscriptsis also performed.

Model (1) is convenient in those problems where the load function has no breaks of the first and second kind and thus does not contradict the principle of continuity of displacements and the principle of differentiation of functions. In analytical calculation methods, model (1) is very effective [5]. However, it must be replaced for problems where the load function $p(x_i)$ has breaks in (1) by an unknown compression function. Thus, the model is also implemented in the form of:

$$u_3^{(k)} = \psi_{3t,3}^{(k)}(z)\gamma_t\,;\;\; t = \overline{1,\,3}\,;$$

$$u_i^{(k)} = v_i - \psi_{3t}^{(k)}(z)\gamma_{t,i} - \psi_{ir}^{(k)}(z)\beta_{ir}\,. \tag{2}$$

The sought-for function $\gamma_3(x_i)$ is smooth and meets the conditions of continuity as well as functions v_i, γ_1, γ_2, β_{ir}.

The functions $\psi_{3t}^{(k)}$, $\psi_{ir}^{(k)}$ modeling the distribution of displacement at coordinate z have the form [5]:

$$\psi_{3t,3}^{(k)} = \int_0^z a_{3333}^{(s)} F_t^{(s)} dz\,;\quad \psi_{33,3}^{(k)} = \int_0^z a_{3333}^{(s)} dz\,;\;\; t = \overline{1,\,2}\,;\quad r = \overline{1,\,4}\,;$$

$$\psi_{ir}^{(k)} = -\int_0^z a_{i3i3}^{(s)} f_{ir}^{(s)} dz \; ; \; \psi_l^{(k)} = \int_0^z \psi_{l,3}^{(s)} dz \; ; \; s = \overline{1, k} \; ; \quad l = \overline{1, 3} \, . \tag{3}$$

In expressions (3), the functions $F_t^{(k)}(z)$ and $f_{ir}^{(k)}(z)$ approximate the distribution the stresses of transverse compression σ_{33} and shear σ_{i3}, respectively, along the plate height. They have the following form[5]:

$$f_{i1}^{(k)} = \Theta_i^{(k)} - \Theta_i^{(n)}(b_n)\varphi_i^{(k)} ; \quad \varphi_i^{(k)} = \int_{-b_n}^{z} A_{iiii}^{(s)} dz \Big/ \int_{-b_n}^{b_n} A_{iiii}^{(p)} dz \; ;$$

$$f_{i(\omega+1)}^{(k)} = \int_{-b_n}^{z} A_{iiii}^{(s)} \int_0^z a_{i3i3}^{(s)} f_{i\omega}^{(k)} dz^2 - \varphi_i^{(k)} \int_{-b_n}^{b_n} A_{iiii}^{(p)} \int_0^z a_{i3i3}^{(s)} f_{i\omega}^{(s)} dz^2 \; ;$$

$$F_t^{(k)} = \int_{-b_n}^{z} (f_{1t}^{(s)} + f_{2t}^{(s)}) dz \; ; \quad k, \, p = \overline{1, \, n} \; ; \;\; s = \overline{1, k} \; ;$$

$$\Theta_i^{(k)} = \varphi_{3i}^{(k)} + \zeta_{0i(\chi)} \varphi_{4i}^{(k)} \; ; \quad \varphi_{3i}^{(k)}(z) = \int_{-b_n}^{z} A_{iiii}^{(s)} \varphi_0^{(s)} dz \; ; \;\; \omega = \overline{1, 3} \; ;$$

$$\varphi_{4i}^{(k)}(z) = \int_{-b_n}^{z} A_{iiii}^{(s)} \int_{-b_n}^{z} dz^2 \; ; \quad \varphi_0^{(k)}(z) = \int_{-b_n}^{z} \int_{-b_n}^{z} a_{3333}^{(s)} dz^2 \, , \tag{4}$$

where $f^{(n)}(b_n)$ is the value of the functionat $z = b_n$; $A_{\alpha\beta\gamma\delta}$ and $a_{\alpha\beta\gamma\delta}$ are the coefficients of Hooke's law $\sigma_{\alpha\beta} = A_{\alpha\beta\gamma\delta} e_{\gamma\delta}$ and $e_{\alpha\beta} = a_{\alpha\beta\gamma\delta} \sigma_{\gamma\delta}$; for transverse isotropy and symmetry layers, $\Theta_1^{(k)} = \Theta_2^{(k)}$, $f_{1r}^{(k)} = f_{2r}^{(k)}$ and $\zeta_{01(\chi)} = \zeta_{02(\chi)} = 0$.

