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## MECHANICAL STRENGTH INCREASING OF ABRASIVE REINFORCED WHEEL

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The mathematical model of the stress-strain state of the abrasive reinforced wheel was developed in this paper, taking into account the anisotropy of its properties. Anisotropy can be reduced by displacing one reinforcement mesh relatively to the other by angle of  $45^\circ$ . The mechanical strength of unreinforced abrasive wheels is determined by centrifugal and bending forces. To determine the centrifugal forces, the theory of elasticity for an orthotropic body is applied. The bending forces that arise in the working wheel were determined during solving the problem of the distribution of deformations in the anisotropic annular plate rigidly fixed along the inner contour. As a result of experimental studies, it was found that stresses reach 8...23 MPa, which can be compared with the ultimate strength of the wheel matrix. The elastic module of the wheel matrix is noticeably greater than the elastic module of the reinforcing mesh, which practically does not perceive the load at the initial stage.

**Keywords:** abrasive reinforced wheel, strength, reinforcing mesh, centrifugal and bending forces, deformation.

### Introduction

Cutting and cleaning abrasive reinforced wheels are widely used not only in construction, but also in mechanical engineering, instrument making, and other sectors of the national economy associated with metal processing. The annual production of wheels is in the hundreds of millions of pieces. So only OJSC «Luga Abrasive Plant» produces more than 300 million pieces per year [1, 2]. Abrasive reinforced wheels operate in combination with hand-held and portable machines [3] with a working speed of 80 m/s and are classified as high-risk tools [4].

### Analysis of publications

Nowadays, a fairly large number of works of study on the processes of metal grinding have been carried out [5, 6]. There are practically no works aimed at increasing the mechanical strength of abrasive reinforced wheels,

which are widely used in cutting and cleaning operations. At the same time, the results of experimental studies on determining the stresses in the abrasive wheel arising during operation are presented [7]. The issues related to the influence of the acting forces on the abrasive wheel, as well as the influence of reinforcement on the strength indicators of the wheel, are considered.

### Purpose of the paper

The purpose of this paper is to develop a mathematical model for calculating the stress-strain state of abrasive reinforced wheels. That will allow us to determine the stresses in the abrasive wheel, taking into account the reinforcing mesh during performing cutting and cleaning operations.

### Research results

The worker's safety is determined by the strength of an abrasive reinforced wheel, which during operation is in a complex stress-strain state under the influence of centrifugal, bending, tangential and normal forces.

During rotation in the abrasive wheel, centrifugal accelerations  $a = (3...20) \cdot 10^4 \text{ m/s}^2$  arise, which lead to the appearance of tensile stresses on the inner contour of the wheel, comparable in magnitude with the ultimate strength of the wheel material.

Bending forces constantly act on the grinding wheels, and can also appear in cutting wheels when they are skewed or pinched [8, 9]. For cleaning wheel, bending forces can be represented by the concentrated force  $F_{bend}$  (Fig. 1) applied to the cutting edge of the wheel 1, perpendicular to its plane and equal in magnitude to the force  $P$ , with which the worker presses the wheel to the cutting surface 2, multiplied by the sinus of the angle of inclination of the wheel  $\alpha$ . In this case, the stresses caused by bending forces can reach values on the inner contour comparable to the ultimate strength of the wheel material.

The analysis showed that in the area of the clamping flange, the stresses caused by tangential and normal forces are much less than the ultimate strength of its material, therefore, we do not take them into account.

The mechanical strength of the wheels during rotation is determined by centrifugal forces. In this regard, for an approximate analysis of the stress state and comparison of experimental data, the theory of elasticity under the assumption that the abrasive wheel is an isotropic body was used.

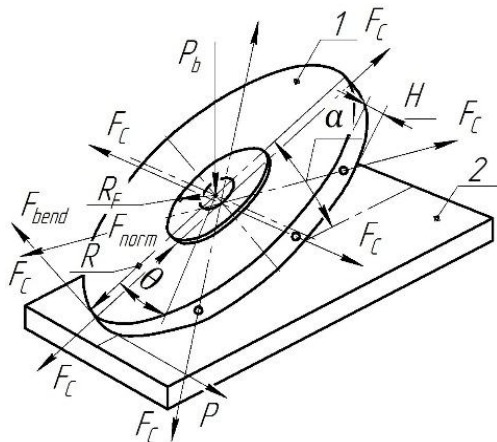


Fig. 1. Forces acting on the cleaning wheel

The equilibrium equation for an elastic wheel, which is characterized by constant thickness and rotates with angular velocity  $\omega$ , has the form [10, 11]:

$$\frac{d}{dr} \cdot \left[ \frac{1}{r} \cdot \frac{d(UR)}{dR} \right] = -\frac{1-\nu^2}{E} \cdot \rho \omega^2 R, \quad (1)$$

where  $U$  – radial displacement of point located at distance  $R$  from the center of the wheel, m;  $\nu$  – Poisson's ratio;  $E$  – module of elasticity for the wheel material, Pa;  $\rho$  – material density, kg/cm<sup>3</sup>.

