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FINITE ELEMENT ANALYSIS OF NONLINEAR DEFORMATION, STABILITY AND VIBRATIONS OF ELASTIC THIN-WALLED STRUCTURES

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Thin-walled shell-type structures are widely used in various branches of technology and industry. Such structures under operating conditions are usually exposed to various loads, including thermomechanical ones. Real shell structures, as a rule, have a complex shapes. To increase reliability, reduce material consumption, for technological reasons, they are designed as inhomogeneous systems in thickness. This causes a great and constant interest of engineers and designers in the problems of investigating the behavior of elastic thin-walled shell structures.

The work is devoted to the method of analysis of geometrically nonlinear deformation, stability, post-buckling behavior and natural vibrations of thin elastic shells of complex shape and structure under the action of static thermomechanical loads. The unified design model has been created on the basis of the developed universal spatial finite element with introduced additional variable parameters. The model takes into account the multilayer material structure and geometric features for structural elements of the thin shell. The shells can be reinforced with ribs and cover plates, weakened by cavities, channels and holes, have sharp bends in the mid-surface.

Such a uniform formulation made it possible to create a unified finite element model of the shells with an inhomogeneous structure. It is shown on a number of problems that the method presented in this article is an effective tool for analyzing geometrically nonlinear deformation, stability, post-buckling behavior and natural vibrations of thin elastic shells of an inhomogeneous structure under the action of static thermomechanical loads.

Key words: thin inhomogeneous shell, universal space finite element, geometrically nonlinear deformation, buckling, vibration, thermo-mechanical load.

Introduction. Thin-walled shell-type structures are widely used in modern construction, mechanical engineering and instrument making, rocket and space technology, and many other industries. Such structures are usually exposed during operation to various loads, including thermomechanical ones. These circumstances are the reason for a great and constant interest of engineers and designers in the problems of analyzing the behavior of shell systems. There are many numerical methods suitable for efficiently solving particular problems [1-8]. Recently, the finite element method has been recognized as one of the most frequently used and effective numerical methods, due to its versatility, physicality and unlimited applicability to complex structures under arbitrary loading.

Real shell designs are usually not limited to the classical canonical forms. They are often structures of complex shapes. To increase reliability, reduce material consumption, for technological reasons, such structures are designed in the form of inhomogeneous systems: smooth and stepwise-varying thickness, reinforced with ribs and cover plates, weakened by holes, cavities and channels, faceted, multilayer. Shells are often subjected to mechanical and thermal stress. In this case, temperature fields can cause significant deformations and affect the shape of buckling and critical loads quantity.

The work is devoted to the method of analysis of geometrically nonlinear deformation, stability, post-buckling behavior and natural vibrations of thin elastic shells of complex shape and structure under the action of static thermomechanical loads.

1. Problem statement and method of its solving. The static problems of the stress-strain state (SSS), stability and post-buckling behavior of a wide class of thin inhomogeneous shells under the action of external mechanical loads and uneven volumetric heating are considered. Determination of natural vibrations of inhomogeneous shells is carried out at each stage of thermomechanical loading, taking into account the prestressed state. This approach allows, within the framework of one algorithm, to determine the critical loads of the shells using both the static and dynamic criterion of buckling. By the inhomogeneity of a shell is meant that (i) its thickness is continuously or stepwise variable and (ii) it consists of combinations of multilayer stacks along the thickness and in plan.

The method for solving static problems of nonlinear deformation and buckling of various shells subject to mechanical and thermal loads has been developed on the basis of the unified methodological positions of the 3-D geometrically nonlinear theory of thermoelasticity and the use of the moment finite-element scheme (MFES). Detailed outline of the method, justification of its reliability, solution of a variety of problems is given in [9, 10]. The shell is modeled by a nonlinear elastic continuum subject to large displacements and small strains whose components are linear functions of stresses. The layers of the shell are considered linear elastic and described by the generalized Duhamel–Neumann law. To develop a finite element model of the shell, we approximate a thin shell by one spatial FE throughout the thickness, which is an efficient approach. So we use the so-called one-layer FE approximation throughout the shell thickness. The difficulties of describing the combined behavior of structural elements with different dimensionality in an inhomogeneous shell are overcome by using the 3-D FEs of the same type to model sections with stepwise-varying thickness. The universal FE is based on an isoparametric spatial FE with polylinear shape functions for coordinates and displacements. Additional variable parameters are introduced to enhance the capabilities of the modified FE.

Two hypotheses are used to describe the SSS of a thin inhomogeneous shell. The nonclassical kinematic hypothesis of deformed straight line: though stretched or shortened during deformation, a straight segment along the thickness remains straight. This segment is not necessarily normal to the mid-surface of the shell. The static hypothesis assumes that the compressive stresses in the fibers of the n-th layer are constant throughout the thickness. The use of this hypothesis does not deprive the stress state of an inhomogeneous shell of its three-dimensional properties. Finite element models, constructed on the basis of the developed modified element and the use of the FEMS, have stable indicators of convergence of solutions, both for thin shells and shells of medium thicknesses.