Using the Lagrange variational principle and the method described in [3, 5], for model (2) we come to the system of differential equations in generalized forces

$$N_{ij},_j = 0 \; (\delta u_i) \; ; \;\; M_{ij,\,ij}^{[3t]} - N_3^{[3t]} = 0 \;\; (\delta\gamma_t) \; ; \;\; i, j = 1, 2 \; ; t = \overline{1, 3} \; ;$$

$$M_{1i,\,i}^{[1r]} - Q_1^{[1r]} + M_{2i,\,i}^{[2r]} - Q_2^{[2r]} = 0 \;\; (\delta\beta_{ir}) \; ; \;\; r = \overline{1, 4} \, , \tag{5}$$

and boundary conditions at the plate and face $x_m = 0$ and $x_m = a_m$

$$\left(N_{im} - N_{im}^*\right) \;\; \delta u_m = 0 \; ; \;\; \left(M_{mm,\,m}^{[3t]} + M_{mj,\,j}^{[3t]} - M_{ml,\,l}^{[3t]*} - Q_m^{[3t]*}\right) \;\; \delta\gamma_t = 0 \; ;$$

$$\left(M_{mm}^{[3t]} - M_{mm}^{[3t]*}\right) \;\; \delta\gamma_t,_m = 0 \; ; \; \left(M_{mm}^{[mr]} + M_{ml}^{[lr]} - M_{mm}^{[mr]*} - M_{ml}^{[lr]*}\right) \delta\,\beta_{ir} = 0. \tag{6}$$

Here, the assigned forces operating at the end faces of the plate are designated by the asterisk.

In equations (5) and (6), the following generalized forces are assumed:

$$\begin{bmatrix} N_{ij}, \, M_{ij}^{[3t]}, \, M_{ij}^{[ir]} \\ Q_i^{[jr]}, \, N_3^{[3t]} \end{bmatrix} = \int_0^h \begin{bmatrix} \sigma_{ij}^{(k)}, \;\; \sigma_{ij}^{(k)} \psi_{3t}^{(k)}, \, \sigma_{ij}^{(k)} \psi_{ir}^{(k)} \\ \sigma_{i3}^{(k)} \psi_{jr}^{(k)},_3 , \;\; \sigma_{33}^{(k)} \psi_{3t}^{(k)},_{33} \end{bmatrix} dz \, . \tag{7}$$

The resolving system of equations in displacement functions is derived from system (5) by substituting expressions of forces (7), with the use of kinematic hypotheses (2), Cauchy relations, and Hooke`s law, and can be presents as

$$L_i^{(c)}(u_i) + L_{ir}^{(c)}(\beta_{ir}) + L_{3t}^{(c)}(\gamma_t) = 0 \; ; \;\; c = \overline{1, 7} \, , \tag{8}$$

where $L_{\alpha r}^{(c)}$ are differential operators described in [5].

Numerical results and their analysis. In order to substantiate the scope of usage of the models offered, depending on the plate size, the following is the problem of plane deformation of the homogeneous isotropic plate ($\nu = 0{,}3$) on a rigid foundation under the influence of a sinusoidal load $p = p_0 \sin(\pi x_1 / a_1)$. The plate was calculated using four variants of the proposed models, which are marked (Table 1 and Fig. 2, 3), respectively, $M_1(1,1)$ is an optimized model with a given load function (1) which contained one unknown function of transverse compression C_1, and one transverse shift function S_1; $M_1(2,2)$ – model (1) with C_2, S_2; $M_2(2,2)$ – the optimized model (2) with no apparent function of load C_2, S_2. $M_3(3,3)$ is a general model [4] for the scheme on (Fig. 1(a)) with C_3 and S_3.

