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THE STABILITY OF LOW-PITCHED VON MISES TRUSSES WITH HORIZONTAL ELASTIC SUPPORTS

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The work's aim is to study the horizontal supports stiffness impact, which simulate the conditions for supporting the domes upper tier on the von Mises trusses' stability. A three-hinged truss' deformed scheme with a concentrated vertical load in the ridge joint was considered. A two transcendental equations' system for the dependence of the load on vertical and horizontal displacements taking into account the rods' compression was obtained. The truss' stability numerical studies were carried out depending on the structure's design geometry.

Keywords: stability; von Mises truss; steel dome; nonlinear displacements; deformational calculation: elastic horizontal supports; the critical load equation; domes' nodal buckling.

1. Introduction

Topicality. Dome coverings are used in the public buildings construction. Such constructions best meet the rational design criterion - minimizing the difference between construction and functional volume [4, 5], along with other design solutions [5].

It should be noted that there are a large number of the solid shell structures stability studies [1, 2, 3, 17, 23]. But in the transition to rod dome systems it is possible to design cylindrical shells and geodesic domes structures with different tiers' variable stiffness to reduce the inclined elements deformation's impact with small uppermost tier angles.

There are also known dome coverings accident studies, which showed the rod domes or shell systems sensitivity to the stability loss with the joints' snap-through effect to the covering's center [22]. It is important that the several rods' stability loss effect which hinged in the dome's node has a caloptic character [6, 7, 9, 17, 19, 20]. Dome node stability loss effect description similar to snap-through effect first was studied in von Mises works (Richard Edler von Mises) [19, 20], which were described a trusses deformation model that consists of two rods or several rods hinged at the ridge and hinged to the base. However, these studies relevance remains, because it is possible, by complicating the von Mises truss physical and mathematical model, to study the sloped rod structures (cylindrical shells and geodesic domes) nodes snap-through effect, by modeling the load at an angle and elastic supports in the ridge node [6,9]. In the next, some similar structural systems nonlinear deformation problems, including the loss of stability, are described in [10, 11, 12, 13, 14, 15, 16]. Some scientists to illustrate the theory of catastrophes cites the von Mises's truss snap-through effect example. The following works were devoted to such phenomena [21, 22]. The researchers paid special attention to the tall two-rod systems stability loss, in which they discovered the skew-symmetric stability loss possibility [7].

An important task is also to avoid, during dome with rods made of thin-walled profiles deformation, local stability loss by choosing the optimal cross-section of the rods [8].

Modeling the dome system upper tier elements support's elasticity with small angles inclination to the horizon will make it possible to predict such system's stability loss. Therefore, the upper tier support elastic conditions influence study on the lower tiers for such rod-systems is relevant. At the same time, it is necessary to study displacements' growing and impact in the horizontal direction. In this case, into the von Mises truss structural model should be added the horizontal elastic supports. Such asymmetrical constructive systems are considered in work [18].

Topical von Mises trusses stability problems, which needs to be further investigated, include the two-rod systems on elastic supports stability problems. The need to solve them is conditioned by the compliance influence of dome upper-tier rods connection to the rods structures lower-tier.

2. Works' purpose

Obtain analytical dependencies and reveal the elastic horizontal supports stiffness influence on the von Mises trusses stability at small inclination angles and at different elastic supports' stiffness, thereby simulating the dome annular elements stiffness effect on the its upper tier stability.

3. Basic research

The von Mises truss design scheme is a three-hinge rod system: the support risers are hinge supported on the foundations; the rods are pivotally connected in the ridge node (Fig. 1).

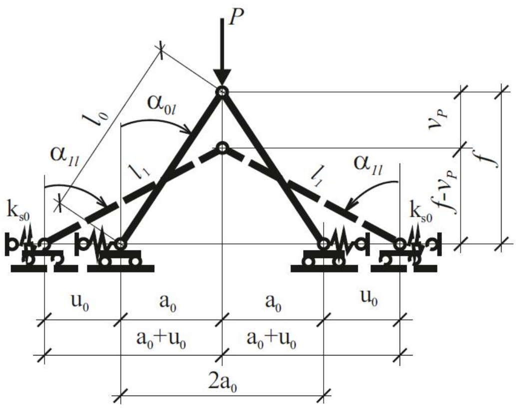

Fig. 1. Three-hinge system design scheme and its deformed scheme

Three-hinge sloping truss (Fig. 1) has rods length l_0, a truss span equals to $2a_0$, the truss height f_0 – the distance between hinge-supports to the ridge joint. The initial support-rod angle from the vertical axis equals α_{0l}. The horizontal elastic support stiffness characteristic is taken as follows – $k_{s0}=T_1/u_{01}$, where T_1- standard force at which an elastic support elongation or contraction occurs by a certain value - u_{01}.