In the problems of natural vibrations of the shell, the presence of prestress in the deformed structure from the action of various static loads is taken into account [11, 12]. The presence of shell prestressing significantly affects the spectrum of natural vibrations. This approach makes it possible to analyze, at each step of thermomechanical loading, small vibrations of the shells relative to the reference deformed state, caused by an arbitrary static load, taking into account large displacements and the presence of a pre-stressed state. Thus, the adopted approach allowed to develop a universal methodology for studying the stress-strain state, stability, post-buckling behavior and vibrations of shell structures of various classes. Their list is determined by the type of structural elements characterizing the shell: constant, smoothly or stepped-variable thickness, ribs, cover plates, inserts, cavities, channels, holes, fractures of the middle surface, multilayer material.

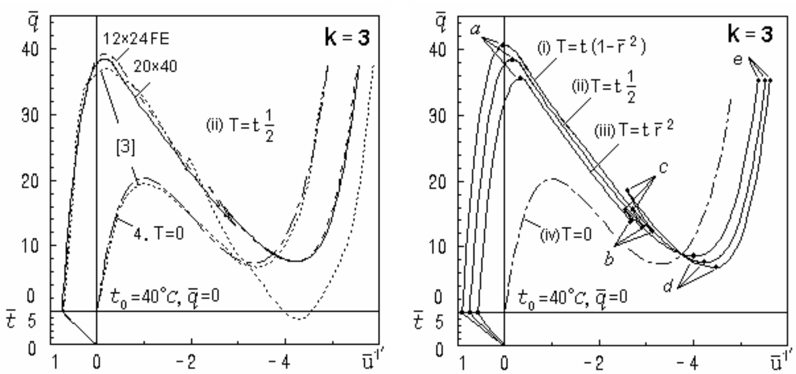
2. Analysis of stability and post-buckling behavior of inhomogeneous shells. The versatility of the developed method requires proving the reliability of solutions for various classes of problems falling within the scope of its application. Justification of the reliability of solutions by studying the convergence of the results and their comparison with known nonlinear solutions has been proven on a number of specially selected problems [9, 10]. The effectiveness and versatility of the method is demonstrated below on several problems of nonlinear deformation and stability of shells of different classes.

2.1. Conical panel in a nonuniform temperature field. The convergence and accuracy of nonlinear solutions are analyzed for a clamped axisymmetric shallow conical panel subject both temperature and mechanical fields. The numerical solution of this problem has been obtained with the use of a variational method by B.Ya. Kantor [3]. The effect of the thermomechanical load on the panel consists of two stages: (i) the SSS of the shell is perturbed by the temperature field $T(t,\bar{r})$, whose parameter t increases to a set value t_0 and (ii) the panel is subjected to pressure, the temperature field remaining constant. Three options of temperature field that is constant throughout the thickness h and uniform or nonuniform along the radius r are examined:

$$\text{(i) } T(t,\bar{r}) = t(1-\bar{r}^2)\,; \quad \text{(ii) } T(t,\bar{r}) = t\frac{1}{2}\,; \quad \text{(iii) } T(t,\bar{r}) = t\bar{r}^2\,.$$

The results are presented in dimensionless form: $k = H/h = 5$, $\bar{t} = t\alpha(a/h)^2 = 5$ (a – radius of support boundary, h – thickness, H – rise, α – coefficient of linear thermal expansion, the material is isotropic).

Good agreement of the solutions for the value of the upper critical load $\bar{q}_{cr}^{up}$ is obtained. The upper critical load $\bar{q}_{cr}^{up}$ is in good agreement with the axisymmetric solution [3] (error is $2{,}86 \div 3{,}75\%$, Fig. 1). A complete coincidence of the diagrams "$q-u$" up to the upper critical point and their gradual divergence in the supercritical area (Fig. 1,a) have been obtained when heating was uniform (variant (i)). Comparison of solutions gives their complete coincidence when only pressure is present (variant $T=0$). The heating causes deformation opposite to that induced by pressure. Therefore, in all the cases of preheating, the stiffness of the panel increases considerably and $\bar{q}_{cr}^{up}$ increases by a factor of 1.75 to 2 compared with the nonheated shell ($T=0$) (Fig. 1,b).



(a) (b)

Fig. 1

2.2. Shells of revolution with linearly varying thickness. The issue of more rationally distributed material over the volume of the structure is considered. The stability of spherical shallow panels with linearly varying thickness clamped at the edge and subjected to pressure q is analyzed. The effect of thickness variation parameter $h(r)$ on the stability of panels is examined using three linear dependencies:

(i) $\bar{h}(\bar{r}) = 1+(b_o-1)\bar{r}$, (ii) $\bar{h}(\bar{r}) = 1+(b_a-1)(1-\bar{r})$, (iii) $\bar{h}(\bar{r}) = b_V$,

where $b_o = h_{\bar{r}=1}/h^*$, $b_a = h_{\bar{r}=0}/h^*$ and $b_V = h_V/h^*$ are parameters of dimensionless thickness $\bar{h} = h/h^*$ along the radius $\bar{r} = r/a$. The value $b_o = b_a = b_V = 1$ corresponds to a panel of constant "base" thickness h^* and volume V^*. In the 1st case, the thickness in the center of the panel takes the "base" value ($h_{\bar{r}=0} = h^*$) and at the edge it is given by b_o. In the 2nd case, the thickness at the edge is of the "base" value ($h_{\bar{r}=1} = h^*$) and in the center it is given by b_a. In the 3rd case, the thickness h_V is determined through the volume of the panel V : $h_V = V/(2\pi HR)$.

