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TRANSIENT CHAOS IN PLATFORM-VIBRATOR WITH SHOCK

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Platform-vibrator with shock is widely used in the construction industry for compacting and molding large concrete products. Its mathematical model, created in our previous work, meets all the basic requirements of shock-vibration technology for the precast concrete production on low-frequency resonant platform-vibrators. This model corresponds to the two-body 2-DOF vibro-impact system with a soft impact. It is strongly nonlinear non-smooth discontinuous system. This is unusual vibro-impact system due to its specific properties. The upper body, with a very large mass, breaks away from the lower body a very short distance, and then falls down onto the soft constraint that causes a soft impact. Then it bounces and falls again, and so on. A soft impact is simulated with nonlinear Hertzian contact force. This model exhibited many unique phenomena inherent in nonlinear non-smooth dynamical systems with varying control parameters. In this paper, we demonstrate the transient chaos in a vibro-impact system. Our finding of transient chaos in platform-vibrator with shock, besides being a remarkable phenomenon by itself, provides an understanding of the dynamical processes that occur in the platform-vibrator when varying the technological mass of the mold with concrete. Phase trajectories, Poincaré maps, graphs of time series and contact forces, Fourier spectra, the largest Lyapunov exponent, and wavelet characteristics are used in numerical investigations to determine the chaotic and periodic phases of the realization. We show both the dependence of the transient chaos on the control parameter value and the sensitive dependence on the initial conditions. We hope that this analysis can help avoid undesirable platform-vibrator behaviour during design and operation due to inappropriate system parameters, since transient chaos may be a dangerous and unwanted state of a vibro-impact system.

Keywords: platform-vibrator, vibro-impact, technological mass, mold with concrete, transient chaos, dependence on initial conditions.

Keep an eye on the potential appearance transient chaos since this phenomenon is an inexhaustible source of challenge and inspiration.

TámasTél [1]

1. Introduction

Platform-vibrator with shock is an equipment widely used in the construction industry for compaction and molding large-sized concrete products. Its appearance is shown in Fig. 1.

In [2], the basic requirements of the shock-vibration technology for the precast concrete production on low-frequency resonant platform-vibrators are described. We have described in detail the creation of a mathematical model of a platform-vibrator that uses shock to produce asymmetric oscillations. It was shown that the created model meets all the basic requirements for a real machine. It provides: T-periodic steady-state movement after passing the transient process; the appropriate value of mold oscillations amplitude A= 0.76 mm; the satisfactory value of the asymmetry coefficient – the ratio of lower acceleration to the upper acceleration $\frac{w_L}{w_U} = 3.6$.


Fig. 1. Appearance of the platform-vibrator with shock that widely used in the construction industry for compaction and molding large concrete products

The created mathematical model corresponds to the two-body 2-DOF vibro-impact system (Fig. 3). It is strongly nonlinear non-smooth discontinuous system. This is unusual vibro-impact system due to its specific properties. The upper body (mold with concrete) with a very large mass breaks away from the lower body (platform-vibrator table with attached rubber gasket) at a very short distance during vibrational motion. Both bodies move separately and then the upper body falls down onto the soft constraint. The impact that occurs is soft one due to the softness and flexibility of the gasket. The soft impact simulation requires special discussion. After comparing simulations by different methods [3], in particular, linear and nonlinear interactive contact forces, we decided to simulate a soft impact with a nonlinear contact force in accordance with the Hertzian quasistatic contact theory [4, 5].

This model turned out to be appropriate for numerical investigations of a variety of chaotic phenomena. It exhibited many unique phenomena inherent in nonlinear non-smooth dynamical systems with varying control parameters [3, 6]. We have observed chaotic motion, boundary and interior crises, crisis-induced intermittency, coexisting regimes in the hysteresis zone, and transient chaos. The exciting frequency, the technological mass of the upper body (mold with concrete), and the stiffness of vibro-isolating spring were chosen as control parameters.

These phenomena are widely discussed in the scientific literature [7-10].

In this paper, we want to demonstrate precisely the transient chaos in a vibro-impact system.

To our knowledge, there were no prior results on transient chaos in platform-vibrator with shock. This type of example is observed for the first time in the literature. Our finding of transient chaos in platform-vibrator with shock, besides being a remarkable phenomenon by itself, provides an understanding of the dynamical processes that occur in the platform-vibrator when varying the technological mass of the mold with concrete.

This phenomenon is often observed in many theoretical, numerical simulation and experimental investigations. Transient chaos is a common phenomenon of many engineering, physical and biological systems. There are many experimental evidence of transient chaos.

Transient chaos arises and finds applications in a wide variety of disciplines such as physics, chemistry, biology, engineering, economics, and even social sciences. There are many works about it in the world scientific literature [11-20]. These articles consider the emergence of transient chaos and its analysis in different dynamical systems in various branches of science.