With the integration of equation (1) two times and the notation  $\tilde{a}_0 = \frac{1}{8} \cdot \rho \omega^2$ , we get:

$$U = A_1 \frac{1-\nu}{E} \cdot R + A_2 \frac{1+\nu}{E} \cdot \frac{1}{R} - \tilde{a}_0 \frac{1-\nu^2}{E} \cdot R^3, \quad (2)$$

where  $A_1$ ,  $A_2$  – integration constants, which are determined from the boundary conditions.

The stresses in the radial  $\sigma_r$  and circumferential  $\sigma_\theta$  directions corresponding to displacements of equation (2), according to [11], have the form:

$$\sigma_r = A_1 - A_2 \frac{1}{R} - \tilde{a}_0 (3+\nu) R^2, \quad (3)$$

$$\sigma_\theta = A_1 + A_2 \frac{1}{R} - \tilde{a}_0 (1+3\nu) R^2.$$

As boundary conditions on the outer and inner contours of the abrasive wheel it is accepted that:

$$\left. \begin{aligned} \sigma_r \Big|_{R=R_0} &= 0, \\ U_r \Big|_{R=R_F} &= 0. \end{aligned} \right\} \quad (4)$$

The boundary conditions take into account the rigid clamping of the wheel along the contour of the clamping flange and the absence of stresses on the cutting edge.

Integration constants  $A_1$  and  $A_2$  are determined from the following system of equations:

$$A_1 \cdot \frac{1-\nu}{1+\nu} \cdot R_F^2 + A_2 = \tilde{a}_0 (1-\nu) R_F^4, \quad (5)$$

$$A_1 \cdot R_0^2 - A_2 = \tilde{a}_0 (3+\nu) R_0^4.$$

After transformation we get:

$$A_1 = \tilde{a}_0 \frac{(1-\nu) \cdot R_F^4 + (3+\nu) \cdot R_0^4}{((1-\nu)/(1+\nu)) \cdot R_F^2 + R_0^2}, \quad (6)$$

$$A_2 = \tilde{a}_0 (1-\nu) R_F^2 \cdot R_0^2 \frac{R_F^2 - ((3+\nu)/(1+\nu)) \cdot R_0^2}{((1-\nu)/(1+\nu)) \cdot R_F^2 + R_0^2}.$$

Now, using equation (3), we can calculate the stresses in the wheel. The inner contour ( $R = R_F$ ) which is the most likely place of destruction represents the greatest interest.

It should be noted that under the influence of overloads, the material of the matrix of the wheel is destroyed, and the stress-strain state in the wheel undergoes significant changes. At the same time, the integrity of the wheel under the action of centrifugal forces can be maintained only by a reinforcing mesh capable of withstanding only radial loads.

It is proposed to install reinforcing mesh (Fig. 2) of the specified configuration [12] which will improve technical characteristics of the wheel.

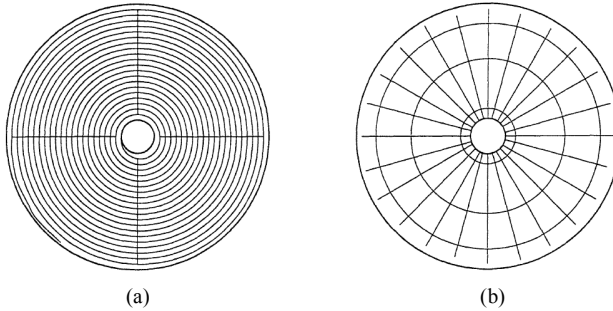


Fig. 2. Wheels reinforcement schemes: (a) tangential and (b) radial

Let's take  $x$  as the relative volumetric content of reinforcing fibers in the body of the wheel. Then, considering the equilibrium of the wheel element and assuming that  $\sigma_\theta = 0$ , and the radial stress is perceived only by oriented fibers, the proportion of which is  $q_1$  from all fiber ( $q_1 \approx 1/2$ ), we have:

$$\frac{d\sigma_r}{dR} + \frac{\sigma_r}{R} = -\frac{1}{q_1 x} \cdot \rho \omega^2 R. \quad (7)$$

Expressing stress through displacement, we get:

$$\frac{d^2 U}{dR^2} + \frac{1}{R} \cdot \frac{dU}{dR} = -\frac{1}{E_{mesh}} \cdot \frac{1}{q_1 x} \cdot \rho \omega^2 R. \quad (8)$$

where  $E_{mesh}$  – elasticity module of the reinforcing mesh material.