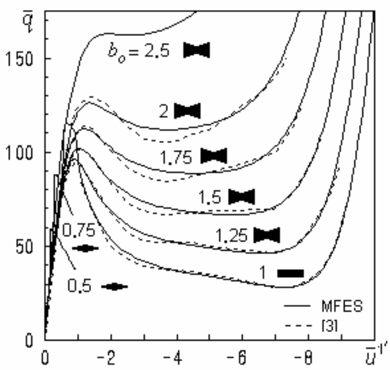


Fig. 2

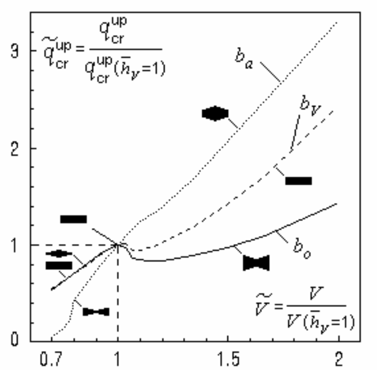
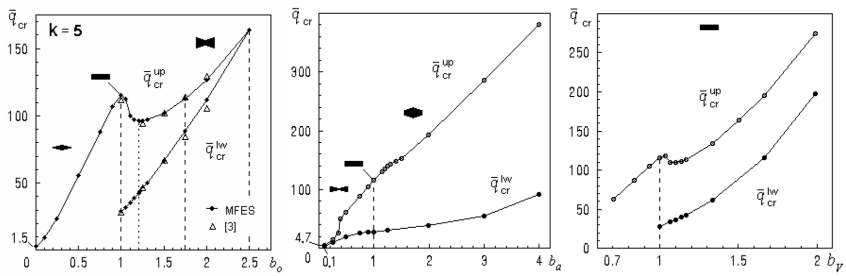


Fig. 3

Comparing the results for panels with the (i) law of variation in thickness with the results from [3] reveals an insignificant difference between the upper $\bar{q}_{cr}^{up}$ critical loads (2.4–3.3%) and between the lower $\bar{q}_{cr}^{lw}$ critical loads (0.2–5.8%) and complete agreement between the curves "$q-u$" on all sections (Fig. 2). For the (i) law, $\bar{q}_{cr}^{up}$ depends nonlinearly on b_o (Fig. 4,a). When $1 < b_o \leq 1{,}75$, an increase in the mass of the panel does not lead to an increase in $\bar{q}_{cr}^{up}$. This nonlinear effect is due to the dependence of the buckling modes on the parameter b_o (Fig. 5-6). For the (ii) law, the dependence of $\bar{q}_{cr}^{up}$ on b_a is nearly linear (Fig. 4,b). For the (iii) law, $\bar{q}_{cr}^{up}$ depends nonlinearly on b_V in a similar manner when $1{,}05 < b_V \leq 1{,}167$ (Fig. 4,c). The material is more rationally used in panels that are thicker in the middle (Fig. 3).



(a) (b) (c)

Fig. 4

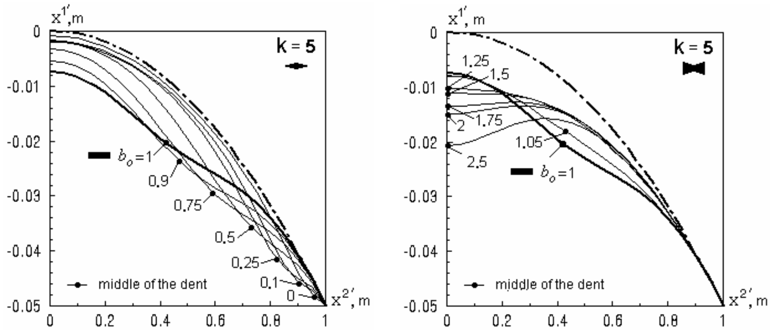
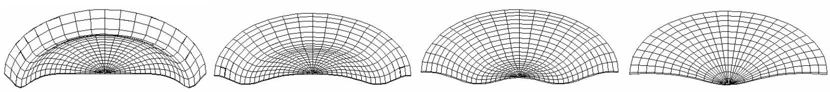


Fig. 5



$0 \le b_o < 0.5$ $(b_o = 0.1$) $0.5 \le b_o \le 1.2$ $(b_o = 0.5$) $b_o = 1$ $(b_o = 1$) $1.2 \le b_o \le 2.5$ $(b_o = 2$)

Fig. 6

2.3. Effect of heating on the buckling of smooth and faceted panels. Curvilinear shapes are what make the manufacturing of shells difficult. In practice, this problem is solved by changing the smooth curvilinear shape with a faceted one, the flat elements of which are made from an assortment of standard sheet products. Let us consider the above-considered spherical panels of constant thickness ($h = 0{,}01$; $0{,}013$; $0{,}02\,\text{m}$). The shell heated to $T^\circ C$ (-10°, 0°, 20°, 100°) is loaded with pressure. The dependences of the relative values $\tilde{q}_{cr}^{up}$ and $\tilde{V}$ are shown in Fig. 8, where $q_{cr}^{up}(h^*, T = 0^\circ)$ is the maximum load of a smooth unheated panel with a thickness h^*.