Truss elements are made from linear elastic material with the deformation module E.

Under the vertical load (P) which effects to the ridge joint the system deforms symmetrically. The ridge node is undergoing displacement - v_p, and the rods length is reduced due to the compression, the longitudinal reduction in length is indicated by $\Delta l_{01} = l_0 - l_1$. In the deformed state the length of each element will be - l_1. Now each inclined rod's deformation will be as follows:

$$\varepsilon_1 = \Delta l_{01} / l_0 = 1 - l_1 / l_0. \tag{1}$$

Under the load action, due to the occurrence in the supports, in addition to vertical reactions, also horizontal reactions in elastic supports, the horizontal supports will move by an amount u_0. The relations between rods angles (α_{0l}) in deformed state depending on horizontal and vertical displacements will be:

$$\tan\alpha_{0l} = \frac{a_0}{f} \rightarrow f = \frac{a_0}{\tan\alpha_{0l}}, \tag{2,a}$$

$$\left(f - v_p\right) = \left(\frac{a_0}{\tan\alpha_{0l}} - v_p\right) = a_0\left(\frac{1}{\tan\alpha_{0l}} - \frac{v_p}{a_0}\right). \tag{2,b}$$

$$\cos(\alpha_{1l}) = \frac{f - v_p}{\sqrt{(a_0+u_0)^2 + (f-v_p)^2}} = \frac{\left((f/a_0) - (v_p/a_0)\right)}{\sqrt{\left(1+(u_0/a_0)\right)^2 + \left((f/a_0)-(v_p/a_0)\right)^2}},$$

$$\sin(\alpha_{1l}) = \frac{a_0 + u_0}{\sqrt{(a_0+u_0)^2 + (f-v_p)^2}} = \frac{\left(1+(u_0/a_0)\right)}{\sqrt{\left(1+(u_0/a_0)\right)^2 + \left((f/a_0)-(v_p/a_0)\right)^2}},$$

$$\tan\left(\alpha_{1l}\right) = \frac{1+(u_0/a_0)}{(f/a_0)-(v_p/a_0)}. \tag{3}$$

From the deformed truss state equations (2, 3) it is composing a deformation's continuity equation through the conjoint vertical displacements at the ridge node under the load:

$$f - v_p = l_{1l}\cos\left(\alpha_{1l}\right), \quad a_0 + u_0 = l_{1l}\sin\left(\alpha_{1l}\right), \quad l_{0l} = a_0 / \sin\left(\alpha_{0l}\right). \tag{4}$$

The deformations in rods by (1) (Fig.1):

$$\varepsilon_1 = \frac{\dfrac{a_0}{\sin(\alpha_{0l})} - \dfrac{a_0 + u_0}{\sin(\alpha_{1l})}}{\dfrac{a_0}{\sin(\alpha_{0l})}} = \left[1 - \left(1 + (u_0/a_0)\right)\frac{\sin(\alpha_{0l})}{\sin(\alpha_{1l})}\right]. \tag{5}$$

The compression forces in rods (N_1) at deformed state under the vertical load has type:

$$N_1 = \varepsilon_1 E A_{cal}; \rightarrow N_1 = \left[1 - \left(1 + (u_0/a_0)\right)\frac{\sin(\alpha_{0l})}{\sin(\alpha_{1l})}\right] E A_{cal}. \tag{6}$$

The forces in the rods, which are due to the balance of the forces in the node:

$$P = 2N_1\cos(\alpha_{1l}). \tag{7}$$

The ratios combination (1, 2, 5) with equation (7) gives an equation (criterion) for the two-rod truss structure on elastic supports operation:

$$\frac{P}{E_0 A_{cal}} = 2\left[1 - \left(1 + (u_0/a_0)\right)\frac{\sin(\alpha_{0l})}{\sin(\alpha_{1l})}\right]\cos(\alpha_{1l});$$