The general integral of equation (8) has the form:

$$U = C_1 \ln R + C_2 - \frac{\omega^2 \rho}{3E_{mesh} q_1 x} \cdot R^3, \quad (9)$$

$$\sigma_r = E_{mesh} C_1 \cdot \frac{1}{R} - \frac{\omega^2 \rho}{3q_1 x} R^2, \quad (10)$$

considering that  $\sigma_r|_{R=R_0} = 0$ , we determine

$$C_1 = \frac{\omega^2 \rho R_0^3}{3E_{mesh} q_1 x} \quad (11)$$

and after substitution equation (11) into equation (10), we find an expression for the stresses experienced by the reinforcing fibers on the inner contour

$$\sigma_r = \frac{\omega^2 \rho (R_0^3 - R_F^3)}{3q_1 x R_F}. \quad (12)$$

Under the condition of acceleration (deceleration) of the wheel, the circumferential acceleration  $a = \frac{dV}{dt} = R \frac{d\omega}{dt}$  causes stresses  $\sigma_r$  in sections  $R = const$ , which can be determined from the equilibrium condition of the wheel part located between the radii  $R$  and  $R_0$ :

$$\sigma_r 2\pi R_F^2 \tilde{H} = \rho \tilde{H} 2\pi \int_{R_F}^{R_0} \frac{d\omega}{dt} R^3 dR; \quad \sigma_r = \frac{\rho}{4} \cdot \frac{d\omega}{dt} \cdot \frac{R_0^4 - R_F^4}{R_F^2}. \quad (13)$$

After testing the samples of abrasive wheels for tension, the obtained experimental and calculated values of stresses in the circumferential and radial directions were compared. It was found that the calculated stresses are more than two times less than the experimental ones, that means they can be used only for a qualitative assessment.

For further analysis, the deformations arising under the action of centrifugal forces in wheels with different reinforcement schemes were determined on a special stand. As a result, it was confirmed that in the directions of the fibers of the reinforcing mesh, the value of deformations is 1,3...1,6 times less. At the same time samples in which the direction of tensile forces coincides with the direction of the fibers of the reinforcing mesh or make angle of  $45^\circ$  (Fig. 3) with them have the maximum and minimum strength. It has been established that, depending on the design, the ultimate strength of the abrasive reinforced wheels is 8...23 MPa.

After studying the stress-strain state, the abrasive wheels on the stand were brought to destruction, which occurs mainly along the radii. As a result, it was found that the abrasive reinforced wheel is an anisotropic body, while orthotropy of its mechanical properties is observed. Anisotropy can be reduced if the

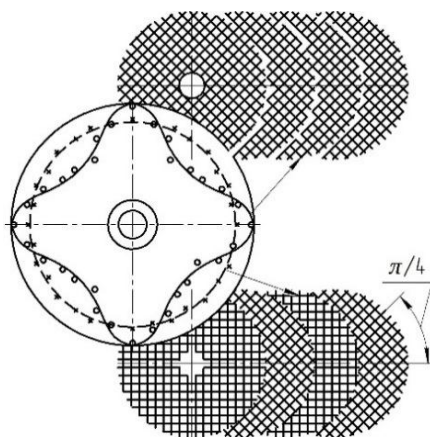


Fig. 3. Diagram of the deformations distribution in the wheel with different orientations of the reinforcing meshes

reinforcing mesh is positioned in such a way that the fibers of one of them are displaced at angle of  $45^\circ$  relative to the fibers of the other. In this case, the wheel has the most equal strength, and its reliability increases. Changing the orientation of the inner meshes in the abrasive reinforced wheels (Fig. 3) allows an increase in tensile strength by an average of 20%, so they can be considered as orthotropic bodies.