The replacement of the curvilinear shape with a faceted one causes a minor alteration of the SSS and $\tilde{q}_{cr}^{up}$ (Fig. 8, curve $T = 0^\circ C$), which increases by 1% as the volume decreases by 2.5% because the panels are shallow. The effect of preheating on the stability of smooth and faceted panels of constant thickness is examined (Fig. 8). In all cases, the critical loads of faceted panels are slightly greater than those of the smooth panel: the upper loads by 3–13,1% and the lower loads by 2,4–7,4%. The investigation of the stress state of the panels shows that the transition to a faceted structure is characterized by a qualitative redistribution of stress fields: from axisymmetric to cyclically symmetric in accordance with the location of the edges.

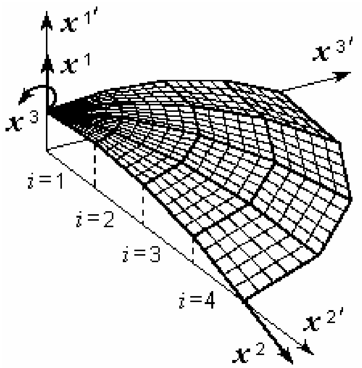


Fig. 7

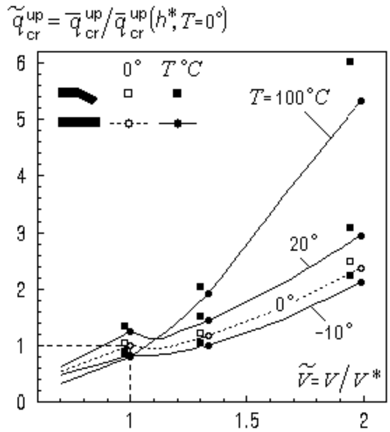


Fig. 8

2.4. Behavior of ribbed shells under the various thermomechanical loadings. The stability of shells, reinforced by ribs, is considered in this paragraph. The object is a shallow square spherical panel with a curvature parameter $K = 2a^2/(Rh)$ (where $K = 32$, thickness of the casing $h = 0{,}01$ m, panel size $a = 60h$, radius $R = 225h$). The panel is hinged at the edges.

Consider panels reinforced, from inside, with two cross ribs (height $h_r = 3h$ without casing thickness h, width $b_r = 2h$, length $a = 60h$) and subjected to pressure and heating (the casing is heated by T_c and the ribs by T_r degrees). The heating of the casing and ribs by 40°C is terminated at $\bar{q} = \bar{q}_{cr}^{up0}$, which is the critical load of the ribbed panel subject to pressure alone ($T = 0^\circ C$). Four cases of thermomechanical loading are examined:

(i) $\bar{q}$, $T_c = 0^\circ$C, $T_r = 0^\circ$C – only pressure (for reference);

(ii) $\bar{q}$, $T_c = 0^\circ$C, $T_r = 40^\circ$C – heating only the ribs;

(iii) $\bar{q}$, $T_c = 40^\circ$C, $T_r = 0^\circ$C – heating only the casing;

(iv) $\bar{q}$, $T_c = 40^\circ$C, $T_r = 40^\circ$C – heating the casing and the ribs.

The pressure causes the ribbed shallow panel to snap through in the middle (Fig. 10, (i)). In all cases with heating the stiffness of the panels increases and there is no buckling. The panel becomes the stiffest when both casing and ribs are simultaneously heated (Fig. 10, (iv)) and least stiff when only ribs (Fig. 10, (ii)) are heated. The instant the terminated heating ($\bar{q} = \bar{q}_{cr}^{up0}$) is represented by the salient point "•" on the curves. With further loading by pressure, the deformation process stabilizes – the curves "$q-u$" merge (Fig. 9). Deformation of heated shells occurs with an insignificant predominance of the membrane component of the deformation energy in comparison with its bending component (Fig. 11).

