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GENERATION OF ENERGY IN CONSOLE PIEZOELECTRIC ENERGY HARVESTERS

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In this work, the oscillations of the cantilever unimorph energy harvester under harmonic loads are investigated. Unimorf console consisting of a brass base and a rectangular piezoelectric element with electroded flat surfaces without and with tip mass is considered.

There is derived the characteristic equation for beam bending oscillations, wave numbers, circular frequencies and natural frequencies are determined. Eigenforms of oscillations are constructed, the dependence of natural frequencies from body size and tip mass is analyzed.

Forced oscillations of the energy harvesters with tip mass at the end at oscillations of the base are studied. The voltage generated on the piezo element plates is determined taking into account the electrical resistance. Due to the voltage and resistance of the conduct line the power of the energy harvester is determined.

Keywords: cantilever energy harvester, passive layer, piezoceramic overlay, characteristic equation, amplitude function, forced oscillations, energy generation, energy harvesting, harvester power.

Introduction. Harvesting of mechanical oscillations energy and its conversion into electrical for the purpose of accumulation for further use (energy harvesting) has already occupied an important place both in mechanical engineering (damping of oscillations with conversion of excess energy into electricity), and in construction and environment as autonomous systems for monitoring and state controlling of the object [2].

Piezo-based devices are one of the most common types of energy harvesters [9]. Under harmonic oscillations, piezoelectric elements produce alternating electric current, showing the greatest efficiency at resonant frequencies. Piezoceramic elements working on bending give a much higher yield of the potential difference compared to the axial load, because in this case we have much greater displacement. One of the most common are cantilever unimorph or bimorph energy converters, consisting of a passive layer and one or two thin symmetrically placed piezoceramic plates [10]. Tip mass at the end is often used to adjust the operating frequency of the element with external one. One of the applications of cantilever piezoelectric elements is the conversion of unnecessary or undesirable oscillations of the structure into electrical energy and its subsequent use for autonomous operation of the monitoring device or accumulation in accumulators or batteries [6]. The operation of the element at resonant frequencies is the most effective, so an important characteristic of the piezoelectric element is the width of the range of operating (natural) frequencies [11].

There are commercially available cantilever bimorphs with the following dimensions: thickness 0.3 - 0.35 mm, length 4 - 100 mm, width 1.6 - 22 mm. Electrodes made of silver (6-10 μm thick) or nickel (1-3 μm thick) are applied to the piezoceramic plate. After applying the electrodes, the piezoelectric element is polarized in a strong constant electric field [10]. The main rod can be made of bronze, brass, stainless steel, nickel foil, graphite, composites, etc. For products with high sensitivity, a piezoelectric element is also used as the main rod. The passive layer increases the mechanical strength, but reduces the amount of displacement. The use of a stainless steel base provides 25% greater strength of the element and is used in cases with high blocking force, such as implanted pacemakers. Epoxy or acrylate glue is used for gluing layers, which provides a strong connection. The thickness of the adhesive layer is 10-15 μm.

An unimorph or bimorph operating in generator mode is often used as a flexible sensor [8]. The generator-type sensor does not require an external power supply to operate. It is designed to convert dynamic deformations into electrical signals with further processing and recording by various de-vices, including energy collection. Unimorph can be used as a stand-alone converter of mechanical energy into electric current, and be a part of a more complex device. It can be connected to the control and management system in two main ways: by the voltage registration circuit or by the charge registration circuit.

This work is devoted to the study of oscillations of cantilever energy harvester at monoharmonic loads. Earlier in [5] the resonant oscillations of piezoceramic cylinders with energy dissipation were studied. Multilayer piezoceramic elements are considered in [7]. Fundamental theory of vibrations is described in [4]. The most up-to-date overview of piezoelectric energy harvesting is found in [9]. Works describing the use of cantilever energy harvesters in bridge structures [1], in pavements [12], in sound energy harvesting [3] should be noted.

