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IMPACT OF PULSE DYNAMIC LOADING DIRECTION AND SURFACE CURVATURE ON THE STRESS-STRAIN STATE OF A THREE-LAYERED SPHERICAL SHELL

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On two size types of the semispherical three-layered structures, impact of the surface curvature and impact of the pulse dynamic loading on the stress-strain state (SSS) of these structures has been investigated. Values of normal stresses and vertical displacements of the structures' bearing layers and distribution of these indicators along the spatial coordinate have been determined. A finite element calculation method implemented within the software calculation complex NASTRAN has been applied.

Key words: three-layered conical shell, loading direction, stress-strain state, finite element model, axisymmetric pulse load.

Introduction and problem statement. Layered shells are often used in objects and facilities related to the space, military industry, innovative machine-building enterprises etc. The extent of using such structures is preconditioned by a number of circumstances, the most important of which is optimization of their design solutions.

To a significant extent, this problem has been positively addressed through theoretical basic provisions described in writings [1,2], reference and other materials [3]. Also, positive effect is given by the results of investigations, which characterize impact of geometric and physical and mechanical properties of such structures on their stress-strain state [4-5].

Extending and enhancing such information could contribute to taking early efficient design decisions, which would prevent negative consequences.

In this respect, assessment of the shell's Gaussian curvature magnitude impact and the dynamic pulse load impact direction on the shell deserve specific attention. Impact of these factors on the layered shell's stress-strain state has been performed using two size types of hemispherical three-layered structures.

Investigation and analysis of obtained results

The efficacy of mentioned factors has been investigated using hemispherical three-layered structures with different surface curvature and other similar geometric and physical and mechanical properties. The pulse dynamic loading of structures also had similar parameters and was applied in the forward and the opposite directions. Values of the structures' bearing layers' spatial displacements (u_1, u_3) and normal stresses (σ_1, σ_3) and their distribution along the spatial coordinate α have been determined. During the analysis, the obtained results were additionally compared to the investigation results of other authors.

In compliance with the hypothesis of thin-layered shells [6], determining SSS parameters of such structures is based on applying a non-lineal theory of structural orthogonal Timoshenko-type shells, in the quadratic approximation. Equation of motion for such structures appear as follows [2]:

а) provided smooth surface $s_{ij} < s_i < s_{ij+1}$

$$\frac{1}{A_2^i}\frac{\partial}{\partial s_i}(A_2^i T_{11}^i) - \psi^i T_{22}^i + k_1^i \overline{T}_{13}^i = \rho_i h_i \frac{\partial^2 u_1^i}{\partial t^2},$$

$$\frac{1}{A_2^i}\frac{\partial}{\partial s_i}(A_2^i \overline{T}_{13}^i) - k_1^i T_{11}^i + k_2^i T_{22}^i + P_3 = \rho_i h_i \frac{\partial^2 u_3^i}{\partial t^2},$$

$$\frac{1}{A_2^i}\frac{\partial}{\partial s_i}(A_2^i M_{11}^i) - \psi^i M_{22}^i + T_{13}^i = \frac{\rho_i h_i^3}{12}\frac{\partial^2 \varphi_1^i}{\partial t^2}, \tag{1}$$

б) on the lines of discontinuity $s = s_j$:

$$\sum_{i=1}^{2} T_{11}^{i\pm} = \rho_j F_j \frac{\partial^2 u_{1j}}{\partial t^2}, \quad \sum_{i=1}^{2} T_{13}^{i\pm} = \rho_j F_j \frac{\partial^2 u_{3j}}{\partial t^2},$$

$$\sum_{i=1}^{2}(M_{11}^{i\pm} \mp h_j T_{11}^{i\pm}) = \rho_j I_{kpj} \frac{\partial^2 \varphi_{1j}}{\partial t^2}. \tag{2}$$

Equations (1), (2) are supplemented by corresponding initial and boundary conditions ($s = s_{j0}, s = s_{jN}$).

Calculations of spatial movements and normal stresses values along the spatial coordinate α has been performed by applying a finite-element modeling adjusted to specific structures.

The finite-element models reflected relation between the potential energy of deformation and the potential of external loads [5]:

$$\Pi = E + W, \tag{3}$$

where E is the potential energy of deformations and W is the potential of external loads.