Table 1

Calculation of plane deformation of an isotropic plate on a rigid foundation

	T	$M_3(3,3)$	$M_1(1,1)$	$M_1(2,2)$	$M_2(2,2)$
	$a_1 = 2h$				
$-10u_3^{\#}$ (Δ,%)	8,35	8,66 (3,66)	8,74 (4,58)	8,59 (2,86)	8,34 (0,1)
$-10\sigma_{11}^{\#}$ (Δ,%)	5,72	7,05 (23,2)	6,75 (17,4)	6,65 (16,3)	6,20 (8,3)
$-10\sigma_{22}^{\#}$ (Δ,%)	4,72	6,32 (33,9)	5,02 (6,25)	5,85 (24,0)	4,86 (2,9)
$-10u_1^{\#}$ (Δ,%)	3,40	3,39 (0,19)	3,34 (1,7)	3,42 (0,6)	3,43 (0,7)
	$a_1 = 3h$				
$-10u_3^{\#}$ (Δ,%)	8,90	9,15 (2,76)	9,13 (2,58)	9,06 (1,73)	8,90 (0)
$-10\sigma_{11}^{\#}$ (Δ,%)	3,13	3,70 (18,1)	3,65 (16,7)	3,68 (17,5)	3,37 (7,8)
$-10\sigma_{22}^{\#}$ (Δ,%)	3,94	4,58 (16,3)	4,10 (3,95)	4,58 (16,1)	4,01 (1,8)
$-10u_1^{\#}$ (Δ,%)	4,79	4,84 (1,0)	4,82 (0,7)	4,83 (0,9)	4,80 (0,2)
	$a_1 = 5h$				
$-10u_3^{\#}$ (Δ,%)	9,07	9,13 (0,7)		9,14 (0,8)	9,07 (0)
$-10\sigma_{11}^{\#}$ (Δ,%)	1,25	1,49 (19)		1,48 (18)	1,34 (7,4)
$-10\sigma_{22}^{\#}$ (Δ,%)	3,37	3,64 (7,9)		3,63 (3,8)	3,40 (1,0)
$-10u_1^{\#}$ (Δ,%)	7,03	7,07 (0,5)		7,08 (0,6)	7,04 (0,1)

Table 1 offers the values of maximum relative displacements $u_\alpha^{\#} = u_\alpha^{\max} E / p_0 h$, and maximum relative stresses $\sigma_{ii}^{\#} = \sigma_{ii}^{\max} / p_0$. The brackets show the errors in comparison with three-dimensional solutions (T) for the problem of flat deformation and for a square plate, which are here and hereafter obtained by the method [6]. The distribution of errors is also shown in the graphs (Fig. 2).

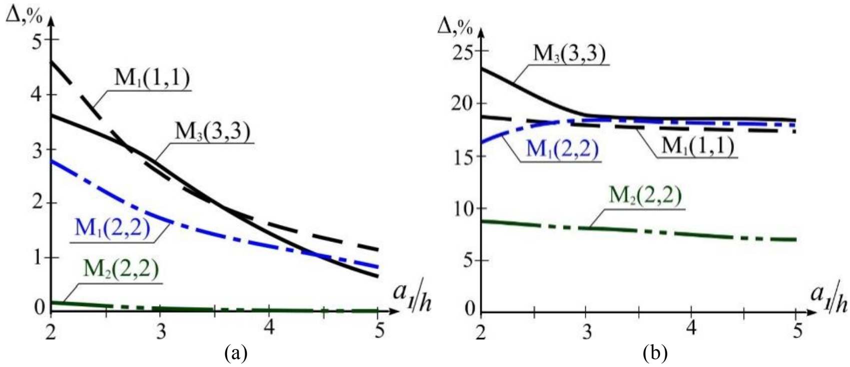

Fig. 2. Distribution of relative errors in a homogeneous isotropic plate:
(a) normal deflections $u_3^{\#}$; (b) maximum stresses $\sigma_{11}^{\#}$