Formulation of the problem. For collaboration work of element with the structure, the natural frequencies of the element must be coordinated with the operating frequency of the structure. This is done by varying the mass and its position at the end of the rod. For the most efficient operation of the converter, a piezoelectric material with a high coefficient of electromechanical conversion is used. The console consists of a metal rod (steel or brass) of rectangular cross-section with relatively low rigidity in the direction of oscillation (fig. 1).

Piezoelectric rectangular thickness-polarized element is attached as pad to the part of the beam that undergoes maximum deformation. At the end, an additional mass is attached in the form of a steel cylinder, which reduces the operating frequency of the element.

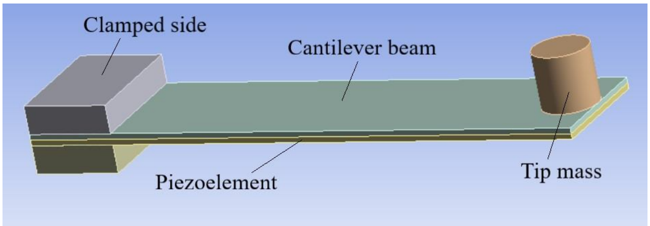

Fig. 1. Cantilever piezoelectric harvester

The calculation is performed in several stages: determination of natural frequencies for different design options; analysis of oscillation forms to determine operating frequencies; studying of harmonic oscillations of the console at operating frequencies; determination of the potential difference generated at the electrodes of the piezoelectric element; determining the power of the energy harvester.

Natural oscillations of the cantilever beam. We consider the transverse oscillations of the rod, which stiffness in vertical direction is much lower than in horizontal. This allows to most effectively use the influence of gravity and cause significant deflections in the rod. The section width is several times less than the length to provide torsional rigidity, so as torsional modes are undesirable.

a) Cantilever beam loaded with its own weight. Given: length l, density ρ, Young's modulus *E*, cross-sectional area *A*, moment of inertia *I*. Differential equation of transverse oscillations of the rod

$$a^2 \frac{d^4 w}{dx^4} + \frac{d^2 w}{dt^2} = 0\,, \tag{1}$$

where $a^2 = \dfrac{EI}{A\rho}$. We use the procedure of separating variables

$$w(x,t) = X(x) \cdot T(t). \tag{2}$$

Substitute (2) into (1) and obtain two differential equations with corresponding solutions:

$$X^{IV} = \frac{\omega^2}{a^2} X = k^4 X, \tag{3}$$

$$X = C_1 \sin kx + C_2 \cos kx + C_3 sh\, kx + C_4 ch\, kx, \tag{4}$$

$$T'' + \omega^2 T = 0\,,\; T = A \sin \omega t + B \cos \omega t\,, \tag{5}$$

where ω is the natural circular frequency of the body oscillations, *k* is the wave number. The coefficients in (4) and (5) are determined from the boundary and initial conditions.

For clamping in $x = 0$ we have BC

$$X(0) = 0\,, \quad X'(0) = \varphi(0) = 0. \tag{6}$$

At the free end

$$M(l) = X''(l) = 0,\; Q(l) = X'''(l) = 0. \tag{7}$$

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General solution

$$X = C_1(\sin kx - sh\,kx - \frac{\sin kl + sh\,kl}{\cos kl + ch\,kl}(\cos kx - ch\,kx)). \quad (8)$$