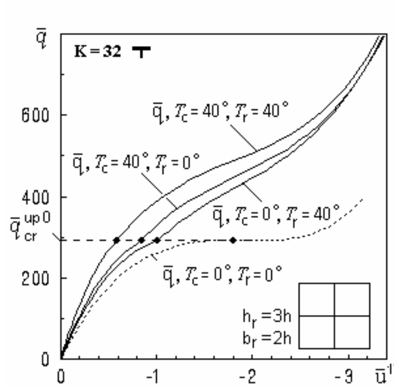


Fig. 9

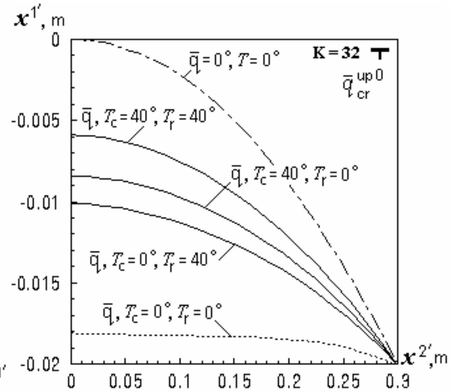


Fig. 10

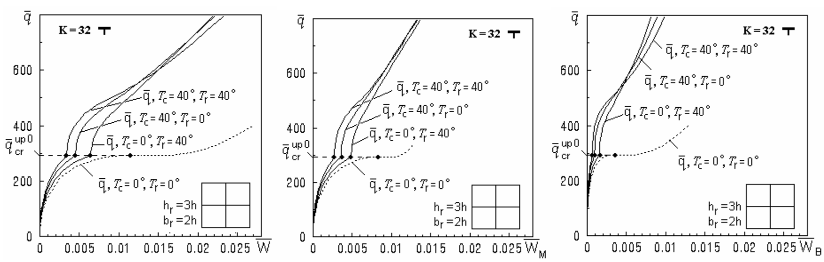


Fig. 11

2.5. Effect of the way of thermomechanical loading on the behavior of shells with channels. The effect of the weakening parameters (channels) on the stability of the considered above panels under thermomechanical loads is investigated. Consider a shallow panel ($K = 32$, $a = 60h$) weakened with four identical cross channels (width $b_{ch} = 2h$, $h_{ch} = 0{,}3h$). The panel is hinged at the edge.

Examination of the eccentric positions of the channels relative to the middle surface of the casing shows a greater weakening effect when they are located on the outer surface "▄▂▄" (Fig. 12). A dash-dotted line marked with "▬" is the solution for a smooth panel, for comparison.

Tree cases of thermomechanical loading are examined for the panel which casing weakened from inside "▀▔▀" (Fig. 13):

(i) pressure (for reference), $T = 0^\circ\mathrm{C}$;

(ii) preheating by $T = 40^\circ\mathrm{C}$ followed by pressure at constant temperature;

(iii) simultaneous pressure and heating ($T = 40^\circ\mathrm{C}$) until the upper critical point of the 1st case $\bar{q} = \bar{q}_{cr}^{up0}$ is reached.

With heating, $\overline{q}_{cr}^{up}$ increases by 17.7% (i) and 68.3% (ii), respectively, in all the cases. When heat and pressure ((iii) case) act simultaneously, $\overline{q}_{cr}^{up}$ increases by 43.0% compared with the case ((ii) case) where they act sequentially. In this case, $\overline{q}_{cr}^{lw}$ increases by a factor of 12. These effects are due to the increased stiffness of the heated shell.

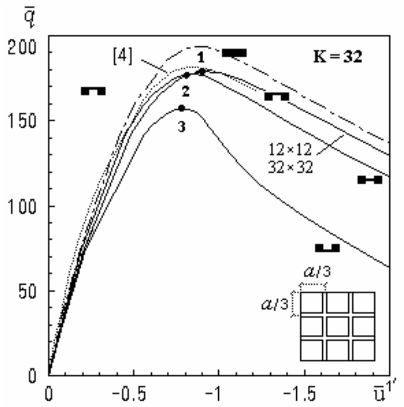


Fig. 12

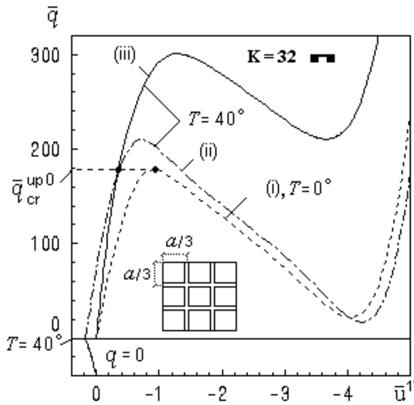


Fig. 13

3. Modal analysis of shell structures. It is known that static loads significantly affect both the stress-strain state of the structure and the dynamic characteristics, which include the frequencies and mode shapes of natural vibrations. Investigation of the effect of static load on vibrations of shells, even of constant thickness, is a complex and insufficiently studied problem of structural mechanics. The results of the study of buckling and vibrations of thin shells of constant and stepped-variable thickness under the action of thermomechanical loads are reproduced below.

3.1. Effect of static loads on natural vibrations of ribbed shells.

Consider the rib-reinforced shallow spherical panel square in plan ($K = 32$, $a = 60h$, $R = 225h$), hinged at the edges, and subject to pressure. The panel is reinforced, from inside, with two cross ribs (height $h_r = 3h$ without casing thickness h, width $b_r = 2h$, length $a = 60h$).

The dependences of the characteristics of natural vibrations on the growth of the static load are obtained.