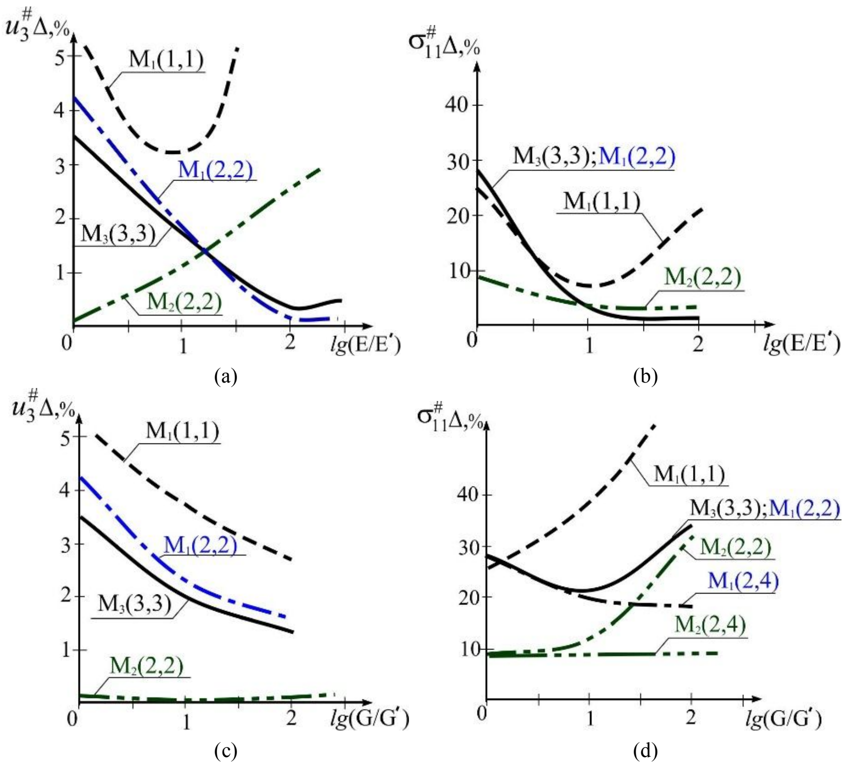

Fig. 3. Distribution of relative errors in a homogeneous transversely isotropic plate:
(a), (c) for normal deflections $u_3^{\#}$; (b), (d) for maximum stresses $\sigma_{11}^{\#}$

As can be seen from the results of the calculation for the isotropic plate (Table 1, Fig. 2), the errors in the calculations decrease along with the decreasing relative thickness. It should be noted that for the optimized plate scheme, the plate thickness in the calculation scheme is doubled. For maximum stresses, the convergence of the models is slightly worse than for displacements (Fig. 2). The best results for an isotropic plate were obtained according to the optimized model (2) with an unknown load function $M_2(2,2)$ which allows calculating very thick plates which are almost arrays.

To substantiate the scope of the proposed models, depending on the physical parameters, the influence of the ratios of the elastic characteristics of the plate on the accuracy of the models is investigated (Fig. 3).

Solutions for a square plate with Navier boundary conditions under the action of a sinusoidal load at slip without friction contact of plate with a rigid foundation were analyzed (Fig. 3, Table 2). Plate material transversely isotropic with $\nu = 0{,}3$; $\nu'' = 0{,}1$; $\nu''/E = \nu'/E'$; $a = 3h$ ($a = 1{,}5H$). Modules of elasticity and shear in the isotropy plane are E, G, and in the perpendicular direction – E', G'.

Table 2

Comparison of maximum deflections $u_3^{\#}$ and maximum stresses $\sigma_{11}^{\#}$ in a square plate with a three-dimensionalsolution

Model	$B1 = 0$; $B2 = 0$		$B1 = 1$; $B2 = 0$		$B1 = 2$; $B2 = 0$		$B1 = 0$; $B2 = 2$	
	$u_3^{\#}$	$\sigma_{11}^{\#}$	$u_3^{\#}$	$\sigma_{11}^{\#}$	$u_3^{\#}$	$\sigma_{11}^{\#}$	$u_3^{\#}$	$\sigma_{11}^{\#}$
$M_1(1,1)$ (Δ, %)	0,906 (6,95)	0,629 (27,2)	6,406 (3,12)	2,245 (6,5)	35,97 (15)	8,40 (22,4)	0,933 (2,73)	0,390 (90)
$M_1(2,2)$ (Δ, %)	0,883 (4,2)	0,631 (27,5)	6,327 (1,85)	2,190 (3,9)	31,43 (0,74)	6,898 (0,52)	0,922 (1,57)	0,289 (41,2)
$M_2(2,2)$ (Δ, %)	0,847 (0,08)	0,540 (9,1)	6,279 (1,08)	2,183 (3,55)	30,40 (2,5)	7,069 (3,0)	0,907 (0,07)	0,270 (32)
$M_3(3,3)$ (Δ, %)	0,877 (3,5)	0,635 (28,4)	6,320 (1,74)	2,189 (3,89)	31,37 (0,54)	6,885 (0,34)	0,920 (1,3)	0,274 (34)
T	0,848	0,495	6,212	2,108	31,98	6,862	0,908	0,205

Table 2 introduces the denomination $\lg(E/E') = B1$, $\lg(G/G') = B2$ and in the transverse-isotropy the relation is true: $\nu''/E = \nu'/E'$ with $\nu'' = 0{,}1$. Are accepted $\nu = 0{,}3$; $\nu' = 0{,}3$ for $B1 = 0$; and $\nu = 0{,}3$; $\nu' = 0{,}01$ for $B1 = 1$; and $\nu = 0{,}3$; $\nu' = 0{,}001$ for $B1 = 2$.