Characteristic equation can be got as determinant of algebraic system (6), (7):

$$1 + \cos r\; ch\, r = 0\,. \quad (9)$$

Roots r_i of equation (9) [4] ($k > 2$) don't depend from beam size:

$$r_i = \{1{,}8751;\; 4{,}6941;\; \frac{2k-1}{2}\pi ...\}. \quad (10)$$

Wave numbers can be got as $k_i = r_i / l$, natural circular frequencies are $\omega_i = k_i^2 a$. Corresponding oscillation frequencies $f_i = \frac{\omega_i}{2\pi}$. The amplitude function X_i (8) is constructed for each k_i. The general solution of equation (1) has the form

$$w(x,t) = \sum_{i=1}^{\infty} X_i(x)(A_i \sin \omega_i t + B_i \cos \omega_i t). \quad (11)$$

b) The natural oscillations of the mass at the end of a cantilever rod. If the ratio of the mass of the beam to the attached mass is small, the mass of the beam can be neglected. The oscillations of the mass at the end of the cantilever rod are described by a differential equation

$$m\frac{d^2 w}{dt^2} = -kw\,. \quad (12)$$

Here $k = \frac{3EI}{l^3}$ is the stiffness factor. Natural frequency

$$f = \frac{1}{2\pi}\sqrt{\frac{3EI}{ml^3}}\,. \quad (13)$$

c) Natural oscillations of the cantilever beam with mass at the end, taking into account the mass of the beam. The deflection for all values of x, except the points of application of the load, satisfies equation (3) with the solution in the form (4) [4]. At attaching the mass at the end of the beam we have a boundary condition

$$Q(l) = EI\frac{d^3 w}{dx^3} = m\frac{d^2 w}{dt^2}$$

or

$$\frac{d^3 X}{dx^3} + \frac{m}{EI}\omega^2 X = 0$$

or

$$\frac{d^3 X}{dx^3} = -\alpha\beta^2 X \;\text{ at }\; x = l\,. \quad (14)$$

Here $\alpha = \dfrac{m}{\rho A l}$; $\beta^2 = r^4 l = \dfrac{\omega^2}{a^2} l = \dfrac{\omega^2 A \rho l}{EI}$.

We supplement (6), (7) with condition (14), write the determinant and obtain the characteristic equation $(r = kl)$:

$$1 + \cos r ch\, r - \alpha r(\sin r ch\, r - sh\, r \cos r) = 0. \tag{15}$$

Roots of equation (15) at $\alpha = 0{,}14$:

$$r_i = \{1{,}67;\ 4{,}33;\ 7{,}38;\ 10{,}46;\ 13{,}56;\ 16{,}67;\ 19{,}78;\ 22{,}91...\}.$$

Fig. 2 shows the dependence of the roots of equation (15) from the coefficient α. At $\alpha = 0$ ($m = 0$) (15) is reduced to (9). With increasing α, the values of the roots change little and at $\alpha = 5$ we have $r_i = \{0{,}8807;\ 3{,}9512;\ 7{,}0833\}$. A larger mass ratio is physically improbable.

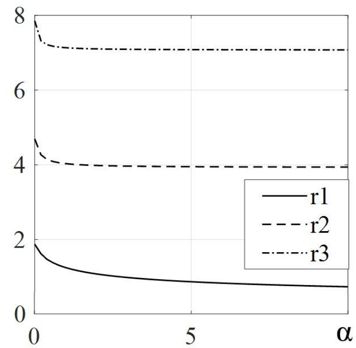

Fig. 2. Roots of equation (15) from the coefficient α

2. Forced oscillations of the converter during oscillations of the base. Consider the oscillations of the base with amplitude *A* and frequency *ω*. The general solution of the problem is $w(x,t) = X(x)\cos\omega t$, where

$$X(x) = C_1 \sin kx + C_2 \cos kx + C_3 sh\, kx + C_4 ch\, kx .$$

For function *X* we have boundary conditions:

$$X(0) = A,\quad X'(0) = 0;\ X''(l) = 0;\ X''' + \alpha\beta^2 X = 0 . \tag{16}$$