After splitting the integral field into the separate elements, equation appears as follows:

$$\Pi = \sum_{e=1}^{E}\left(E^{(e)} - W^{(e)}\right) = \sum_{e=1}^{E}\pi^{(e)}. \tag{4}$$

The global stiffness matrix $[K]$ and the global column vector $\{F\}$ in the matrix equation

$$[K]\{U\} = \{F\} \tag{5}$$

correspond to correlations:

$$[K] = \sum_{e=1}^{E} \left[k^{(e)} \right], \quad (6)$$

$$\{F\} = -\sum_{e=1}^{E} \left\{ f^{(e)} \right\}. \quad (7)$$

Two layered hemispheres (fig. 1) have been analyzed with the clamped footing and diameters (D_1=0.30 m, D_2=0.60 m), which had similar bearing layers' thickness (h_1=h_3=0.010 m), polymeric filler with h_2=0.020 m thickness, reinforced with the discrete stiffening rings rigidly bound to the bearing layers. Each hemisphere consisted of five rings located in points, which have been calculated according to the dependence below: $\alpha_k = k * \Delta\alpha$, $k = \overline{1,5}$, $\Delta\alpha = \dfrac{\pi}{12}$.

Shells with the Young's moduli ratio of the filler and shell sheathing materials E_1/E_T =500 have been studied. Values E_1=E_3 were 70 GPa, E_T=0.14 GPa, and other values were as below: μ_1=μ_3=0.3, ρ_1=ρ_3=2.7*10^3 kg/m^3.

The pulse load distribution $P(t)$ was as below:

$$P(t) = A * \sin\frac{\pi t}{T}[\eta(t) - \eta(t-T)], \quad (8)$$

where T is load duration; t is time interval; A is the pulse load amplitude. The following loading parameters have been accepted: A = 10^6 Pa; T = 50*10^{-6} s.

Distribution of displacements and stresses along the spatial coordinate α was calculated according to the direct transient dynamic process algorithm within the time interval $0 \le t \le 10T$. Calculations have been performed by the software complex Nastran. The time interval step duration was $0.25 \cdot 10^{-6}$s, and the total number of steps was 200.

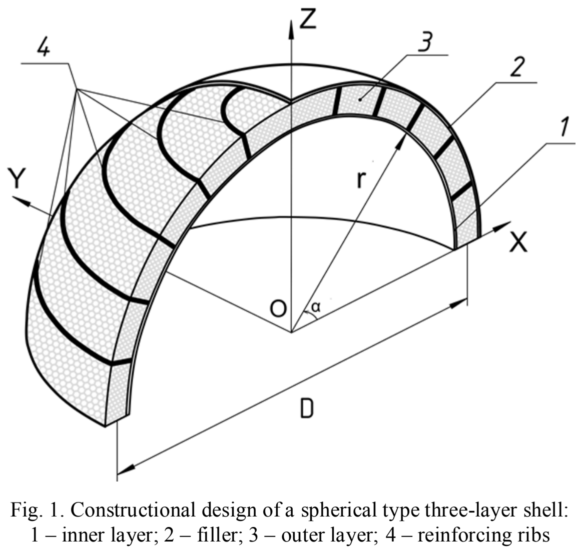


Fig. 1. Constructional design of a spherical type three-layer shell:
1 – inner layer; 2 – filler; 3 – outer layer; 4 – reinforcing ribs

Choice of the finite element model type was conditioned by the goal of obtaining reliable and precise results. The finite element model of the structure with the diameter D = 0.3 m (Fig. 2) had 83250 finite elements and 99906 nodes, and with the diameter D = 0.6 m it includes 22200 such elements and 26646 nodes, respectively.

The bearing layers' displacement results related to the analyzed structures under the internal pulse load impact are shown in Table 1.

As seen, vertical displacements of both structures reach their maximums near the poles of the structures at $\alpha = \frac{\pi}{2}$.