$$\frac{P}{E_0 A_{cal}} = 2\left[1 - \frac{\left(1 + (u_0/a_0)\right)\sin(\alpha_{0l})}{\dfrac{\left(1 + (u_0/a_0)\right)}{\sqrt{\left(1 + (u_0/a_0)\right)^2 + \left((f/a_0) - (v_p/a_0)\right)^2}}}\right]\frac{(f/a_0) - (v_p/a_0)}{\sqrt{\left(1 + \dfrac{u_0}{a_0}\right)^2 + \left(\dfrac{f}{a_0} - \dfrac{v_p}{a_0}\right)^2}};$$

$$\frac{P}{E_0 A_{cal}} = 2\left(\frac{f}{a_0} - \frac{v_p}{a_0}\right)\left[\frac{1}{\sqrt{\left(1 + (u_0/a_0)\right)^2 + \left((f/a_0) - (v_p/a_0)\right)^2}} - \sin(\alpha_{0l})\right]. \tag{7}$$

However, the horizontal displacements u_0 depends on thrust ($T_{KS} = k_{s0} u_0$). From the forces balance at the ridge node:

$$T_{KS} = k_{s0} u_0; \quad T_{KS} = N_1 \sin(\alpha_{1l}).$$

$$P = 2N_1\cos(\alpha_{1l}). \rightarrow N_1 = \frac{P}{2\cos(\alpha_{1l})}.$$

$$T_{KS} = k_{s0} u_0 = \frac{P}{2}\tan(\alpha_{1l}) = \frac{P}{2}\frac{1 + (u_0/a_0)}{(f/a_0) - (v_p/a_0)}. \tag{8}$$

From the equilibrium equation of the moment's sum that arise due to external forces-reactions in the semi-arch support in the deformed state, the following equation can also be obtained:

$$k_{s0}u_0\left((f/a_0)-(v_p/a_0)\right)-\frac{P}{2}\left(1+(u_0/a_0)\right)=0;$$

$$\frac{u_0}{a_0}\left((f/a_0)-(v_p/a_0)\right)-\frac{P}{2k_{s0}a_0}\left(1+(u_0/a_0)\right)=0;$$

$$\left((f/a_0)-(v_p/a_0)\right)\frac{u_0}{a_0}=\frac{P}{2k_{s0}a_0}\left(1+(u_0/a_0)\right). \qquad (9)$$

Next, we will find the relationship between the supports horizontal displacements and the ridge node vertical displacements under the action of the force P according to equation (8.9).

$$\left(1+(u_0/a_0)\right)=\frac{P}{2\left(k_{s0}a_0\right)}\frac{\left(1+(u_0/a_0)\right)}{\left((f/a_0)-(v_p/a_0)\right)}+1;$$

$$\left(1+(u_0/a_0)\right)\left[1-\frac{P}{2\left(k_{s0}a_0\right)\left((f/a_0)-(v_p/a_0)\right)}\right]=1;$$

$$\left(1+(u_0/a_0)\right)=\frac{1}{\left[1-\dfrac{P}{2\left(k_{s0}a_0\right)\left((f/a_0)-(v_p/a_0)\right)}\right]}. \qquad (10)$$

Thus, the condition that the structure is quickly geometrically unchanged is the condition:

$$1-\frac{P}{2\left(k_{s0}a_0\right)\left((f/a_0)-(v_p/a_0)\right)}>0, \qquad (11,a)$$

$$\frac{P}{2\left(k_{s0}a_0\right)\left((f/a_0)-(v_p/a_0)\right)}<1. \qquad (11,b)$$

That is, the elastic support rigidity must be sufficient to fulfill the last condition so that the structure does not instantly turn into a mechanism.

Equation (8.9) implies another equation for the vertical and horizontal displacements dependence on the vertical load:

$$\frac{P}{2\left(k_{s0}a_0\right)}=\left[\frac{(u_0/a_0)}{\left(1+(u_0/a_0)\right)}\right]\left((f/a_0)-(v_p/a_0)\right). \qquad (12)$$

Combining equation (12) with equation (7), we obtain a system of two transcendental equations with two unknown relative displacements $((v_p/a_0),(u_0/a_0))$:

$$\begin{cases} \dfrac{P}{E_0A_{cal}}=2\left(\dfrac{f}{a_0}-\dfrac{v_p}{a_0}\right)\left[\dfrac{1}{\sqrt{\left(1+(u_0/a_0)\right)^2+\left((f/a_0)-(v_p/a_0)\right)^2}}-\sin(\alpha_{0l})\right] \\ \dfrac{P}{2\left(k_{s0}a_0\right)}=\left[\dfrac{(u_0/a_0)}{\left(1+(u_0/a_0)\right)}\right]\left((f/a_0)-(v_p/a_0)\right) \end{cases}. \qquad (13)$$

Thus, the two nonlinear equations system for the two-rod hinge system deformation calculation with a certain geometry on elastic supports with two unknown displacements v_p, u_0 under a load with a force P and the rods constant stiffness $E_0 A_{cal}$ and the supports stiffness k_{s0}.