The coefficients C_i in general solution are got from (16) and are equal to:

$$C_1 = A/2[\, ch\, kl[\, ch\, kl \sin kl + 2\alpha kl ch\, kl \cos kl + sh\, kl \cos kl]/$$
$$/(1 + \cos kl ch\, kl - \alpha_1 kl(\sin kl ch\, kl - sh\, kl \cos kl));$$

$$C_2 = \frac{-C_1(\sin kl + sh\, kl) + A\, ch\, kl}{\cos kl + ch\, kl};\ C_3 = -C_1;\ C_4 = A - C_2 . \tag{17}$$

3. Characteristics of the unimorph. We consider a two-layer cantilever beam of length *l*, consisting of a metal layer with a Young's modulus E_1, density ρ_1 and cross section $b_1 \times h_1$ and a piezoceramic layer with a Young's modulus E_2, density ρ_2 and cross section $b_2 \times h_2$. We will average the material characteristics:

$$E^* = \frac{E_1 A_1 + E_2 A_2}{A_1 + A_2},\ \rho^* = \frac{\rho_1 A_1 + \rho_2 A_2}{A_1 + A_2}. \tag{18}$$

The coordinate of the neutral layer is $z_{0C} = \dfrac{E_1 A_1 z_{01} + E_2 A_2 z_{02}}{E_1 A_1 + E_2 A_2}$.

Reduced areas $A_1^* = \dfrac{E_1 A_1}{E^*}$, $b_1^* = \dfrac{A_1^*}{h_1}$, $A_2^* = \dfrac{E_2 A_2}{E^*}$, $b_2^* = \dfrac{A_2^*}{h_1}$.

Moments of inertia $I_{y1}^* = \dfrac{b_1^* h_1^3}{12}$, $I_{z1}^* = \dfrac{b_1^{*3} h_1}{12}$, $I_{y2}^* = \dfrac{b_2^* h_2^3}{12}$, $I_{z2}^* = \dfrac{b_2^{*3} h_2}{12}$,

$$I_y^* = I_{y1}^* + z_{1C}^2 A_1^* + I_{y2}^* + z_{2C}^2 A_2^*,\ I_z^* = I_{z1}^* + I_{z2}^*. \tag{19}$$

Equations (18), (19) are a set of characteristics required for the application of the above formulas.

4. Determination of electromotive force and power of a harvester. Electrical boundary conditions are applied to the electrodes located on the upper and lower surfaces of the piezoceramic element. For thin plates, we believe [5] that the electric potential inside the body varies linearly:

$$E_z = -\frac{d\varphi}{dz} = -\frac{V(t)}{h}. \tag{20}$$

Here $V(t)$ is the required electromotive force of the transducer (potential difference at the electrodes). We can use the hypothesis of flat sections and assume that the deformations in the cross-sectional plane are small. Linear deformation in the direction of the axis of the rod is $\varepsilon_x = -z\dfrac{\partial^2 w}{\partial x^2}$. We have

$$\sigma_x = -c_{11} z \frac{\partial^2 w}{\partial x^2} - e_{13}\frac{V}{h_2};\quad D_z = -e_{31} z \frac{\partial^2 w}{\partial x^2} - \varepsilon_{33}\frac{V}{h_2}. \tag{21}$$

Total charge on the lower electrode

$$Q = \int_A D_z dA = -e_{31} b_2 z_{\max} \left.\frac{dw}{dx}\right|_{x=l} - \varepsilon_{33}\frac{Vlb_2}{h_2}. \tag{22}$$

Generated current can be written through Ohm’s law

$$I = \omega Q = \frac{V}{R}. \tag{23}$$

We substitute (22) into (23) and find the generated potential difference:

$$V = \frac{e_{31} h_2\, z_{\max} X'(l)}{\varepsilon_{33} l + h_2/(b_2 R\omega)}. \tag{24}$$

Unimorph power as an electric current generator

$$P = IV = \frac{V^2}{R}. \tag{25}$$

For an AC electrical circuit the time average output power

$$\overline{P} = \frac{1}{\tau}\int_0^\tau V(t) I(t) dt = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} VI \sin^2(\omega t) dt = \frac{1}{2\pi} VI \int_0^{2\pi} \sin^2(\tau) d\tau = \frac{VI}{2} = \frac{V^2}{2R}.$$