The change in the elastic modules of the material of the wheel as an anisotropic body is described by the dependence:

$$E(\theta) = \frac{E_1 E_2}{E_1 \cos^4 \theta + ((4E_1 E_2)/E_{12} - E_1 - E_2) \sin^2 \theta \cos^2 \theta + E_2 \sin^4 \theta}, \quad (14)$$

where  $E_1$ ,  $E_2$ ,  $E_{12}$  – tensile modules in directions making angles  $\pi/2$ ,  $\pi$ ,  $\pi/4$  with reinforcing mesh fibers.

For the orthotropic wheel we obtain the stress distribution from the action of centrifugal forces using the stress function:

$$\begin{aligned} & \frac{1}{E_\theta} \frac{\partial^4 F_C}{\partial R^4} + \left( \frac{1}{G_{r\theta}} - \frac{2\nu_r}{E_r} \right) \frac{1}{R^2} \frac{\partial^4 F_C}{\partial R^2 \partial \theta^2} + \frac{1}{E_r} \frac{1}{R^4} \frac{\partial^4 F_C}{\partial \theta^4} + \\ & + \frac{2}{E_\theta} \frac{1}{R} \frac{\partial^3 F_C}{\partial R^3} - \left( \frac{1}{G_{r\theta}} - \frac{2\nu_r}{E_r} \right) \frac{1}{R^3} \frac{\partial^3 F_C}{\partial r \partial \theta^2} - \frac{1}{E_r} \frac{1}{R^2} \frac{\partial^2 F_C}{\partial R^2} + \\ & + \left( 2 \frac{1-\nu_r}{E_r} + \frac{1}{G_{r\theta}} \right) \frac{1}{R^4} \frac{\partial^2 F_C}{\partial \theta^2} + \frac{1}{E_r} \frac{1}{R^3} \frac{\partial F_C}{\partial R} = \\ & = - \left[ \frac{1-\nu_\theta}{E_\theta} \frac{\partial^2}{\partial R^2} + \frac{1-\nu_r}{E_r} \frac{1}{R^2} \frac{\partial^2 J}{\partial \theta^2} + \left( \frac{2}{E_\theta} - \frac{1-\nu_r}{E_r} \right) \frac{1}{R} \frac{\partial J}{\partial R} \right], \quad (15) \end{aligned}$$

where  $E_r$ ,  $E_\theta$  – tensile modules in main directions  $R$  and  $\theta$ , Pa;  $J = -\rho\omega^2 \cdot R^2 / 2$  – moment of inertia;  $G_{r\theta}$  – shear module for main directions, Pa.

Considering that the load from centrifugal forces is symmetrical, the stress function will take the form:

$$F_C = A_0 + B_0 R^2 + CR^{1+K} + DR^{1-K} + \frac{\rho\omega^2}{2} (3-K-2\nu_\theta) R^2, \quad (16)$$

where  $K = \sqrt{E_\theta/E_r}$ ;  $\nu_r = \nu_\theta = \nu$  – Poisson's ratio.

Using relations from the theory of elasticity [10]:

$$\begin{aligned} \sigma_r^c &= \frac{1}{R} \frac{\partial F_C}{\partial R} + \frac{1}{R^2} \frac{\partial^2 F_C}{\partial \theta^2} + J; \\ \sigma_\theta^c &= \frac{\partial^2 F_C}{\partial R^2} + J; \\ U_r &= R \left( \frac{\sigma_\theta^c}{E_\theta} - \frac{\nu_r}{E_r} \sigma_r^c \right), \quad (17) \end{aligned}$$

we define the stress components and the projection of the displacement:

$$\sigma_r^c = C(1+K)R^{K-1} + D(1-K)R^{-K-1} - \rho\omega^2 \frac{3+v_0}{9-K^2} R^2, \quad (18)$$

$$\sigma_\theta^c = C(1+K)KR^{K-1} + D(1-K)KR^{-K-1} - \rho\omega^2 \frac{K^2+3v_0}{9-K^2} R^2, \quad (19)$$

$$U_r^c = \frac{C}{E_0}(1+K)(K-v_0)R^K + \frac{D}{E_0}(1-K)R^{-K} - \frac{\rho\omega^2}{E_0} \frac{K^2-v_0^2}{9-K^2}. \quad (20)$$