To solve the equations system (13), we introduce a dimensionless relative vertical and horizontal displacements parameter (ψ_{vu}).

$$\psi_{vu} = \frac{\left((f/a_0)-(v_p/a_0)\right)}{\left(1+(u_0/a_0)\right)} = \frac{f}{a_0}\frac{\left(1-(v_p/f)\right)}{\left(1+(u_0/a_0)\right)}. \tag{14}$$

The relative displacements parameter (ψ_{vu}) is a rod's inclination angle changes trigonometric function:

$$\tan(\alpha_{1l}) = \psi_{vu} = \frac{\left((f/a_0)-(v_p/a_0)\right)}{\left(1+(u_0/a_0)\right)} = \tan(\alpha_{0l})\frac{\left(1-(v_p/f)\right)}{\left(1+(u_0/a_0)\right)}.$$

In addition, we have dependencies:

$$\left(1+(u_0/a_0)\right)\psi_{vu} = \left((f/a_0)-(v_p/a_0)\right) = (f/a_0)\left(1-(v_p/f)\right).$$

$$(v_p/f) = 1-(a_0/f)\left(1+(u_0/a_0)\right)\psi_{vu}. \tag{15}$$

Now, the given equations system has single common reduced argument (ψ_{vu})

$$\begin{cases} \dfrac{P}{E_0 A_{cal}} = 2\dfrac{\left(\dfrac{f}{a_0}-\dfrac{\mathrm{v}_p}{a_0}\right)}{\left(1+\dfrac{u_0}{a_0}\right)}\left[\dfrac{1}{\sqrt{1+\dfrac{\left((f/a_0)-(v_p/a_0)\right)^2}{\left(1+(u_0/a_0)\right)^2}}}-\left(1+\dfrac{u_0}{a_0}\right)\sin(\alpha_{0l})\right], \\ \dfrac{P}{E_0 A_{cal}} = \dfrac{2(k_{s0}a_0)}{E_0 A_{cal}}\left(\dfrac{u_0}{a_0}\right)\left[\dfrac{\left((f/a_0)-(v_p/a_0)\right)}{\left(1+(u_0/a_0)\right)}\right] \end{cases}$$

$$\begin{cases} \dfrac{P}{E_0 A_{cal}} = 2\psi_{uv}\left[\dfrac{1}{\sqrt{1+\psi_{uv}^2}}-\left(1+(u_0/a_0)\right)\sin(\alpha_{0l})\right] \\ \dfrac{P}{E_0 A_{cal}} = 2\psi_{uv}\dfrac{(k_{s0}a_0)}{E_0 A_{cal}}\left(\dfrac{u_0}{a_0}\right) \end{cases}. \tag{16}$$

Eliminating the load parameter ($P/(E_0 A_{cal})$) from the two equations, we have the equation:

$$\frac{2(k_{s0}a_0)}{E_0 A_{cal}}\left(\frac{u_0}{a_0}\right)\psi_{uv} = 2\psi_{uv}\left[\frac{1}{\sqrt{1+\psi_{uv}^2}}-\left(1+\frac{u_0}{a_0}\right)\sin(\alpha_{0l})\right],$$

$$\frac{(k_{s0}a_0)}{E_0 A_{cal}}\left(\frac{u_0}{a_0}\right)=\left[\frac{1}{\sqrt{1+\psi_{uv}^2}}-\left(1+\frac{u_0}{a_0}\right)\sin(\alpha_{0l})\right].$$

Next:

$$\frac{u_0}{a_0}\left[\frac{k_{s0}a_0}{E_0 A_{cal}}+\sin(\alpha_{0l})\right]=\frac{1}{\sqrt{1+\psi_{uv}^2}}-\sin(\alpha_{0l}).$$