Now we can calculate stresses and electrical induction by (21), using second derivative from w:

$$\frac{d^2w}{dx^2} = X'' = k^2\left(-C_1 \sin kx - C_2 \cos kx + C_3 sh\, kx + C_4 ch\, kx\right).$$

5. Analysis of the results. A brass cantilever rod with a Young's modulus $E = 95\,\text{GPa}$ and density $\rho = 8730\,\text{kg}/\text{m}^3$ is considered. The piezoceramic plate is made of PZT5A piezoceramics, for which $E = 52\,\text{GPa}$, $\rho = 7800\,\text{kg}/\text{m}^3$, $e_{13} = -5{,}2$, $\varepsilon_{33} = 663\,\varepsilon_0$.

All analytical formulas were duplicated using finite-difference approximations of the second order of accuracy. The deviation between the results was 1% at $n = 200$ breakpoints along the length of the rod.

Figure 3 shows the dependence of the resonant frequencies of the rod with a cross section of the brass base $10 \times 1\,\text{mm}$ and the piezoelectric element $10 \times 0{,}3\,\text{mm}$ with attached mass $m = 6{,}2\,\text{g}$, which corresponds to the mass of the steel cylinder in size $10 \times 10\,\text{mm}$, from the length of the rod. We see that the first resonance is in the range from 2,5 kHz at $L = 4\,\text{mm}$ to 57 Hz at $L = 60\,\text{mm}$. With increasing length coefficient α varies from 13,9 to 0,88. At $L < 15\,\text{mm}$ the second and third resonances lie in the ultrasonic range. Increasing the length leads to decreasing in the resonant frequency along the curve close to the quadratic hyperbola, what corresponds to the physical laws.

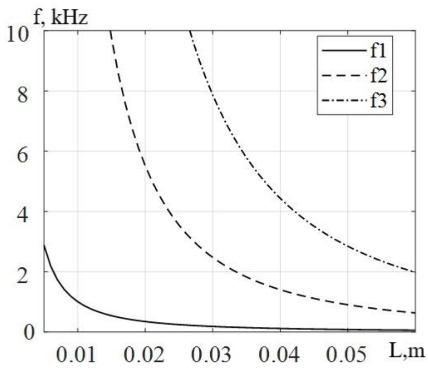

Fig. 3. Natural frequencies of the rod with the attached mass from the length

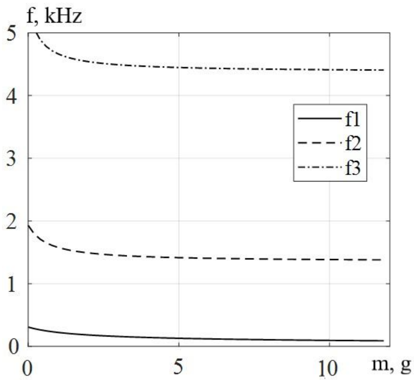

Fig. 4. Natural frequencies of a rod of constant lengthfrom the tip mass

Figure 4 illustrates the dependence of the resonant frequencies of the rod with a cross section $10 \times 1\,\text{mm}$ of the brass base and the piezoelectric element with section $10 \times 0{,}3\,\text{mm}$ and length $L = 40\,\text{mm}$ from the value of the attached mass. In this case coefficient α varies from 0 to 2,66. The first resonance varies from 307 Hz at $m = 0$ to 89 Hz at $m = 12\,\text{g}$. The second and third resonances lie in the sound range. Therefore, the presence of the attached mass can reduce the first resonant frequency on 70%, the second and third on 30%.