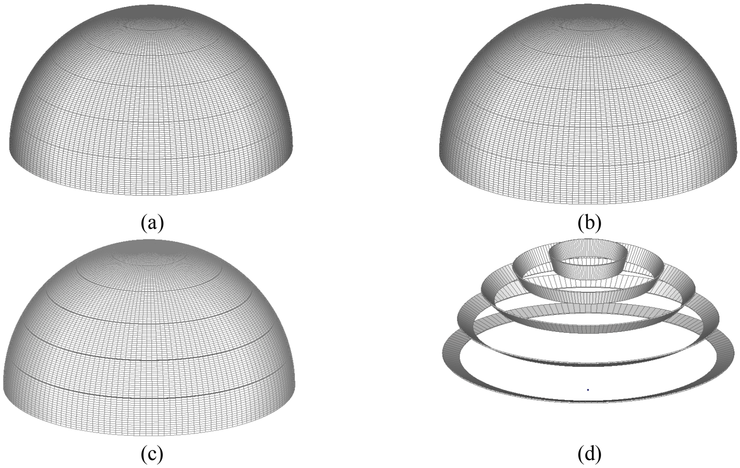


Fig. 2. Finite-element model of the three-layered hemispherical shell:
(a) – inner load-bearing sheathing; (b) – same, outer; (c) – polymeric filler; (d) – reinforcing rings

Provided the inner loading, maximal displacements of the structure having lesser diameter (D=0.3 m) have been detected at 8.25T, and having bigger diameter (D=0.6 m) – at 7.7T. Absolute value of the inner layer displacement (u_1^i) in the structure with the smaller diameter appeared to be greater. It has far exceeded the displacement value of a similar layer within the structure (D=0.6 m).

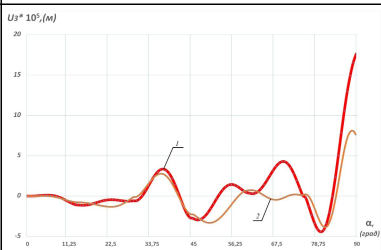
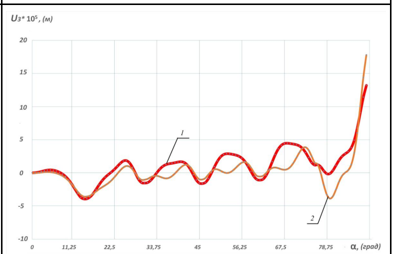
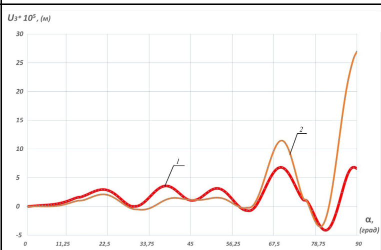
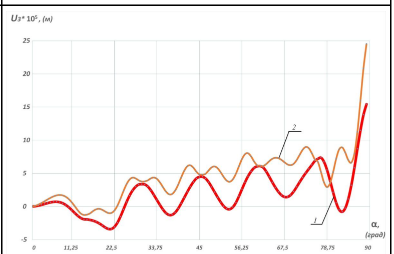
Provided the outer loading, displacements of the outer layer of both structures appeared greater. At that, its absolute value in the structure D = 0.3 m appeared practically similar to that in the structure D = 0.6 m. Its maximal value in the lesser diameter structure has been detected at 10T, and in the structure D=0.6 m – at 8.3T. Provided this loading direction, maximal displacement value has considerably exceeded displacement caused by the inner loading in both structures.

The structure with the lesser diameter appeared more stressed (Fig. 3). Maximal stress values in the inner layer of this structure exceeded for over 20% its values detected in the bigger diameter structure.

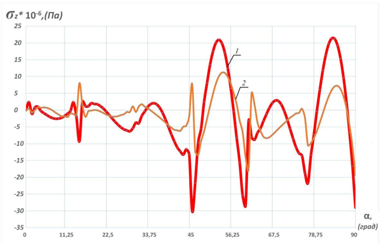
First natural frequency of the structure D = 0.3 m was 2740 Hz, and of the structure D = 0.6 m it was 1417 Hz.

Table 1

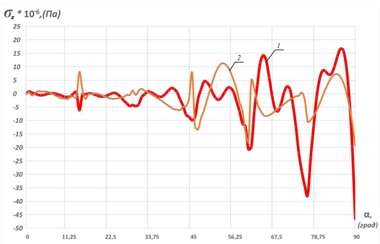
Maximal vertical movements u_1^i of the sheathing shells with the polymeric filler E_1/E_T=500 at time (T)

Pulse dynamic loading	Hemispherical three-layer structures	
	D = 0,3 m	D = 0,6 m
Inner	 8,25T	 7,7T
Outer	 10T	 8,3T

1 – shell inner layer; 2 – shell outer layer.