Constants  $C$  and  $D$  are determined from the boundary conditions:

$$U_r^c \Big|_{R=R_F} = 0; \quad \sigma_r^c \Big|_{R=R_0} = 0, \quad (21)$$

$$C = \frac{\begin{vmatrix} \rho\omega^2 \frac{K^2-v_0^2}{9-K^2} R_F^3 & (K-1)(K-v_0)R_F^{-k} \\ \rho\omega^2 \frac{3+v_0^2}{9-K^2} R_0^2 & (1-K)R_0^{-K-1} \end{vmatrix}}{(1-K^2) \left[ R_F^K R_0^{-K-1} (K-v_0) + R_F^K R_0^{K-1} (K+v_0) \right]}, \quad (22)$$

$$D = \frac{\begin{vmatrix} (1-K)(K-v_0)R_F^k & \rho\omega^2 \frac{K^2-v_0^2}{9-K^2} R_F^3 \\ (1+K)R_0^{K-1} & \rho\omega^2 \frac{3+v_0^2}{9-K^2} R_0^2 \end{vmatrix}}{(1-K^2) \left[ R_F^K R_0^{-K-1} (K-v_0) + R_F^{-K} R_0^{K-1} (K+v_0) \right]}.$$

Dependences (18) and (19), taking into account equation (22), describe the stress state of a rotating abrasive reinforced wheel.

In the process of experimental verification of theoretical calculations physical and mechanical characteristics of wheels (Table 1) with different reinforcement schemes were determined.

Table 1

Wheel type	Tensile module			Poisson's ratio $\nu$	Tensile strength $\sigma$ , MPa	Breaking angular velocity $\omega$ , $s^{-1}$
	$E_1$ , MPa	$E_2$ , MPa	$E_{12}$ , MPa			
41–400×4×32 (one mesh) for cutting mountain rocks	4394	4725	3855	0,2	7,4	602
41–300×3×32 (two meshes) for cutting the metal	9550	11040	9124	0,2	18,5	760
27–230×6×22,23 (four meshes) for cleaning the metal	10440	10870	9540	0,2	23,0	1060

The total stresses arising in brittle bodies are calculated using the equation  $\sigma_{total}^c = \sigma_r^U - \nu\sigma_\theta^c$ . Based on this, we write down the condition for maintaining the integrity of the wheel under the action of centrifugal forces:

$$\sigma_{total}^c = \sigma_r^c - \nu\sigma_\theta^c \leq k_{zp}\sigma_0. \quad (23)$$

The dependence of total stresses  $\sigma_{total}^c$  on the angular velocity  $\omega$  recorded during the destruction of the wheels is presented in Fig. 4.

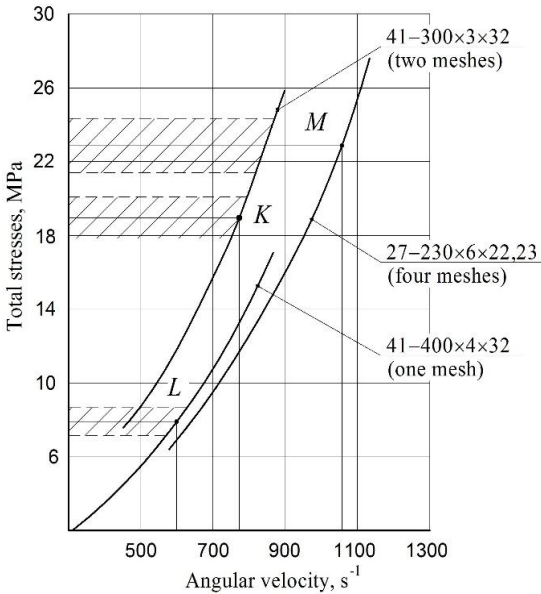


Fig. 4. Dependences of stress change on angular velocity

The points  $K$ ,  $L$ ,  $M$ , marked on the curves, correspond to the destruction of the wheels. Comparison of the calculated and experimental data shows that their ratio does not exceed 8%. During the calculation of the stress state arising under the action of bending forces, the wheel can be considered as closed annular plate, clamped along the inner contour and loaded with concentrated force along the outer contour.

As is known from the methodology for calculating stresses [11] arising in the wheel under the action of centrifugal forces, the differential equation of bending in polar coordinates has the form [13]:

$$D\Delta\Delta W = 0, \quad (24)$$



where  $D = \frac{E\tilde{H}^3}{12(1-\nu^2)}$  – cylindrical stiffness, N·m;  $\tilde{H}$  – plate thickness, m;  $E$  – elastic module, Pa;  $\nu$  – Poisson's ratio;  $W$  – deflection, m;  $\Delta = \frac{\partial^2}{\partial R^2} + \frac{1}{R} + \frac{\partial}{\partial R} + \frac{1}{R^2} \frac{\partial^2}{\partial \theta^2}$  – second order Laplace operator.