Approximate solutions were compared with three-dimensional solutions (*T*). It is shown that in especially thick square plates, for example with $a/H = 1{,}25$ (see Fig. 2), when G/G' increases, the number of transverse shear functions must also increase: with $G/G' < 100$ two shear functions $S_{ri} = 2$ are required in each of the orthogonal directions x_i; and if $100 \le G/G' \le 500$ – $S_{ri} = 3 \div 4$. As ratio E/E' increases, the number of functions of transverse compression must also grow C_t: if $E/E' \le 10$ using one function $C_t = 1$ is enough, and if $10 \le E/E' \le 1000$ two functions of compression are required. $C_t = 2$. The plate with the ratio $a/H = 1{,}5$ requires the same number of unknown functions, whereas the plate with the ratio $a/H = 2{,}5$ it is possible to reduce the number of unknown functions. Note that the errors of stresses σ_{11} are greater than for displacements u_α (see Fig. 3).

The results obtained by the M_2 model are more accurate than by the M_1 and M_3 models, at with the same number of unknown functions in the models.

Conclusions. As can be seen above from the results of the test problems calculations, the constructed mathematical model allows us to obtain results that are qualitatively and quantitatively close to three-dimensional solutions. The model can be used to calculate the SSS of significantly thick plates $(a/H = 1{,}5)$, with a wide range of changes to the parameters of the relative transtropy in the layer ($1 \le E/E' \le 500$, $1 \le G/G' \le 500$) and significant differences in the stiffness of the individual layers ($E^{(k)}/E^{(k+1)} = 10^3 \div 10^5$).

It should be noted that the results of calculations with the usage of an optimized approach to the formation of the design scheme of the plate that employs the models of the unflexural SSS (M_1, M_2) and the general model M_3 describing both flexural and unflexural SSS in a given plate are quite close. However, the optimized approach with models M_1, M_2 allows to obtain reliable results with fewer required functions and with less general order of the differentiation of the calculated system of equations.

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УТОЧНЕНІ МОДЕЛІ В ЗАДАЧАХ ДЕФОРМУВАННЯ БАГАТОШАРОВИХ ПЛИТ НА ЖОРСТКІЙ ОСНОВІ

Аналіз та оцінка напружено-деформованого стану (НДС) багатошарових плит на жорстких основах при дії стаціонарного поперечного навантаження є актуальною задачею, оскільки до неї зводяться розрахунки міцності та деформативності різного роду однорідних та багатошарових покриттів. Це розрахунок дорожнього одягу на відносно жорстких мостових конструкціях, або на недеформівномупідстилаючому шарі, захисних багатошарових покриттів плоских елементів конструкцій більшої жорсткості, ніж нокриття, деталей тощо. Об’єднання матеріалів з ізотропними та трансверсально-ізотропними фізичними характеристиками в багатошаровий пакет дозволяє створювати багатофункціональні конструкції. НДС таких конструкцій, зважаючи на їх структурну неоднорідність та відносно низьку поперечну жорсткість окремих шарів, суттєво пов’язаний з впливом деформацій поперечного зсуву та деформацій поперечного обтиснення. Тому актуальною є задача уточненого моделювання НДС плит, яка б враховувала ці види деформацій. Ґрунтуючись на розкладанні НДС плити на згинові та беззгинові складові, пропонується оптимізація розрахункової схеми деформування прямокутної багатошарової плити на жорсткій основі. Суть оптимізації полягає в розгляді такої розрахункової схеми плити, в якій НДС плити повністю описувався б лише одною складовою, а саме беззгиновою складовою НДС. Для цього замість реальної конструкції багатошарової плити, що деформується без відриву від основи, пропонується розглядати розрахункову схему плити, яка утворена симетричною добудовою відносно поверхні контакту даної плити з основою. У цьому випадку плита буде двосторонньо симетрично навантаженою відносно серединної поверхні плити, а товщина плити збільшиться вдвоє. НДС плити буде беззгиновим, що суттєво спрощує його моделювання. Для беззгинового НДС побудовані в пружній постановці двовимірні, високого ступеня ітераційного наближення, але тривимірні за характером відображення НДС моделі деформування багатошарових прямокутних плит на жорсткій основі з ізотропними та трансверсально-ізотропними шарами, які достатньо повно враховують деформації поперечного зсуву та поперечного обтиснення при поперечному навантаженні плити. Розв’язанням тестових задач деформування ізотропних та трансверсально-ізотропних плит на жорсткій основі з ковзким та жорстким контактом з основою, та порівнянням розв’язків з отриманими за відомими методиками точними тривимірними розв’язками цих задач, дано оцінку точності запропонованих уточнених моделей. Встановлено межі допустимих параметрів пружних характеристик трансверсально-ізотропних плит для застосування запропонованих моделей.