In a large article [11] with excellent Figures, a solid analysis of transient chaos in optomechanics is given. The authors find that transient chaos, besides being a physically meaningful phenomenon by itself, provides a resolution of breakdown of quantum-classical correspondence.

The transient chaos regime in a two-dimensional system with discrete time (Hénon map) is considered in [12] by Russian authors from Saratov State University.

In [13], transient chaos in fractional Bloch equations is described. The authors believe that it is very important to study the non-linear Bloch equation in order to better understand the conditions that affect the development of chaos.

In [15], hidden transient chaotic attractors of Rabinovich-Fabrikant system are considered.

The authors believe that the doubly transient chaotic behavior analyzed in [16] is both surprising and significant.

In [17], the authors show the important role of chaotic transients in Celestial Mechanics through the Sitnikov problem.

In [19], the authors have presented the interesting phenomena of transient chaos in a system of three, four and six globally coupled nearly conservative Hamiltonian Duffing oscillators. They have also presented the experimental evidence of transient chaos.

It is shown in [20] that chaotic saddles are responsible for chaotic transients and intermittency in high-dimensional spatiotemporal chaotic systems.

These articles often use the term chaotic saddle. There is an object in the phase space, the chaotic saddle, that is responsible for transient chaos [17]. In a large, comprehensive tutorial [18], they are defined in this way. A nonattracting set, which exists in phase space and is responsible for chaos, is a well-defined fractal, although it is more rarefied than chaotic attractors. This type of chaos is called transient chaos, and the underlying nonattracting set in invertible systems is a chaotic saddle.

Two known scientists who have studied the transient chaos for many years have published a large comprehensive research monograph [21].

They define the transient chaos in such manner. Transient chaos is a phenomenon exhibited by deterministic nonlinear dynamical systems, wherein trajectories starting from randomly chosen initial conditions appear chaotic up to certain time, and then switch over, often quite abruptly, into a final periodic state that governs all the rest of the signal. Then they clarify: transient chaos is the form of chaos due to nonattracting chaotic sets in the phase space. And once again they emphasize: “We accept the definition, used throughout the book, that transient chaos is the dynamics associated with nonattracting chaotic sets”.

The difference between sustained and transient chaos lies in the actual value of average lifetime $\langle \; \rangle$. It is infinite for sustained chaos, but finite for transient one. The lifetime of a transient chaos strongly depends on the initial condition. The average lifetime can be obtained from an ensemble of several observations, although for individual observations, the actual lengths of transients depend sensitively on initial conditions: nearby trajectories typically have drastically different lifetimes. The sensitive dependence on the initial conditions is the basic feature of chaotic dynamics.

It was discovered by the famous scientist E. Lorenz in 1963. He was a theoretical meteorologist. He simulated atmospheric flows and obtained an unexpected result that led him to a powerful insight about the way nature works: small changes in initial data can have large consequences. The idea came to be known as the “butterfly effect”. He titled his paper "Predictability: Does the Flap of a Butterfly's Wings in Brazil Set Off a Tornado in Texas?" [22]. And the butterfly effect, i.e., sensitive dependence on initial conditions, has a profound corollary: forecasting the future can be nearly impossible.

Fig. 2 shows the phase trajectories for the Lorenz model. The trajectory outlines a figure, which shape resembles the two butterfly wings. The system goes through a completely predictable loop in one wing and then makes a transition from one wing to another, always unexpectedly and unpredictably.

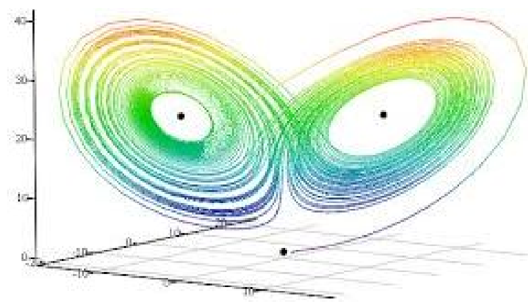


Fig. 2. Two butterfly wings – phase trajectories for Lorenz model. They served as the basis for the term “butterfly effect”, meaning the sensitive dependence of chaotic motion on initial conditions – the main feature of chaotic dynamics

Transient chaos often precedes the birth of permanent chaos.

In [15], the author writes that transient chaos is ubiquitous in chaotic systems. The author warns that the dynamics on systems with chaotic transients can be unpredictable even finally the system falls into a very simple motion. So, he notes that transient chaos can be quite disastrous and therefore unwanted, and it can be the cause of catastrophic developments in a dynamic system. Therefore, control of transient chaos can be desirable in some cases.

A systematic investigation of transient chaos began in the late 1970s. A comprehensive investigation of transient chaos originated in 1983 from the discovery that chaotic transients arise typically in systems passing through a type of global bifurcation called crisis [23].