Internal forces in the plate (Fig. 3) are expressed in terms of deflection in following way:

$$M_r = -D \left[ \frac{\partial^2 W}{\partial R^2} + \frac{\nu}{R} \left( \frac{\partial W}{\partial R} + \frac{1}{R} \frac{\partial^2 W}{\partial \theta^2} \right) \right], \quad (25)$$

$$M_\theta = -D \left[ \nu \frac{\partial^2 W}{\partial R^2} + \frac{1}{R} \frac{\partial W}{\partial R} + \frac{1}{R^2} \frac{\partial^2 W}{\partial \theta^2} \right], \quad (26)$$

$$M_{r\theta} = (1-\nu) D \left[ \frac{1}{R} \frac{\partial^2 W}{\partial R \partial \theta} - \frac{1}{R^2} \frac{\partial^2 W}{\partial \theta^2} \right], \quad (27)$$

$$V_r = Q_r - \frac{1}{R} \frac{\partial M_{r\theta}}{\partial \theta} = -D \left[ \frac{\partial}{\partial R} \Delta W + (1-\nu) \frac{\partial^2}{\partial R \partial \theta} \left( \frac{1}{R} \frac{\partial W}{\partial \theta} \right) \right], \quad (28)$$

where  $M_r$ ,  $M_\theta$ ,  $M_{r\theta}$  – bending moments, N·m;  $V_r$  – support reaction per unit length of the plate contour, N/m;  $Q_r$  – transverse force, N.

The maximum stresses in magnitude are achieved in each section on the surface of the plate and are expressed through internal forces using the ratios:

$$\sigma_r^{bend} = \frac{6M_r}{\tilde{H}^2}; \quad \sigma_\theta^{bend} = \frac{6M_\theta}{\tilde{H}^2}; \quad \sigma_{r\theta}^{bend} = \frac{6M_{r\theta}}{\tilde{H}^2}. \quad (29)$$

We represent the solution of equation (24) in the form of a Fourier series [14]:

$$W = F_0(R) + \sum_{m=1}^{\infty} F_m(R) \cos m\theta, \quad (30)$$

where  $F_m(R)$  – function of the variable  $R$ , which is determined by substituting equation (30) into equation (24) and has the form:

$$F_0(R) = A_0 R + B_0 R^2 + C_0 \ln R + D_0 R^2 \ln R, \quad (31)$$

$$F_1(R) = A_1 R + B_1 R^{-1} + C_1 R^3 + D_1 R \ln R, \quad (32)$$

$$F_m(R) = A_m R^m + B_m R^{-m} + C_m R^{m+2} + D_m R^{-m+2}, \quad (33)$$

where  $A_m$ ,  $B_m$ ,  $C_m$ ,  $D_m$  ( $m = 0, 1, 2, \dots$ ) – arbitrary constants determined in the case under consideration from the following boundary conditions:

$$W \Big|_{R=R_f} = 0, \quad (34)$$

$$\frac{dW}{dR} \Big|_{R=R_f} = 0, \quad (35)$$

$$M_r|_{R=R_0} = 0, \quad (36)$$

$$V_r|_{R=R_2} = \frac{F_{bend}}{\pi R_0^2} \left( 1 + \sum_{m=1}^{\infty} \cos m\theta \right). \quad (37)$$

Performing the corresponding operations of differentiation over series in equation (30), we obtain a Fourier series expansion for the functions  $\frac{dW}{dR}(R, \theta)$ ,  $M_r(R, \theta)$ ,  $V_r(R, \theta)$ ,  $M_\theta(R, \theta)$ ,  $M_{r\theta}(R, \theta)$ :

$$\frac{dW}{dR} = \varphi_0(R) + \sum_{m=1}^{\infty} \varphi_m(R) \cos m\theta, \quad (38)$$

$$M_r = \mu_0(R) + \sum_{m=1}^{\infty} \mu_m(R) \cos m\theta, \quad (39)$$

$$V_r = v_0(R) + \sum_{m=1}^{\infty} v_m(R) \cos m\theta, \quad (40)$$

$$M_\theta = \psi_0(R) + \sum_{m=1}^{\infty} \psi_m(R) \cos m\theta, \quad (41)$$

$$M_{r\theta} = \eta_0(R) + \sum_{m=1}^{\infty} \eta_m(R) \cos m\theta, \quad (42)$$

where  $\varphi_m$ ,  $\mu_m$ ,  $v_m$ ,  $\psi_m$ ,  $\eta_m$  – functions that include arbitrary constants.