Finally, from the last equation, a recurrent formula for determining horizontal and vertical displacements was obtained depending on the displacements relative parameter (ψ_{uv}):

$$\left(\frac{u_0}{a_0}\right)=\frac{\left[1\Big/\sqrt{1+\psi_{uv}^2}\right]-\sin(\alpha_{0l})}{\left(\frac{k_{s0}a_0}{E_0 A_{cal}}+\sin(\alpha_{0l})\right)}, \tag{17,a}$$

$$\left(\frac{u_0}{a_0}\right)+1=\frac{\left[1\Big/\sqrt{1+\psi_{uv}^2}\right]-\sin(\alpha_{0l})}{\left(\frac{k_{s0}a_0}{E_0 A_{cal}}+\sin(\alpha_{0l})\right)}+1. \tag{17,b}$$

Thus, combining the recurrent formula (17, b) with the first equation from the equations system (16), we can obtain a criterion for the two-rod system deformation on elastic supports under symmetric nonlinear deformation:

$$\frac{P}{E_0 A_{cal}}=2\psi_{uv}\left\{\frac{1}{\sqrt{1+\psi_{uv}^2}}-\left[\frac{\frac{1}{\sqrt{1+\psi_{uv}^2}}-\sin(\alpha_{0l})}{\frac{k_{s0}a_0}{E_0 A_{cal}}+\sin(\alpha_{0l})}+1\right]\sin(\alpha_{0l})\right\}. \tag{18}$$

The resulting equation (18) is a criterion for the sloped two-rod truss stability of the Mises truss type on elastic supports. It should be noted that equation (18) is an analytical equation for direct calculation in the critical force relative coordinates. Moreover, the obtained equations (17, b) and (15) are recurrent formulas for the transition from the displacements relative parameter (ψ_{uv}) according to formula (14) to relative displacements ((v_p/a_0) , (u_0/a_0)).

At Fig. 2 is shown the dependency of $\dfrac{P}{E_0 A_{cal}}$ from the relative vertical displacements $\dfrac{v_p}{f}$ with α_{0l1} =80°. Where: 1 - a diagram when $\dfrac{k_{s0}a_0}{E_0 A_{cal}}$ =1; 2 – a diagram when $\dfrac{k_{s0}a_0}{E_0 A_{cal}}$ = 2; 3 - a diagram when $\dfrac{k_{s0}a_0}{E_0 A_{cal}}$ =5; 4 – a diagram when $\dfrac{k_{s0}a_0}{E_0 A_{cal}}$ =15.

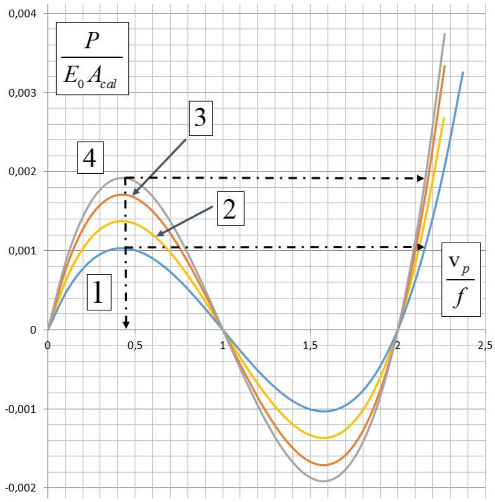

Fig. 2. Numerical studies of von Mises truss with horizontal elastic supports results by formula (18) with the displacements parameters by formulas (15, 17)

4. **The numerical studies results**

In order to determine certain regularities of elastic double-rod trusses' on elastic supports nonlinear deformation under a vertical load in the ridge node numerical studies were carried out. Thus, it was found that the truss' inclined rods angle, as well as the elastic supports stiffness, affects the ridge node vertical displacements dependence on the vertical load.

When manufacturing a truss structure from an elastic material without elastic supports (u_0 / a_0=0) that is a rigid horizontal support, at small rods inclination angles, the obtained expression (16, the first equation) goes over to the traditional solution given in [19, 20].

$$\frac{P}{EA_{cal}} = 2\left(\frac{1}{\tan\alpha_{0l}} - \frac{v_p}{a_0}\right)\left[\frac{1}{\sqrt{1+\left((1/\tan\alpha_{0l}) - (v_p/a_0)\right)^2}} - \sin(\alpha_{0l})\right]. \quad (19)$$

At the first researches stage, the elastic supports rigidity influence was determined at a fixed initial angle α_{0l1} =80° and α_{0l1} =82.5° (Fig. 2, Fig. 3).