In [21] in 2011, the authors regret: “In spite of the experimental works and the several experiments carried out in the last 20 years, it is possible that due to the limited awareness of the phenomena of transient chaos even among researchers in the nonlinear-dynamics community, transiently chaotic signals were considered to be uninterpretable and were discarded". Therefore, we believe that knowledge of transient chaos can be particularly important and useful due to the growing number of applications in various fields of science and engineering based on or motivated by nonlinear dynamics.

Thus, our study of transient chaos in an unusual vibro-impact system may be interesting from three points of view. Firstly, it adds information to fundamental knowledge of the phenomena that occur in nonlinear dynamical systems. Secondly, it shows the behavior of a specific vibro-impact system (platform-vibrator with shock) with varying the control parameter. Thirdly, it allows to point out at what values of the control parameter an undesirable and possibly dangerous state, such as permanent and transient chaos, can occur.

So, the goals of this paper are:

• to demonstrate the transient chaos and its dependence on the values of the control parameter;

• graphically show the strong dependence of the state of a nonlinear non-smooth discontinuous vibro-impact system on the initial conditions by example of transient chaos and coexisting regimes;

• to show an unwanted range of a control parameter for platform-vibrator with shock, in which dangerous phenomena can occur.

2. Brief description of platform-vibrator mathematical model

The two-mass platform-vibrator with shock is one of the successful solutions for vibration equipment that implements shock-vibration technology for concrete mixtures compaction and reinforced products molding [23].

The creation of platform-vibrator mathematical model was described in detail in our papers [2, 3, 6]. Now we have to repeat the basic statements required to understand its dynamical behaviour.

We accept such a design scheme for platform-vibrator with shock (Fig. 3).

Exciting force $F(t) = P\cos(\omega t + \varphi_0)$, its period is $T = 2\pi/\omega$.

The platform table with mass m_1 is attached to the base by linear vibration isolating spring of stiffness k_1 and a linear dashpot with damping factor c_1. Exciting external periodic force $F(t)$ is generated by electric motors mounted under the table. Elastic rubber gasket with thickness h and stiffness k_0 is attached to the table. A linear dashpot with damping factor c_0 is placed between the table and the mold. Mold with concrete with mass m_2 is installed on the gasket but is not fastened both to the gasket and

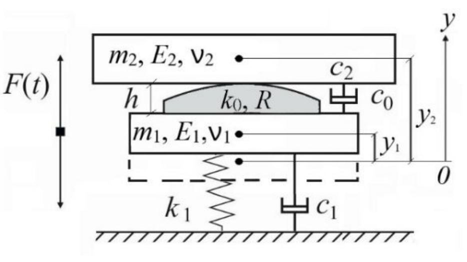


Fig. 3. Design scheme for platform-vibrator with shock. Platform table with attached rubber gasket is attached to the base with a linear vibro-isolating spring. The mold with concrete is installed on the gasket without fastening

to the table. So, it can tear herself away from the gasket and bounce. The machine starts its movement when the electric motors begin their work. First, the table and the mold move vertically together. Then the mold comes off from the gasket. The table and the mold are moving separately until the mold falls down onto the rubber gasket. Impact occurs. The bodies move together again until the mold comes off the gasket and so on.

The created mathematical model corresponds to the two-body 2-DOF vibro-impact system. It is strongly nonlinear non-smooth discontinuous system. It has some specific properties, namely: the upper body with very large mass breaks away from the lower body at a very short distance during vibrational motion; both bodies move separately; the upper body falls down onto the soft constraint; the impact that occurs is soft one due to the softness and flexibility of the constraint.

Vibro-impact movement of the platform includes both joint movement during impact and separate motion between impacts. The equations of this movement are:

$$\ddot{y}_1 = g\chi - \omega_1^2 y_1 - 2\xi_1\omega_1\dot{y}_1 + \frac{1}{m_1}F(t) + H(z)\left\{2\xi_0\omega_2\chi\dot{y}_1 - \omega_2^2\chi[h-(y_2-y_1)] - \frac{1}{m_1}F_{con}(z)\right\},$$

$$\ddot{y}_2 = -g - 2\xi_2\omega_2\dot{y}_2 + H(z)\left\{\omega_2^2\left[h-(y_2-y_1)\right] - 2\xi_0\omega_2\dot{y}_1 + \frac{1}{m_2}F_{con}(z)\right\}. \quad (1)$$

The initial conditions are:

at $t=0$ we have $\varphi_0 = 0$, $y_1 = 0$, $\dot{y}_1 = 0$, $y_2 = h - \lambda_0$, $\dot{y}_2 = 0$. (2)

The static deformation of the gasket is: $\lambda_0 = m_2 g/k_0$, g is the acceleration due to gravity.