Let us analyze the forms of oscillations for the above-described element when the base oscillates with an amplitude A = 1 mm. Graphs of amplitude functions are shown in Fig. 5. Relevant natural frequencies

$$f_i = \{153;\ 1812;\ 5724;\ 11860;\ 20227\}\ \text{Hz}\,.$$

The number of extremum points corresponds to the mode of oscillations + 1. Frequencies after the third are ultrasonic. Considering that the oscillations of building structures and mechanisms are mostly low-frequency, we conclude that harvesting of energy from the construction is possible only on the first resonance.

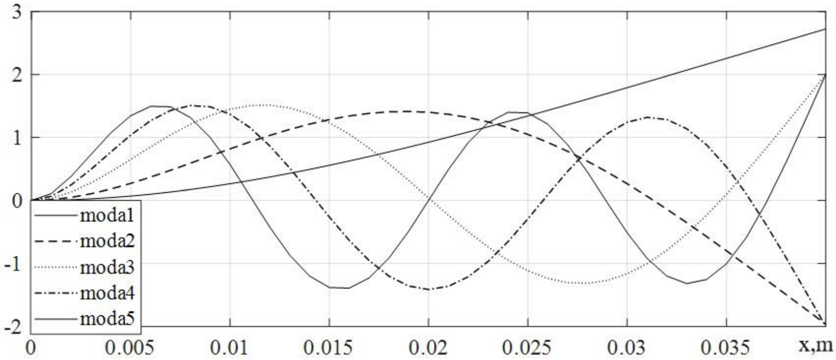

Fig. 5. Forms of oscillations for the first five modes

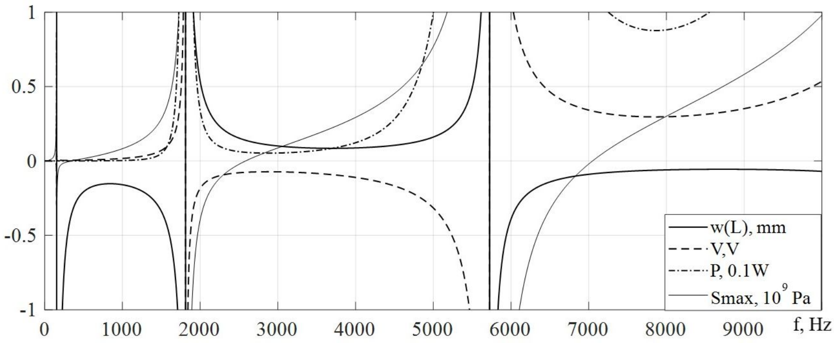

Fig. 6. Deflection, potential difference, power and maximum stress in the element depending on the perturbation frequency

Let us analyze the forced oscillations of the described above element at electrical resistance $R = 1\ \text{Ohm}$. Fig. 6 shows the deflection curves of the rod end, corresponding generated potential difference on electrodes and the power of the element. At $f = 0$ the deflections are equal to 1 mm, which corresponds to the perturbation of the base. From 0 to 300 Hz we have an increase in displacement with changing of vibrations phase, which corresponds to the first resonance. From 500 to 1500 Hz displacement are near 0,15 mm. Maximum stresses in a piezoelement occur on the bottom fibers near fixed end.

Let's analyze the dependence of the generated voltage and power of the element from the electrical resistance R, which varies from 0 to 10 Ohms at $f = 160$ Hz (Fig. 7). It is almost linear, what allows us to say that at higher resistance we get higher generated voltage and power of the element. But at the same time losses increase in a circle, and a considerable part of energy turns to thermal. Therefore, the optimal parameters for the electrical circuit require special research.