Taking into account equations (38)...(42) and (30), boundary conditions equations (34)...(37) are reduced to the following systems of linear equations relatively to unknowns  $A_m$ ,  $B_m$ ,  $C_m$ ,  $D_m$  ( $m = 0, 1, 2, \dots$ ):

$$F_m(R_1) = 0; \quad \varphi_m(R_1) = 0; \quad \mu_m(R_2) = 0; \quad v_m(R_0) = \begin{cases} \frac{F_{bend}}{2\pi R_0}, & m = 0 \\ \frac{F_{bend}}{\pi R_0}, & m \neq 0 \end{cases}. \quad (43)$$

Having determined from equation (43) arbitrary constants for each harmonic of the series (30), we can use formulas equations (25)...(27) to calculate the internal forces in the abrasive wheel, and then using equation (29) – stresses caused by bending. The results of the action of various force factors are summed up and the main stresses are calculated:

$$\sigma_{total} = \left( \sigma_r^c + \sigma_r^{iend} \right) - \nu \left( \sigma_\theta^c + \sigma_\theta^{bend} \right). \quad (44)$$

The developed model of the stress-strain state of an abrasive reinforced wheel allows predicting its strength characteristics, which will ensure safe and reliable exploitation.

### Conclusions

As a result of theoretical and experimental studies it was determined that an abrasive reinforced wheel is an anisotropic body. The main force factors affecting the strength of abrasive wheels are bending and centrifugal forces,

while for grinding wheels the influence of bending and centrifugal forces should be taken into account and for cutting wheels – centrifugal forces.

The greatest danger from the point of view of breaking the wheel is its working part at the contour of the clamping flange of the drive machine. The elastic modulus of the wheel ligament significantly exceeds the elastic modulus of the reinforcing mesh, therefore, it practically does not perceive the load at the initial stage of loading. The role of the reinforcing mesh is to maintain the integrity of the wheel after the formation of cracks in it.

To increase the strength of the wheel, it is advisable to additionally reinforce with meshes of small diameter at the contour of the clamping flange of the drive machine, and also to increase the scatter of the characteristics of elasticity and strength of the polymer matrix of the wheel. The developed algorithm for calculating the strength indicators of abrasive reinforced wheels allows you to calculate forces that arise in it, which makes it possible to predict their reliability and safer operation.

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#### **ПІДВИЩЕННЯ МЕХАНІЧНОЇ МІЦНОСТІ АБРАЗИВНОГО АРМОВАНОГО КРУГА**

Механічна міцність неармованих абразивних кругів визначається відцентровими та згинальними силами, але їх розподіл при армуванні невідомий. Припущено, що напруження розподіляються рівномірно, але порівняння розрахунків за теорією пружності та реальних характеристик на спеціальному стенді показало повну розбіжність. Проведення випробувань кругів на розтяг дозволило порівняти результати напружень у коловому та радіальному напрямках. Встановлено, що армований круг є анізотропним тілом. Анізотропію можна зменшити, змістивши одну армуючу сітку відносно іншої на кут  $45^\circ$ . У роботі розроблено математичну модель напружено-деформованого стану абразивного армованого круга з урахуванням анізотропії його властивостей. Для визначення відцентрових сил застосовується теорія пружності для ортотропного тіла. Згинальні сили, що виникають у працюючому крузі, визначалися при вирішенні задачі про розподіл деформацій в кільцевій анізотропній пластині, жорстко закріпленої по внутрішньому контуру. В результаті експериментальних досліджень встановлено, що напруження досягають  $8..23$  МПа, що можна порівняти з межею міцності матриці круга. Модуль пружності матриці круга помітно більше модуля пружності армуючої сітки, яка практично не сприймає навантаження на початковому етапі. Розроблена математична модель показників міцності абразивних армованих кругів дозволяє прогнозувати їхню надійність та безпечну експлуатацію.