Here the standard notations are introduced:

$$\frac{k_1}{m_1} = \omega_1^2,\ \frac{k_0}{m_2} = \omega_2^2,\ \frac{c_0}{m_2} = 2\xi_0\omega_2,\ \frac{c_1}{m_1} = 2\xi_1\omega_1,\ \frac{c_2}{m_2} = 2\xi_2\omega_2,\ \frac{m_2}{m_1} = \chi. \quad (3)$$

$H(z)$ is Heaviside step function relatively bodies' rapprochement $z = h - (y_2 - y_1)$. $F_{con}(z)$ is contact interactive force that simulates an impact and acts only during an impact.

The damping forces are taken to be proportional to the first degree of velocity: $F_{damp\ 1} = c_1\dot{y}_1$, $F_{damp\,0} = c_0\dot{y}_1$. The influence of the concrete mixture can be taken into account as some additional damping $c_2\dot{y}_2$.

In the two-body model, the masses are concentrated in the mass centers of both bodies. Parameters y_1 and y_2 are the coordinates of these centers for the lower body (platform table) and the upper body (mold with concrete) respectively in the selected coordinate system. The origin of coordinate y is chosen in the table centre in the state of static equilibrium.

The model numerical parameters are listed in Table 1.

Table 1

Numerical parameters of platform-vibrator with shock

Mass of table m_1 , kg	7400	Damping ratio of dashpot in spring ξ_1	0.5
Mass of mold with con. m_2 ,kg	15000	Damping ratio of dashpot in gasket ξ_0	0.02
Stiffness of rub.gask. k_0 , N·m^{-1}	$3.0 \cdot 10^8$	Damping ratio in concrete mixture ξ_2	0.03
Stiffness of spring k_1, N·m^{-1}	$2.6 \cdot 10^7$	Elastic modulus of mold E_2 , N·m^{-2}	$2 \cdot 10^{11}$
Poisson's ratio of rub.gask. ν_1	0.4	Elastic modulus of rub.gask. E_1 , N·m^{-2}	$3.5 \cdot 10^6$
Poisson's ratio of mold ν_2	0.3	Amplitude of exciting force P,N	$2.44 \cdot 10^5$
Thickness of gasket h, m	0.0275	Frequency of exciting force ω, Hz	25
Radius of gasket R, m	5		

We simulate a soft impact using nonlinear contact Hertzian force in accordance with quasistatic contact Hertz's theory [4,5].

$$F_{con}(z) = K[z(t)]^{3/2}, \quad K = \frac{4}{3}\frac{q}{(\delta_1+\delta_2)\sqrt{A+B}}, \quad \delta_1 = \frac{1-\nu_1^2}{E_1\pi}, \quad \delta_2 = \frac{1-\nu_2^2}{E_2\pi}. \quad (4)$$

Here $z(t)$ is the rapprochement of the bodies, as before, $z = (y_2 - y_1) - h$, when $(y_2 - y_1) \leq h$; ν_i and E_i − Poisson's ratios and Young's moduli of elasticity for both bodies; A, B, q − are constants characterizing the local geometry of the contact zone. The gasket surface is flat, but we consider it as a sphere of the large radius R. Then in the collision of a plane (mold) and a sphere (rubber gasket) $A = B = 1/2R$, q=0.318.

3. Transient chaoswhen the technological mass m_2 is varied

As we have already written in the Introduction, this model exhibited many unique phenomena inherent in nonlinear non-smooth dynamical systems with varying control parameters. When the technological mass m_2 of the upper body (mold with concrete) was chosen as the control parameter and varied, we observed transient chaos. Transient chaos is known as chaos with finite lifetime. When a transient chaos is observed in the system, the trajectory is first chaotic for some time and then becomes periodic for the same value of the control parameter [13]. In [14], the authors note that a typical occurrence of the transient chaos is in the periodic windows inside the chaotic region. "Periodic windows, *in spite of their name,* are in fact parameter regions in which transient chaos is typically present" [21].

Let's see how the largest Lyapunov exponent behaves when varying the control parameter (Fig. 4, 5). In Fig.4 its behavior is shown in the wide control parameter range. Fig. 5 is the portion of this graph that is inside the oval on a larger scale. The first thing that catches your eye is the presence of coexisting modes that exist in this narrow range of the control parameter. They are show ninyellow.