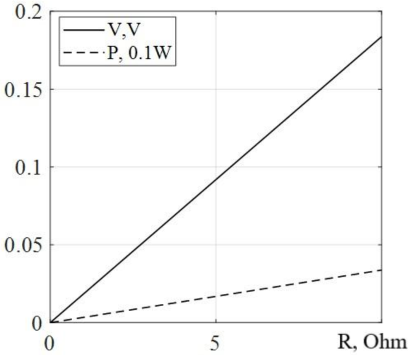

Fig. 7. Generated potential difference and power depending from external resistance

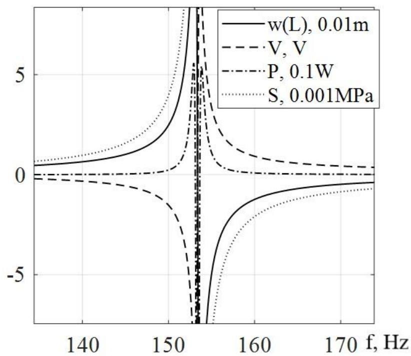

Fig. 8. Deflection, potential difference, power and stress near the first resonance

We can analyze real electromechanical state of the element at electrical resistance $R = 50$ Ohms near the first resonance with Fig. 8. To get real parameters of harvester on resonance frequency we take into account dissipation of energy in the body through losses tangents. Consider all material characteristic complex with small imaginary part: $c_{ij} = c_{ij}(1 + c''i)$ and so on, where we for example will take $c'' = \tan\alpha = 1\% = 0.01$. In nonrezonance range the potential difference and deflections are not zero, but the power is small. At $f = 115$ Hz we have $\overline{P} = 0,5$ mW and $\sigma_{\max} = 22$ MPa, and they increase near the resonance.

Conclusion. The proposed approach makes it possible to calculate cantileverunimorphic energy harvesters, for which $h/b < 0,2$, $b/L < 0,2$, since such dimensions can be assumed to fulfill the hypothesis of flat sections, the linearity of the potential difference in the thickness of the element and use the long eamsbending relations. The generated voltage is proportional to the angle of rotation of the beamend and the most remoted fiber from neutral axis. In the denominator we have two terms, one of which is responsible for the electrical conductivity of the element and is proportional to length, other is inversely proportional to the resistance and frequency. At low resistance and frequency, the second term is much larger than the first, and the voltage value is not high. If resistance increases, the voltage and power of the element increase proportionally to *R*. At $f = 115\ \text{Hz}$ we have power of the energy harvester $\overline{P} = 0{,}5\ \text{mW}$, and in the vicinity of resonances it increases. Between the first and second resonance the power is approximately 3,7mW. At transition to the ultrasonic zone, the power of the energy collector increases significantly. So cantilever harvester is resonant device and works at defined frequensies. Detailed analysis of oscillations in the resonant mode should include damping of oscillations in the material.

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ГЕНЕРАЦІЯ ЕНЕРГІЇ В КОНСОЛЬНОМУ ЗБІРНИКУ ЕНЕРГІЇ

Збір енергії механічних коливань та перетворення їх в електричну енергію за допомогою пристроїв, що працюють наоснові п'єзоефекту, набув широкого поширення. Це стало можливим завдяки створенню п'єзоелектричних матеріалів з високим коефіцієнтом

перетворення енергії та поширенню мініатюрних пристроїв, для живлення яких достатньо потужності в кілька міліват.

В цій роботі досліджуються коливання стержневого консольного уніморфного збірника енергії при гармонічних навантаженнях. Розглядається двошаровий стержень, що складається з латунної основи та прямокутного п’єзоелектричного елемента з електродованими плоскими поверхнями без маси та з приєднаною масою. Товщина шарів значно менша за ширину, а ширина значно менша за довжину, що дає змогу використовувати гіпотезу плоских перерізів та припущення про лінійність розподілу різниці потенціалів по товщині елементу, а також використовувати співвідношення згину довгих стержнів.

В роботі виводиться характеристичне рівняння для стержня при згині, визначаються хвильові числа, кругові частоти та власні частоти для консолі. Проводиться усереднення матеріальних характеристик по площі перерізу. Будуються власні форми коливань, проводиться аналіз залежності власних частот від розмірів тіла та приєднаної маси.