**Ключові слова:** абразивний армований круг, міцність, армуюча сітка, відцентрові та згинальні сили, деформація.

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#### **MECHANICAL STRENGTH INCREASING OF ABRASIVE REINFORCED WHEEL**

The mechanical strength of unreinforced abrasive wheels is determined by centrifugal and bending forces, but their distribution during reinforcement is unknown. It was assumed that the stresses are distributed evenly, but a comparison of calculations on the theory of elasticity and real characteristics on a special stand showed complete discrepancy. Tensile tests of the wheels made it possible to compare the stresses results in the circumferential and radial directions. Was found that the reinforced wheel is an anisotropic body. Anisotropy can be reduced by displacing one reinforcement mesh relatively to the other by angle of  $45^\circ$ . In this paper, a mathematical model of the stress-strain state of the abrasive reinforced wheel was developed, taking into account the anisotropy of its properties. To determine the centrifugal forces, the theory of elasticity for an orthotropic body is applied. The bending forces that arise in the working wheel were determined during solving the problem of the distribution of deformations in the anisotropic annular plate rigidly fixed along the inner contour. As a result of experimental studies, it was found that stresses reach  $8..23$  MPa, which can be compared with the ultimate strength of the wheel matrix. The elastic module of the wheel matrix is noticeably greater than the elastic module of the reinforcing mesh, which practically does not perceive the load at the initial stage. The developed mathematical model of the strength indicators for abrasive reinforced wheels makes it possible to predict their reliability and safe operation.

**Keywords:** abrasive reinforced wheel, strength, reinforcing mesh, centrifugal and bending forces, deformation.

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### **ПОВЫШЕНИЕ МЕХАНИЧЕСКОЙ ПРОЧНОСТИ АБРАЗИВНОГО АРМИРОВАННОГО КРУГА**

Механическая прочность неармированных абразивных кругов определяется центробежными и изгибающими силами, но их распределение при армировании неизвестно. Предположено, что напряжения распределяются равномерно, но сравнение расчетов по теории упругости и реальных характеристик на специальном стенде показало полное расхождение. Проведение испытаний кругов на растяжение позволило сравнить результаты напряжений в окружном и радиальном направлениях. Установлено, что армированный круг является анизотропным телом. Анизотропию можно уменьшить, сместив одну армирующую сетку относительно другой на угол  $45^\circ$ . В работе разработана математическая модель напряженно-деформированного состояния абразивного армированного круга с учетом анизотропии его свойств. Для определения центробежных сил применяется теория упругости для ортотропного тела. Изгибающие силы, возникающие в работающем круге, определялись при решении задачи о распределении деформаций в анизотропной кольцевой пластине, жестко закрепленной по внутреннему контуру. В результате экспериментальных исследований установлено, что напряжения достигают  $8...23$  МПа, что можно сравнить с пределом прочности матрицы круга. Модуль упругости матрицы круга заметно больше модуля упругости армирующей сетки, которая практически не воспринимает нагрузку на начальном этапе. Разработанная математическая модель показателей прочности абразивных армированных кругов позволяет прогнозировать их надежность и безопасную эксплуатацию.

**Ключевые слова:** абразивный армированный круг, прочность, армирующая сетка, центробежные и изгибающие силы, деформация.

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*У роботі розроблено математичну модель напружено-деформованого стану абразивного армованого круга з урахуванням анізотропії його властивостей. Для визначення відцентрових сил застосовується теорія пружності для ортотропного тіла. Згинальні сили, що виникають у працюючому крузі, визначалися при вирішенні задачі про розподіл деформацій в кільцевій анізотропній пластині, жорстко закріпленій по внутрішньому контуру. Модуль пружності матриці круга помітно більше модуля пружності армуючої сітки, яка практично не сприймає навантаження на початковому етапі.*

Табл. 1. Іл. 4. Бібліогр. 14.

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*The mathematical model of the stress-strain state of the abrasive reinforced wheel was developed in this paper, taking into account the anisotropy of its properties. To determine the centrifugal forces, the theory of elasticity for an orthotropic body is applied. The bending forces that arise in the working wheel were determined during solving the problem of the distribution of deformations in the anisotropic annular plate rigidly fixed along the inner contour. The elastic module of the wheel matrix is noticeably greater than the elastic module of the reinforcing mesh, which practically does not perceive the load at the initial stage.*

Table 1. Fig. 4. Ref. 14.

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*В работе разработана математическая модель напряженно-деформированного состояния абразивного армированного круга с учетом анизотропии его свойств. Для определения центробежных сил применяется теория упругости для ортотропного тела. Изгибющие силы, возникающие в работающем круге, определялись при решении задачи о распределении деформаций в анизотропной кольцевой пластине, жестко закрепленной по внутреннему контуру. Модуль упругости матрицы круга заметно больше модуля упругости армирующей сетки, которая практически не воспринимает нагрузку на начальном этапе.*

Табл. 1. Ил. 4. Библиогр. 14.

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