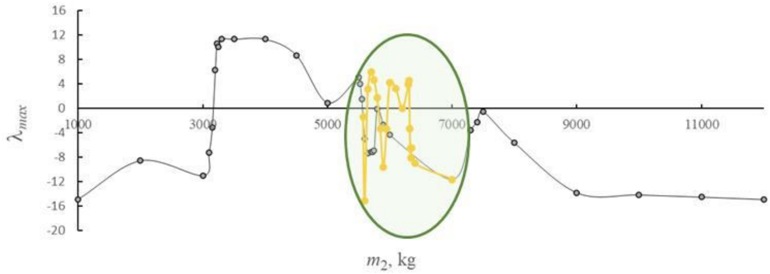


Fig. 4. Dependence of the largest Lyapunov exponent on the technological mass m_2

We emphasize once again that coexisting regimes can arise when the control parameter is constant, but the initial conditions are different. We have shown these coexisting regimes in more detail in [6]. Here we observe the hysteresis effect, that is, the jump phenomenon [25].

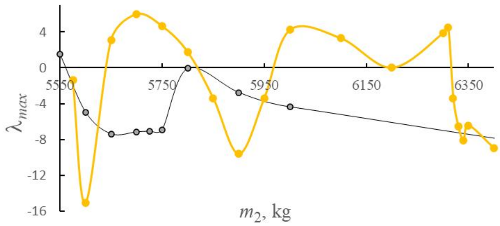


Fig. 5. Dependence of the largest Lyapunov exponent on the technological mass m_2 in narrow range of control parameter (inside the oval). In the coexisting regime, we see the alternation of Lyapunov exponent sign, i.e. the alternation of chaotic and periodic modes

It is known that the positive sign of the largest Lyapunov exponent determines chaotic dynamics.

Its negative sign gives hope for the periodic motions. We can believe that areas of negative Lyapunov exponent signs correspond to the periodic windows inside the chaotic region, cannot we? We observed transient chaos precisely in the region of periodic windows. We emphasize that transient chaos has a different form for different values of the control parameter and initial conditions, and also its lifetime is different.

When the initial conditions are chosen in the state of permanent chaos for $m_2 = 6000$ kg, we get transient chaos in a narrow range of the control parameter values. Chaotic vibrations, arising at certain system parameters values, degenerate into a periodic subharmonic (2,2)-regime after some time. (2,2)-regime is the regime with period $2T$ and 2 impacts per cycle. In Fig.6, we show pronounced transient chaos for $m_2 = 6330$ kg. Time series for the upper body (mold with concrete), contact force, and phase trajectories for both bodies are shown.

The figures of the time series (Fig. 6 (a)) and the contact force (Fig. 6 (b)) clearly show how the chaotic regime suddenly turns into a periodic one. Phase trajectories in the periodic phase, overlapped with the corresponding trajectories in chaotic phase, are shown in red in Fig. 6 (c), (d).

A natural question is whether there is actually chaos in the seemingly chaotic signals observed over finite time scales [21]. Measurement of the lifetime distribution, the escape rate, and the average lifetime (see sec. 5) may give one of the quantitative characteristic. Another paramount characteristic is the Lyapunov exponent. One should measure, for example, the Lyapunov exponents and check whether at least one of the exponents is positive. Determination of dynamical invariant such as the Lyapunov exponent and its positive sign can be considered as one of the chaos criteria.

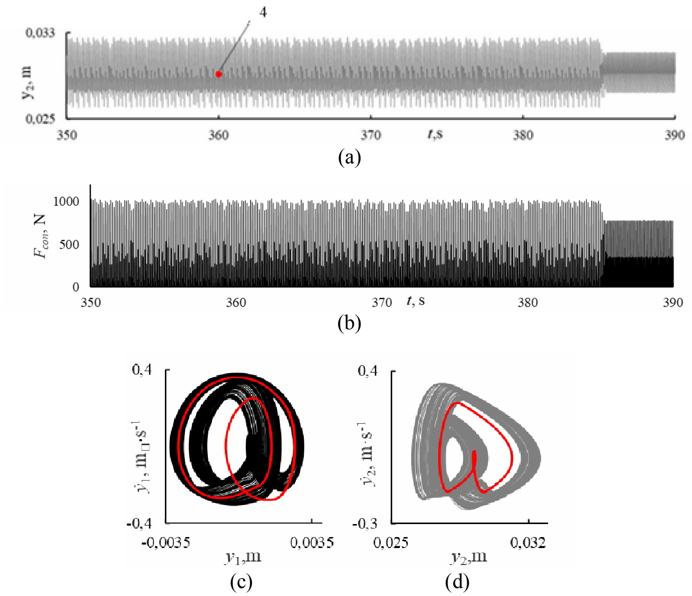


Fig. 6. (a) Time histories for upper body; (b) Hertz contact force; (c), (d) phase trajectories for m_2=6330 kg (trajectory initiated from permanent chaos at m_2 =6000 kg in red point 1 (Fig. 12))

Analysis of the largest Lyapunov exponent λ_{max} over a quite a long time helps to determine the existence of transient chaos. Its sign is positive for chaotic motion, then after a long procedure, the exponent converges to a negative value, which is typical for periodic movement. In Fig. 7 it is clearly seen that when the initial time is t_0=350 s, after some time (4.9 s) the largest Lyapunov exponent λ_{max} crosses the abscissa axis and becomes negative. We emphasize that the value of the control parameter remains the same.