At Fig. 3 is shown the dependency of $P/E_0 A_{cal}$ from relative vertical displacements v_p/f with α_{0l1} =80°. Where: 1 - a diagram when $(k_{s0}a_0)/(E_0 A_{cal})$ =1; 2 – a diagram when $(k_{s0}a_0)/(E_0 A_{cal})$ = 2; 3 – a diagram when $(k_{s0}a_0)/(E_0 A_{cal})$ =5; 4 – a diagram when $(k_{s0}a_0)/(E_0 A_{cal})$ =15.

It is shown that the general deformation's nature is similar to the traditional von Mises truss deformation without elastic horizontal supports - the truss' ridge node is snapping through the horizontal axis, but under a vertical load, which depends on the horizontal supports stiffness.

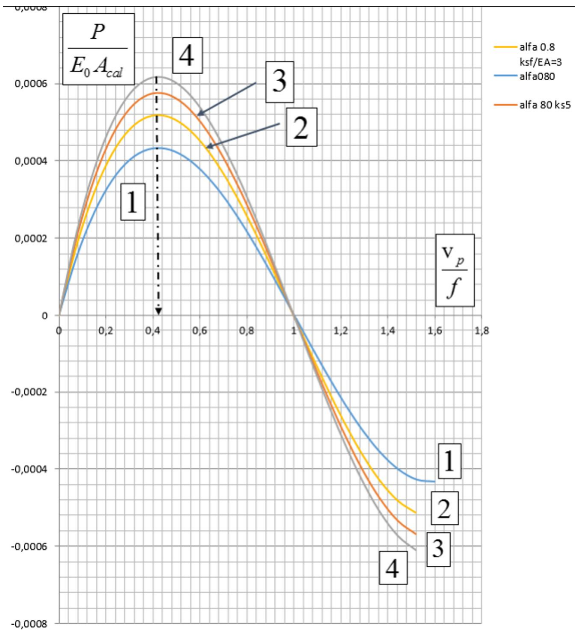

Fig. 3. Numerical studies results of the von Mises truss with elastic horizontal supports deformation according to equation (18) with displacement parameters according to formulas (15.17)

It has been established that the lower the horizontal supports rigidity, the lower the truss critical buckling force. The critical force value corresponds to such a load at which a slight decrease in the vertical force does not lead to the return the truss to the design position. It was also found that at an initial angle value α_{0l1} =80° (Fig. 2), an increase in the elastic supports relative stiffness from $(k_{s0}a_0)/(E_0A_{cal})=1{,}0$ to $(k_{s0}a_0)/(E_0A_{cal})=2{,}0$ increases the critical force reduced to the rods stiffness by almost 1.4 times. But with a further increase in the relative horizontal elastic supports stiffness from $(k_{s0}a_0)/(E_0A_{cal})=2$ to $(k_{s0}a_0)/(E_0A_{cal})=15$ an additional increase in the critical reduced force can be achieved, but such an increase in the horizontal supports stiffness is apparently not economically feasible. The same effect is observed at the initial design support rods angle α_{0l1} =82,5° (Fig. 3).

Thus, by constructive measures by creating additional horizontal rigid elements in the domes belts, or by increasing the elements' cross-sections for lower tiers, it is possible to increase the domes upper tier stability, which will ensure the architectural form and structural safety.

At the second stage, to determine an inclination angle influence on the truss with elastic supports stability, the studies were carried out at a fixed elastic supports stiffness: it is assumed that the relative elastic supports stiffness is $(k_{s0}a_0)/(E_0A_{cal})=2$. The studies were carried out at the truss' elastic supporting elements deflection angles from the vertical axis: α_{0l1} =75°; α_{0l2} =77,5; α_{0l3}=80°; α_{0l4} =82,5°. The research results are shown at Fig. 4.

At Fig.4 is shown the dependency of P/E_0A_{cal} from the relative vertical displacements parameter v_p/f and with $(k_{s0}a_0)/(E_0A_{cal}) = 2$. Where: 1 - a diagram when α_{0l1} =75°; 2- a diagram when α_{0l2} = 77,5; 3 – a diagram when α_{0l3}=80°, 4 – a diagram when α_{0l4} =82,5°.