Ключові слова: уточнена модель, плита багатошарова, жорстка основа, поперечний зсув, поперечне обтиснення.

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SPECIFIED MODELS IN THE PROBLEMS OF THE DEFORMATION OF MULTILAYER PLATES ON A RIGID FOUNDATIONS

The analysis and estimation of the stress-strain state (SSS) of multilayered plates on a rigid foundation under the action of stationary transverse loading is an urgent task. As its includes the calculations of strength and deformability of various homogeneous and multilayer coatings. This is the calculation of pavement on relatively rigid bridge structures, or on a non-deformable underlying layer, protective multilayer coatings of flat elements of structures of greater rigidity than roofing, parts and more. Combining materials with isotropic and transversely isotropic physical characteristics into a multilayer package allows you to create multifunctional structures. The SSS of such structures, due to their structural heterogeneity and relatively low transverse stiffness of the individual layers, is significantly associated with the influence of transverse shear deformations and transverse compression deformations. Therefore, the problem of refined modeling of SSS plates, which would take into account these types of deformations, is urgent. Based on the decomposition of the SSS plate into flexural and unflexuralcomponents, it is proposed to optimize the design scheme of deformation of a rectangular multilayer plate on a rigid foundation. The essence of optimization is to consider such a calculation scheme of the plate, in which the SSS of the plate would be fully described by only one component, namely the unflexural component of SSS. To do this, instead of the actual design of the multilayer plate, which is deformed without separation from the foundation, it is proposed to consider the design scheme of the plate, which is formed by symmetrical completion relative to the contact surface of the plate with the foundation. In this case, the plate will be bilaterally symmetrically loaded relative to the middle surface of the plate, and the thickness of the plate will double. SSS plate will be unflexural, which greatly simplifies its modeling. For unflexural SSS, a two-dimensional, high-degree iterative approximation, but three-dimensional models of deformation of multilayer rectangular plate on a rigid foundation with isotropic and transverse-isotropic layers are constructed in an elastic formulation.That models takes full account deformationof transverseshearand transverse compression at transverse loading of a plate. By solving the test problems of deformation of isotropic and transversely isotropic plates on a rigid foundation with sliding and rigid contact with the foundation, and comparing the solutions with the exact three-dimensional solutions of these problems obtained by known methods, the accuracy of the proposed refined models is estimated.The limits of admissible parameters of elastic characteristics of transversely isotropic plates for application of the offered models are established.

Keywords: refined model, multilayered plate, rigid foundation, transverse shear, transverse compression.

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Для дослідження напружено-деформованого стану (НДС) багатошарових товстих плит на жорсткій основі досліджено застосування побудованих уточнених моделей беззгинового НДС. Розрахунками підтверджено ефективність і точність методики моделювання, яка дозволяє отримати розв'язки близькі до тривимірних.
Табл. 2. Іл. 3. Бібліогр. 6 назв.

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Gurtovyi O.G., Tynchuk S.O., Ugrin L.S. **Specified models in the problems of the deformation of multilayer plates on a rigid foundations**// Strength of Materials and Theory of Structures: Scientific-&-Technical collectedarticles – Kyiv: KNUBA, 2022. – Issue 109. – P. 331-341.
For research of stress-strain state (SSS) of multilayered thick plates on a rigid foundation are investigated the use of constructed refined models of unflexural SSS. Calculations confirm the efficiency and accuracy of such approach, which allows one to obtain solutions close to three-dimensional ones.
Tabl. 2. Fig. 3. Ref. 6.

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Для исследования напряженно деформированного состояния (НДС) многослойных толстых плит на жестком основании исследовано применение построенных уточненных моделей безызгибного НДС. Расчетами подтверждена эффективность и точность методики моделирования, позволяющая получить решения близкие к трехмерным.
Табл. 2. Ил. 3. Библиогр. 6 назв.

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