(a)



(b)

Fig. 3. Normal stress of bearing layers of the median surface of sheathings under the inner loading: (a) – shell D = 0.3 m; (b) – same D = 0.6 m

Conclusions

Value of the Gaussian curvature of the layered shell structures impacts their stress-strain state. Increase of the surface curvature of spherical structures under the impact of the dynamic pulse load increases the displacement and stress of their bearing layers.

In a three-layered hemispherical structure, the diameter decrease from 0.6 m to 0.3 m decreases the displacement of the structure's bearing layers. At that, this became more significantly apparent under the outer loading of the structure.

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ВПЛИВ НАПРЯМКУ ДІЇ ІМПУЛЬСНОГО ДИНАМІЧНОГО НАВАНТАЖЕННЯ І КРИВИЗНИ ПОВЕРХНІ НА НАПРУЖЕННО – ДЕФОРМОВАНИЙ СТАН ТРИШАРОВОЇ СФЕРИЧНОЇ ОБОЛОНКИ

На двох типових розмірах напівсферичних тришарових структур досліджено вплив кривизни поверхні і напрям дії імпульсного динамічного навантаження на напружено-деформований стан цих структур. Визначені величини нормальних напружень і вертикальних переміщень несучих шарів структур і розподіл цих показників по просторовій координаті. Визначено вплив кривизни поверхні тришарової півсферичної оболонки на її напружено-деформований стан при внутрішньому і зовнішньому імпульсному навантаженні. Враховано фізико-механічні характеристики полімерного заповнювача армованого дискретними ребрами.

Ключові слова: тришарова сферична оболонка, напрям дії навантаження, напружено-деформований стан, скінчено-елементна модель, імпульсне внутрішнє вісесиметричне навантаження.

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Values of normal stresses and vertical displacements of the structures' bearing layers and distribution of these indicators along the spatial coordinate have been determined. Distribution of displacements' and stresses' magnitudes along the spatial coordinate α was determined by the software complex Nastran through the direct transient dynamic process algorithm within the time interval $0 \leq t \leq 10T$. The time interval step duration was $0.25*10^{-6}$ s and the total number of steps

was 200. The detailed and accurate calculation results have determined the choice of the solid finite element type.

Value of the Gaussian curvature of the layered shell structures impacts their stress-strain state. Increase of the surface curvature of spherical structures under the impact of the dynamic pulse load increases the displacement and stress of their bearing layers.

In addition to the vertical displacements and normal stresses of the bearing layers of the analyzed semispherical three-layered structures, their first natural frequency (f_1) was also calculated.

Keywords: three-layered conical shell, loading direction, stress-strain state, finite element model, axisymmetric pulse load.

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Гайдайчук В.В., Котенко К.Е., Рувін О.Г. **Вплив напрямку дії імпульсного динамічного навантаження і кривизни поверхні на напружено-деформований стан тришарової сферичної оболонки** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2022. – Вип. 108. – С. 171–177.
У статті приведено результати оцінки напружено-деформованого стану тришарової сферичної оболонки, враховуючі її структурну особливість, фізико-механічні характеристики полімерного наповнювача, армованого дискретними ребрами.
Табл. 1. Іл. 3. Бібліогр. 6 назв.

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Gaidaichuk V.V., Kotenko K.E., Ruvin O.G. **Impact of pulse dynamic loading direction and surface curvature on the stress-strain state of a three-layered spherical shell** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles – Kyiv: KNUBA, 2022. – Issue 108. – P. 171–177.
On two size types of the semispherical three-layered structures, impact of the surface curvature and impact of the pulse dynamic loading on the stress-strain state (SSS) of these structures has been investigated. Values of normal stresses and vertical displacements of the structures' bearing layers and distribution of these indicators along the spatial coordinate have been determined. A finite element calculation method implemented within the software calculation complex NASTRAN has been applied.
Tabl. 1. Fig. 3. Ref. 6.

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