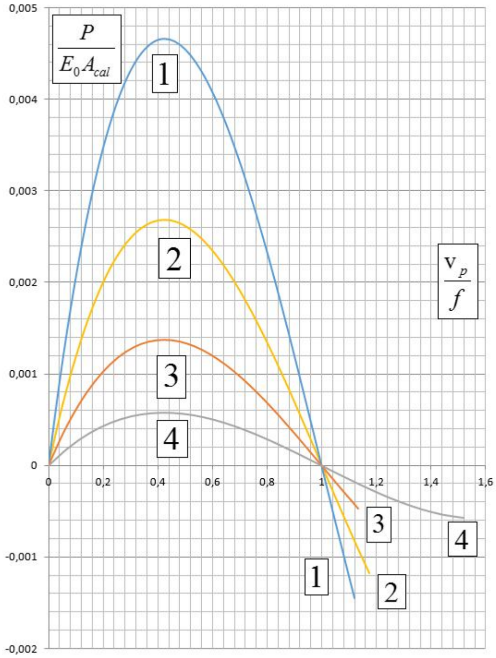

Fig. 4. Numerical studies results of the von Mises truss with elastic horizontal supports deformation according to equation (18)

Studies have confirmed an additional essential sensitivity to the two-rod three-hinged trusses stability loss from the initial inclination angle and at a certain horizontal supports stiffness.

The presented results analysis (Fig. 2) showed that on snapping through the neutral axis which connecting the supports, the rods are stretched. It was also noted that, regardless from small inclination angles, at a given stiffness ratio of the rods and horizontal supports, the truss nonlinear deformation occurs rather quickly. In the stability loss case, any small reduction in the critical load may not lead to a stable state, but will lead to an increase in vertical and horizontal displacements. Therefore, a more detailed dependence graph vertical displacements on the load at different stiffness of horizontal supports is shown in Fig. 3 and Fig. 4.

5. Conclusions. Scientific novelty and practical significance

1. The horizontal supports elasticity influence which is simulating the domes upper tier supporting conditions on the Mises trusses stability has been investigated. A three-hinged truss deformed scheme when a concentrated vertical load is applied to the ridge node is considered. An analytical method is used to obtain a generalized equation for the three-hinged trusses stability criterion to determine the critical load depending on the design system parameters: the rods' inclination angle, the rods' stiffness, and the horizontal elastic supports' stiffness. A two transcendental equations system for the load dependence on vertical and horizontal displacements is obtained, taking into account the rods' compression. Such equations system analytical solution using a generalized parameter - an inclined rods variable angle's tangent, made it possible to obtain one equation for the vertical load dependence on the vertical and symmetric horizontal supports displacement. The truss' numerical stability studies depending on the design geometry of a structure was carried out.
2. The dependence analytical expression of load on the structure reduced to the rod's stiffness on the rods' inclination angle and the supports' horizontal stiffness was obtained. The inclined double-rod three-hinged trusses nonlinear deformation's nature, depending on the elastic supports stiffness and the rods angle was confirmed. It has been established that the inclined two-rod three-hinged systems' deformation nature has the node snapping-through effect. It was found that, depending on the horizontal supports rigidity, and with a decrease in the rods angle, the relative critical load reduced value significantly decreases.
3. On the three-hinged inclined two-rod truss with elastic horizontal supports deformed scheme's theoretical studies basis, was obtained a generalized analytical solution to determine the critical load for such systems. The horizontal supports' elasticity influence regularities on a decrease in the critical load value and a decrease in such systems stability was obtained.
4. The obtained generalized analytical solution models the dome system's annular elements stiffness through the horizontal supports stiffness and determines the lower tier's elements stiffness effect on the dome's uppermost tier structural system's stability.

The obtained analytical equation and a numerical studies has a practical importance, since they allow one to determine the dome's annular elements rigidity's rational design parameters to ensure the upper tier stability.