Comparison the curves "load – deflection" ("$\overline{q} - u$") (Fig. 14) and "load – frequency" ("$\overline{q} - \omega_1$") (Fig. 15) for smooth ("▬") and ribbed ("┳") panels shows the following. The panel mass increase by 19.3% due to the setting of two ribs increases the critical load value $\overline{q}_{cr}^{up}$ by 1.5 times in comparison with a smooth panel and leads to a decrease of the vibration frequency ω_1 in the initial state by 7.5%. The frequency ω_1 for the ribbed panel becomes higher than for the smooth one when the load parameter value $\overline{q} \geq 80$.

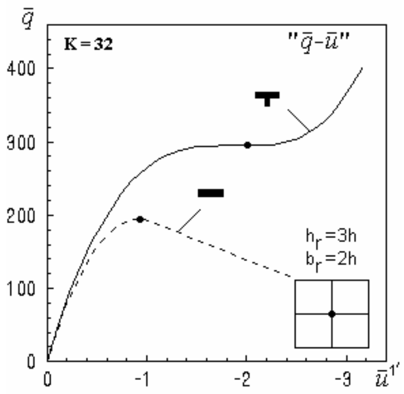


Fig. 14

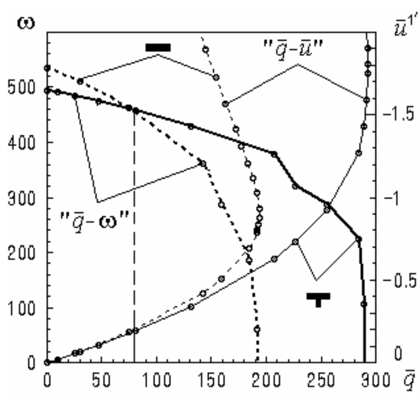
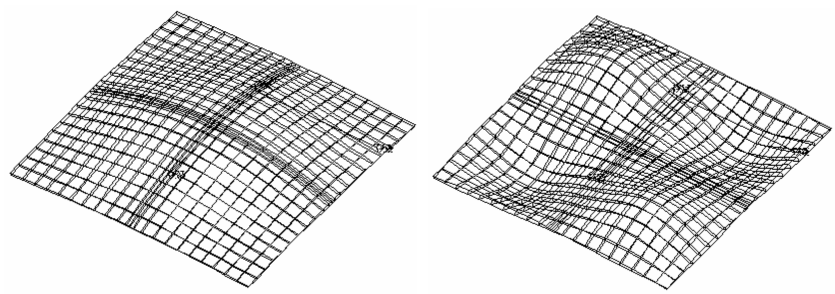


Fig. 15

The vibration mode of the ribbed panel corresponding to the frequency ω_1 has a simple form (Fig. 16,a) in the prebuckling domain. The vibration mode near the critical load is characterized by skew-symmetric deformation with a maximum amplitude in the center of the quarters (Fig. 16,b).



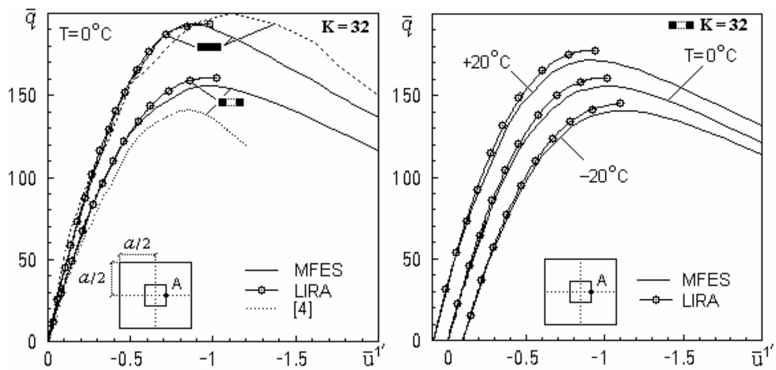
(a) (b)

Fig. 16

3.2. Effect of thermomechanical loading on buckling and vibrations of a panel with a hole. Consider the shallow spherical panel square in plan ($K = 32$, $a = 60h$, $R = 225h$), hinged at the edges. The panel has a central square hole of width $b_{hl} = 12h$. The effect of three cases of preheating ($T = -20^\circ, 0^\circ, 20^\circ \text{C}$) on the stability and vibrations of the shell is analyzed.

The result of a smooth panel calculations ("▬") serves as the basis for the analysis of the influence of the hole geometric features ("■::■") on the behavior of a shallow shell. The deflection of the smooth panel is calculated at its center where as the deflection of the panel with a hole is determined at the point A.