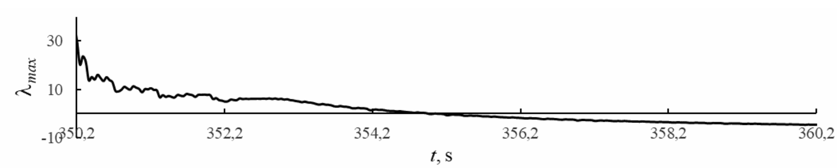


Fig. 7. Convergence of the largest Lyapunov exponent to negative value during the transient chaos: m_2 = 6330 kg, start from permanent chaos at m_2 =6000 kg in red point 1 (Fig. 12)

Since transient chaos is a fairly new concept, an interesting and "capricious" phenomenon, we want to show it in more detail.

To determine whether the transients are truly chaotic, one therefore needs more information than the mere positivity of the Lyapunov exponent. Qualitatively, the visual appearance of the signal can be helpful: about chaotic nonattracting sets trajectories should be complicated. This is, nonetheless, only a hint. A property uniquely indicating the chaotic nature of the transients is the *irregular* dependence of lifetimes on initial conditions [21], as illustrated by Table 3, 4, 5 in Sec. 5.

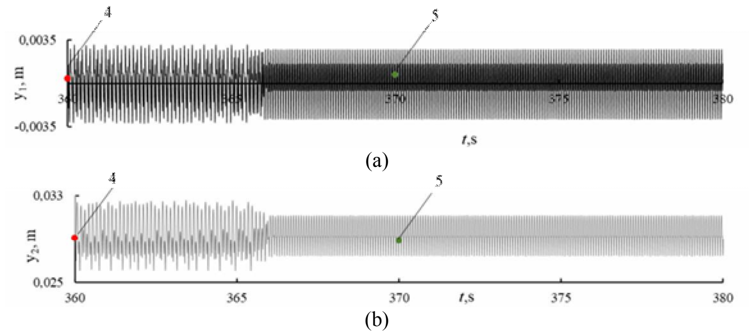


Fig. 8. Time series: (a) for lower body; (b) for upper body.
m_2= 6330 kg; trajectory initiated from phase of transient chaos at m_2= 6330 kg in red point 4 in Fig. 5(a)

We show in detail the transient chaos that we observe when the technological mass m_2 = 6330 kg, and the initial conditions are chosen in the same state of the vibro-impact system in red point 4 in Fig. 6 (a). Time series are depicted in Fig. 8. Fig. 8(a) shows the time series for the lower body (platform table) in black; Fig. 8(b) - for the upper body (mold with concrete) in grey.

We see very well how chaotic trajectories abruptly turn into periodic ones, which then exist all the time.

Fig. 9 shows the phase trajectories for the upper body in grey and for the lower body in black for area of chaotic motion (Fig. 9 (a)) and for area of periodic motion (Fig. 9 (b)). The corresponding Poincaré maps are depicted in Fig. 9 (c), (d)).

Phase trajectories and Poincaré maps have the typical forms for chaotic and periodic movements. Phase trajectories are closed curves for periodic motion and open curves (hence tangles of curves) for chaotic one. The Poincaré map for the periodic mode is several separate dots – two dots for regime with period $2T$. The Poincaré map for a chaotic regime is a set of dots of an undefined shape. Often this set has the fractal structure.

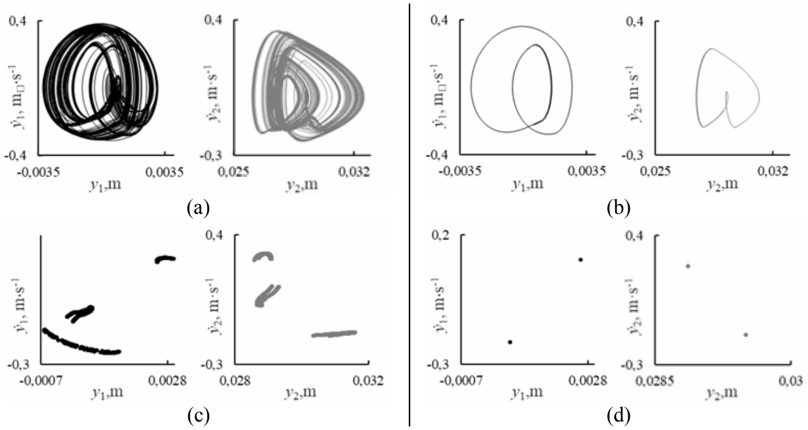


Fig. 9. The phase trajectories: for lower body in black, for upper body in grey, for chaotic phase (a), for periodic phase (b). Poincaré maps for both bodies in chaotic part of signal (c), in periodic part of signal (d).m_2= 6330 kg. Start for left panel is in red point 4, for right panel - in green point 5 in Fig. 8

In Fig. 10 Fourier spectra (Fig. 10 (a), (b)) and the graph of contact impact force F_{con} (Fig.10 (c)) are depicted.