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СТІЙКІСТЬ ПОЛОГИХ ФЕРМ ФОН МІЗЕСА ПРИ ПРУЖНИХ ГОРИЗОНТАЛЬНИХ ОПОРАХ

Анотація. Мета. Метою роботи є дослідження впливу на стійкість ферм фон Мізеса пружності горизонтальних опор, які моделюють умови спирання верхнього ярусу куполів. **Методика.** Розглянуто деформовану схему тришарнірної ферми при прикладанні в гребеневому вузлі зосередженого вертикального навантаження. Аналітичним методом отримано узагальнене рівняння критерію стійкості тришарнірних ферм для визначення критичного навантаження в залежності від параметрів розрахункової системи: кута нахилу стрижнів, жорсткості стрижнів, жорсткості горизонтальних пружних опор. Отримана система двох трансцендентних рівнянь залежності навантаження від вертикальних і горизонтальних переміщень з урахуванням обтиску стрижнів. Аналітичне рішення такої системи рівнянь через узагальнений параметр – змінний тангенс кута нахилу похилих стрижнів, дозволив отримати одне рівняння залежності вертикального навантаження від вертикального та симетричного горизонтального переміщення опор. Проведено числові дослідження стійкості ферми в залежності від проектної геометрії конструкції. **Результати.** Отримано аналітичний вираз залежності приведеного до жорсткості стрижня навантаження на конструкцію в залежності від кута нахилу стрижнів і горизонтальної жорсткості опор. Підтверджено нелінійний характер деформування положистих двострижневих тришарнірних ферм в залежності від жорсткості пружних опор і кута нахилу стрижнів. Встановлено, що характер деформування двострижневих положистих тришарнірних систем має ефект проклацування гребеневого вузлу. Встановлено, що разом зі зменшенням кута нахилу стрижнів знижується відносне приведене значення критичного навантаження в залежності від жорсткості горизонтальних опор. **Наукова новизна.** На підставі теоретичних досліджень деформованої схеми тришарнірної двострижневої похилої ферми з пружними горизонтальними опорами отримано узагальнене аналітичне рішення таких систем. Отримано закономірності впливу пружності горизонтальних опор на стійкість таких систем. Узагальнене аналітичне рішення моделює через жорсткість горизонтальних опор жорсткість кільцевих елементів купольної системи, та визначає загальний вплив жорсткості елементів нижнього ярусу на стійкість конструктивної системи самого верхнього ярусу купола. **Практична значимість.** Отримане аналітичне рівняння дозволяє визначити раціональні конструктивні параметри кільцевих елементів куполу, для забезпечення стійкості верхнього ярусу.

Ключові слова: стійкість; ферма фон Мізеса, сталевий купол; нелінійні переміщення; деформаційний розрахунок, пружні горизонтальні опори; рівняння критичного навантаження; моделювання; вузлова стійкість куполів.

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THE STABILITY OF LOW-PITCHED VON MISES TRUSSES WITH HORIZONTAL ELASTIC SUPPORTS

Abstract. Purpose. The work’s aim is to study the horizontal supports stiffness impact, which simulate the conditions for supporting the domes upper tier on the von Mises trusses' stability. **Methodology.** A three-hinged truss' deformed scheme under applying a concentrated vertical load in the ridge joint was considered. An analytical method was used to obtain a generalized equation for the three-hinged trusses' stability criterion to determine the critical load depending on the design system's parameters such as the rods' inclination angle, the rods' stiffness, and the horizontal elastic supports stiffness. A two transcendental equations' system for the dependence of the load on vertical and horizontal displacements taking into account the rods' compression was obtained. Such equations' system's analytical solution through a generalized parameter - a variable rods' angle tangent, made it possible to obtain one equation for the dependence of the vertical load on the vertical and symmetric horizontal supports' displacement. The truss' stability numerical studies were carried out depending on the structure's design geometry. **Findings.** An analytical expression of the dependence for the load on the structure, which was reduced to the rod's stiffness depending on the rods' angle to the horizontal stiffness of the supports, was obtained. The low-pitched double-rod three-hinged trusses' nonlinear deformation nature depending on the elastic supports' stiffness and the rods' angle was confirmed. It was found that with the two-rod low-pitched three-hinged systems' nonlinear deformation nature the ridge joint's snap-through effect takes place. It was found that the relative reduced critical load value decreases along with the rods' inclination angle decrease depending on the horizontal supports' stiffness. **Scientific innovation.** On the theoretical studies basis of the three-hinged two-rod low-pitched trusses with elastic horizontal supports deformed scheme a generalized analytical equation for the such systems' solution was obtained. The generalized analytical solution models the dome system annular elements stiffness through the horizontal supports' stiffness and determines the general lower tier elements stiffness effect on the dome uppermost tier structural system stability. **Practical value.** The obtained analytical equation makes it possible to determine the dome annular elements rational design parameters to ensure the upper tier stability.

Keywords: stability; von Mises truss; steel dome; nonlinear displacements; deformational calculation: elastic horizontal supports; the critical load equation; domes' nodal buckling.

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Fig. 4. Ref. 22.

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