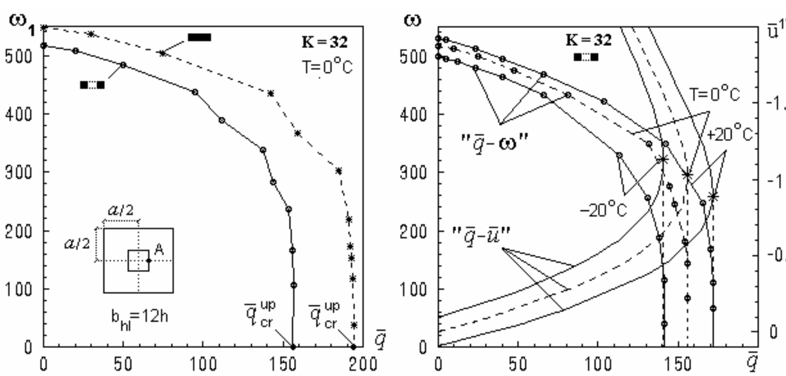
The accuracy of calculations in solving static stability problems has been determined by a comparative analysis of the two solutions. The first one has been obtained by the authors using the MFES while the second one has been obtained using the LIRA software [13] (Fig. 17). Under the pressure alone ($T = 0^\circ \mathrm{C}$), the weakening of the smooth panel reduces the critical load $\overline{q}_{cr}^{up}$ by 19.5%. (Fig. 17,a). Pre-cooling and pre-heating leads to a change of the critical load $\overline{q}_{cr}^{up}$ by -9,97 and 9,78% compared to the corresponding unheated panel ($T = 0^\circ \mathrm{C}$) for the shell with the hole (Fig. 17,b).



(a) (b)

Fig. 17

The "$\overline{q} - \omega_1$" curves look similar for smooth and perforated panels subject to pressure alone ($T = 0^\circ \mathrm{C}$) (Fig. 18,a). Load quantities at which natural vibrations are calculated, shown in the figure by circles. For all heating cases, the "$\overline{q} - \omega_1$" curves look similar as well (Fig. 18,b).

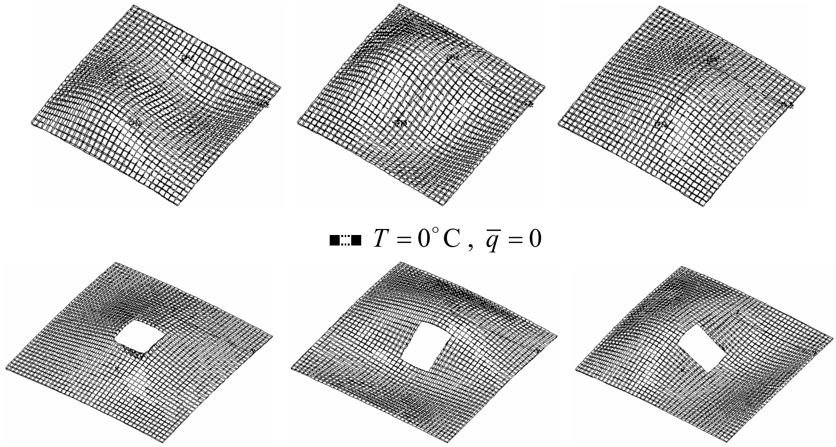


(a) (b)

Fig. 18

▬ $T = 0^\circ\mathrm{C}$, $\bar{q} = 0$

■::■ $T = 0^\circ\mathrm{C}$, $\bar{q} = 0$



ω_1 ω_2 ω_3

Fig. 19

In the initial state ($T = 0^\circ\mathrm{C}$, $\bar{q} = 0$) the frequencies ω_1 and ω_2 are double for a smooth shell, and frequencies ω_2 and ω_3 are double for a panel with a hole. Therefore, the mode shapes differ for the respective shells (Fig. 19). So, the mode shapes that correspond to double frequencies ω_1 and ω_2 of the smooth panel are conjugate, and the mode that corresponds to the frequency ω_3 is characterized by the oscillation of its central part. The opposite nature of the mode shapes is observed for a panel with a hole. The applied pressure causes a restructuring of both the frequency multiples and the vibration modes. During loading, the vibration modes are transformed in accordance with the change in multiple frequencies [12].

Conclusions.

The method for analysis of geometrically nonlinear deformation, stability, post-buckling behavior and natural vibrations of thin elastic shells of complex shape and structure under the action of static thermomechanical loads is presented. It is based on the geometrically nonlinear equations of 3-D thermoelasticity, the finite element formulation of the problem in increments, and the use of the moment finite-element scheme. The prestressed state of the deformable shell is taken into account at each stage of thermomechanical loading when carrying out modal analysis of the shell.

We show that the method presented in this article makes it possible to analyze effectively the behavior of a wide class of thin elastic shells of an inhomogeneous structure under the action of various static thermo-force loads.

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СКІНЧЕННОЕЛЕМЕНТНИЙ АНАЛІЗ НЕЛІНІЙНОГО ДЕФОРМУВАННЯ, СТІЙКОСТІ ТА КОЛИВАНЬ ПРУЖНИХ ТОНКОСТІННИХ КОНСТРУКЦІЙ