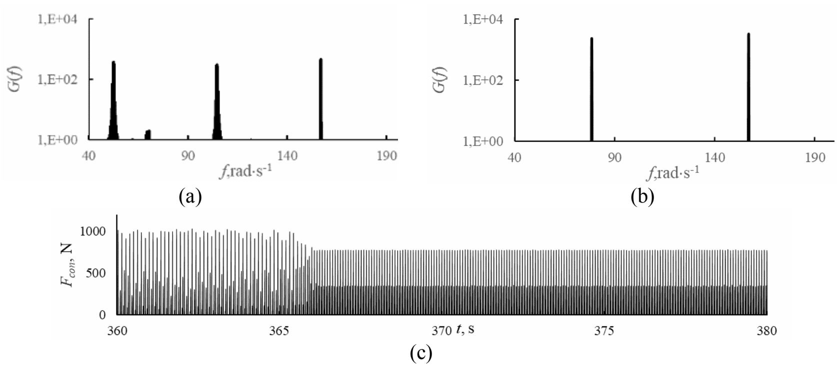
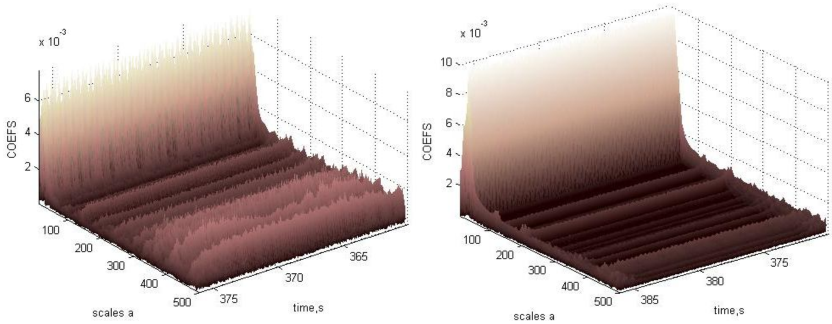


Fig. 10. Fourier spectra for chaotic (a) (start from red point 4 in Fig. 8) and periodic (b) (start from green point 5) parts of signal. Contact impact force F_{con} (c) (start from red point 4 in Fig. 8). m_2= 6330 kg

The Fourier spectrum for the periodic mode is separate clear “sticks” for several frequencies, but for the chaotic mode, it shows many weak frequencies and becomes more broad and continuous.

The graph of contact impact force F_{con} demonstrates a clear sudden boundary between the regions of chaotic and periodic motions in the same mode with the same value of the control parameter and the same initial conditions.

Fig. 11 shows the surfaces of wavelet coefficients [29, 30] for the lower body (the table of the platform-vibrator) in the same motion mode. They are obtained using CWT (Continuous Wavelet Transform) software from Matlab with Morlet wavelet.



(a) (b)

Fig. 11.Surfaces of wavelet coefficients for the lower body in: chaotic(a) (start from red point 4 in Fig. 8) and periodic (b) (start from green point 5) parts of signal. m_2= 6330 kg

Fig. 11(a) clearly shows that the frequency components in chaotic motion are not constant in time; they change over time. Indeed, this is typical of chaotic motion – the presence of many different frequencies that vary over time. This fact is also reflected in the Fourier spectrum (Fig. 10 (a)). On the contrary, the frequency components of the periodic movement do not change in time, they are constant over time. This is clearly seen in Fig. 11 (b) and in the Fourier spectrum in Fig. 10 (b).

The graphs in Fig. 8 - 11 confirm that transient in platform-vibrator is truly chaotic.

All these charts help to understand and feel the phenomenon of transient chaos, because they brightly demonstrate it from different sides.

4. Dependence of transient chaos on control parameter value

The form and lifetime of transient chaos τ depends both on the control parameter value and on the initial conditions.

Table 2 shows this dependence on the control parameter value, when initial conditions are chosen in state of permanent chaos at m_2=6000 kg in red point 1. Since the concept of initial conditions is very important for the transient chaos understanding, we show in Fig. 12 more graphically the points in the permanent chaos at m_2 = 6000 kg, which are chosen as the starting points. They are shown in red (point 1), in yellow (point 2), and in blue (point 3).