Досліджуються вимушені коливання збірника енергії з масою на кінці при заданих коливаннях бази. Формується рівняння пружної лінії консолі, визначаються максимальні прогини та кути повороту. Визначається згенерована на обкладках п’єзоелемента напруга з врахуванням опору зовнішнього кола. Через напругу та опір провідної лінії знаходиться потужність збірника енергії. Будуються криві залежності напруги та потужності від частоти навантаження та зовнішнього опору. Встановлено, що напруга і потужність елемента змінюються пропорційно до *R*. Максимальна потужність збірника енергії виникає в околі резонансів, а до першого резонансу потужність практично нульова. Між першим і другим резонансом потужність складає приблизно 1,5 mW. При переході в ультразвукову зону потужність збірника енергії значно зростає. Аналіз роботи перетворювача на резонансних частотах вимагає врахування демпфування коливань в матеріалі.

Ключові слова: консольний збірник енергії, пасивний шар, п’єзокерамічна накладка, вимушені коливання, ґенерація енергії, збір енергії, різниця потенціалів на обкладках, потужність збірника енергії.

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GENERATION OF ENERGY IN CONSOLE PIEZOELECTRIC ENERGY HARVESTERS

Energy harvesting of mechanical vibrations and their conversion into electrical energy using piezoelectric devices has become widespread. This has been made possible by the creation of high-energy piezoelectric materials and the proliferation of miniature devices with a few milliwatts of power.

In this work, the oscillations of the rod cantilever bimorph energy harvester under harmonic loads are investigated. A two-layer rod consisting of a brass base and a rectangular piezoelectric element with electroded flat surfaces without and with tip mass is considered. The thickness of the layers is much less than the width and the width is much less than the length, which allows us to use the hypothesis of flat sections and assumptions of the potential difference linearity by thickness of the element, as well as beams bending relations.

There is derived the characteristic equation for beam bending oscillations, the wave numbers, circular frequencies and natural frequencies are determined. There is carried out The averaging of material characteristics over the cross-sectional area. Eigenforms of oscillations are constructed, the dependence of natural frequencies from body size and tip mass is analyzed.

The next step is to study the forced oscillations of the energy harvesters with tip mass at the end at given oscillations of the base. The equation of the elastic line of the console is formed, the maximum deflections and angles of rotation are determined. The voltage generated on the piezo element plates is determined taking into account the electrical resistance. Due to the voltage and resistance of the conduct line the power of the energy harvester is determined. Curves of voltage and power dependence from load frequency and external resistance are constructed. It is established that the voltage and power of the element change in proportion to R. The maximum power of the energy collector occurs in the vicinity of resonances, and before the first resonance the power is almost zero. Between the first and second resonance, the power is approximately 1,5 mW. During the transition to the ultrasonic zone, the power of the energy collector increases significantly.Analysis of the harvester operation at resonant frequencies requires consideration of the damping of oscillations in the material.

Keywords: cantilever energy harvester, passive layer, piezoceramic overlay, characteristic equation, amplitude function, forced oscillations, energy generation, energy harvesting, potential difference on the plates, power of the energy collector.

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Григор'єваЛ.О., ІваненкоП.О., КорбаковО.Ф. **Ґенерація енергії в консольному збірнику енергії** // Опірматеріалів і теоріяспоруд: наук.-тех. збірн. – К.: КНУБА, 2022. – Вип.108. – С. 337-348. – Англ.
Досліджуються коливання консольних п'єзокерамічних збірників енергії. Визначаються їх робочі частоти, зґенерована напруга та потужність.
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The oscillations of cantilever piezoceramic energy collectors are investigated. Their operating frequencies, generated voltage and power are determined.
Fig. 8. Ref. 12.

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Исследуются колебания консольных пьезокерамических сборников энергии. Определяются их рабочие частоты, сгенерированное напряжение и мощность.
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