Актуальність. Тонкостінні конструкції оболонкового типу широко використовуються в будівництві та різних галузях техніки. В умовах експлуатації такі конструкції зазвичай піддаються впливу різних навантажень, в тому числі і термосилових. Реальні оболонкові конструкції, як правило, є конструкціями складної форми, які для підвищення надійності, зниження матеріаломісткості, з технологічних міркувань проектуються у вигляді неоднорідних по товщині оболонкових систем. Це обумовлює великий і постійний інтерес інженерів і конструкторів до задач дослідження поведінки пружних тонкостінних оболонкових конструкцій. **Мета роботи.** Робота присвячена методиці аналізу геометрично нелінійного деформування, стійкості, закритичної поведінки і власних коливань тонких пружних оболонок складної форми і структури при дії статичних термосилових навантажень. На базі розробленого універсального просторового скінченного елемента з введеними додатковими змінними параметрами побудована розрахункова модель, яка враховує геометричні особливості конструктивних елементів і неоднорідність матеріалу тонкої оболонки (змінність товщини, злами і гранований обшивки, ребра, накладки, виїмки, отвори, вставки, багатошарову структуру матеріалу). **Результати.** Застосований уніфікований підхід дозволив створити єдину розрахункову скінченно-елементну модель оболонки неоднорідної структуру. На низці прикладів показано, що метод, наведений у цій статті, дозволяє ефективно досліджувати геометрично нелінійне деформування, стійкість, закритичну поведінку і власні коливання тонких пружних оболонок неоднорідної структури при дії статичних термосилових навантажень.

Ключові слова: тонка неоднорідна оболонка, універсальний просторовий скінченний елемент, геометрично нелінійне деформування, стійкість, коливання, термосилове навантаження.

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КОНЕЧНО-ЭЛЕМЕНТНЫЙ АНАЛИЗ НЕЛИНЕЙНОГО ДЕФОРМИРОВАНИЯ, УСТОЙЧИВОСТИ И КОЛЕБАНИЙ УПРУГИХ ТОНКОСТЕННЫХ КОНСТРУКЦИЙ

Актуальность. Тонкостенные конструкции оболочечного типа широко используются в строительстве и различных отраслях техники. В условиях эксплуатации такие конструкции обычно подвергаются воздействию различных нагрузок, в том числе и термосиловых. Реальные оболочечные конструкции, как правило, являются конструкциями сложной формы, которые для повышения надежности, снижения материалоемкости, по технологическим соображениям проектируются в виде неоднородных по толщине оболочечных систем. Это обуславливает большой и постоянный интерес инженеров и конструкторов к задачам исследования поведения упругих тонкостенных оболочечных конструкций. ***Цель работы.*** Работа посвящена методике анализа геометрически нелинейного деформирования, устойчивости, закритического поведения и собственных колебаний тонких упругих оболочек сложной формы и структуры при действии статических термосиловых нагрузок. На базе разработанного универсального пространственного конечного элемента с введенными дополнительными переменными параметрами построена расчетная модель, которая учитывает геометрические особенности конструктивных элементов и неоднородность материала тонкой оболочки (переменность толщины, изломы и граненость обшивки, ребра, накладки, выемки, отверстия, вставки, многослойную структуру материала). ***Результаты.*** Примененный унифицированный подход позволил создать единую расчетную конечно-элементную модель оболочки неоднородной структуры. На ряде примеров показано, что метод, приведенный в настоящей статье, позволяет эффективно исследовать геометрически нелинейное деформирование, устойчивость, закритическое поведение и собственные колебания тонких упругих оболочек неоднородной структуры при действии статических термосиловых нагрузок.

Ключевые слова: тонкая неоднородная оболочка, универсальный пространственный конечный элемент, геометрически нелинейное деформирование, устойчивость, колебания, термосиловая нагрузка.

УДК 539.3
Кривенко О.П., Ворона Ю.В., Козак А.А. **Скінченноелементний аналіз нелінійного деформування, стійкості та коливань пружних тонкостінних конструкцій** / Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2021. – Вип. 107. – С. 20-34. – Англ.

Наведено метод аналізу геометрично нелінійного деформування, стійкості, закритичної поведінки і власних коливань тонких пружних оболонок складної форми і структури при дії статичних термосилових навантажень. На низці задач продемонстровано ефективність методу.

Табл. 0. Іл. 19. Бібліогр. 13 назв.

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Krivenko O.P., Vorona Yu.V., Kozak A.A. **Finite element analysis of nonlinear deformation, stability and vibrations of elastic thin-walled structures** / Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – K.: KNUBA, 2021. – Issue 107. – P. 20-34.
The method for analyzing geometrically nonlinear deformation, stability, post-buckling behavior and natural vibrations of thin elastic shells of complex shape and structure under the action of static thermomechanical loads is presented. The effectiveness of the method was demonstrated on a number of problems.
Tabl. 0. Fig. 19. Ref. 13.

УДК 539.3
Кривенко О.П., Ворона Ю.В., Козак А.А. **Конечно-элементный анализ нелинейного деформирования, устойчивости и колебаний упругих тонкостенных конструкций** / Сопротивление материалов и теория сооружений: науч.-тех. сборн. – К.: КНУСА, 2021. – Вип. 107. – С. 20-34. – Англ.
Приведен метод анализа геометрически нелинейного деформирования, устойчивости, закритических поведения и собственных колебаний тонких упругих оболочек сложной формы и структуры при действии статических термосиловых нагрузок. На ряде задач продемонстрирована эффективность метода.
Табл. 0. Ил. 19. Библиогр. 13 назв.

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