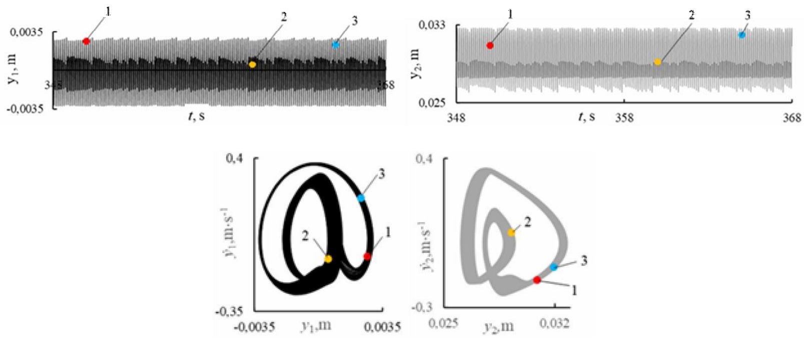


Fig. 12. Permanent chaos at m_2= 6000 kg; its start was chosen in chaos at m_2= 5800 kg

Table 2

The form and lifetime of transient chaos τ for various values of the control parameter when choosing the initial conditions in a state of permanent chaos at m_2 = 6000 kg in red point 1 in Fig. 12

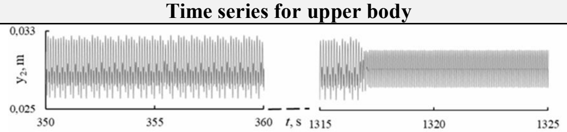
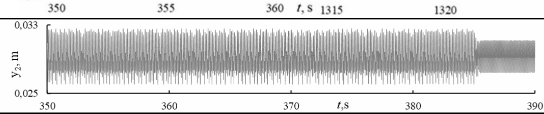
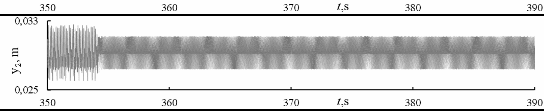
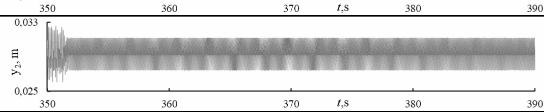
m_2, kg	τ, s	Time series for upper body
6300	967.2	
6330	35.5	
6340	4.5	
6360	1.8	

Table 2 clearly demonstrates how strongly the appearance of transient chaos and its lifetime change with a change in the control parameter. For m_2 = 6300 kg we see a very long chaotic transient. The transient time becomes so long that the system stays in a chaotic state for any practical time. Generally, the lifetime of the transient could be extremely long [15]. For m_2 = 6340 kg it becomes short, for m_2 = 6360 kg it becomes very short. The asymptotics is established quickly. The transient chaos of short average lifetime may be difficult to identify[21].But since these cases are a continuation of the previous ones, we hope that we can treat these short regions as transient chaos.

5. Dependence of transient chaos lifetime τ on initial conditions

The duration of the transient chaotic oscillations depends sensitively on the initial state of the system [1,21]. In other words, the dependence of transient chaos lifetime on the initial conditions is also very strong. Table 3 shows the transient chaos lifetime for different control parameter values with different initial conditions. The initial conditions for the three left columns correspond to red point 1, yellow point 2, and blue point 3 in permanent chaos at m_2= 6000 kg (Fig. 12). The rest of starting points are taken in the same mode. Average lifetime values $\langle T \rangle$ are the result of the averaging of these twelve realizations; they are shown in the farright column.

Table 3

Transient chaos lifetime τ for different values of the control parameter for different initial conditions

Mass m_2, kg	**Lifetime of transient chaos τ, s initial conditions in point**												$\langle T \rangle$,s
	1	2	3	4	5	6	7	8	9	10	11	12	
6300	967.2	401.6	759.1	1955.4	720.3	936	96.1	1604.6	190.4	256.8	887.6	498.9	772.8
6310	261.0	1.4	519.5	696.7	585.4	711.1	287	95.9	368.5	10.3	38.0	72.2	303.9
6320	109.1	34.3	159.8	35.9	309.8	34.3	45.9	89.7	181.0	6.9	147.5	10.1	97.0
6330	35.5	15.8	9.3	24.3	51.9	5.7	105	34.4	26.7	15.8	8.1	4.6	28.1
6340	4.5	5.9	1.2	38.1	5.9	15.8	1.9	7.6	16.3	2.5	6.5	7.4	9.6
6350	2.5	0.6	8.1	6.5	9.4	0.8	3.0	8.6	4.9	11.9	3.4	9.5	5.8
6360	1.8	0.8	2.1	8.2	6.5	5.3	3.3	4.3	3.1	0.54	4.4	2.6	3.6
6370	1.5	0.5	1.8	1.2	2.3	0.9	1.0	1.9	2.1	0.7	1.5	2.8	1.5
6380	3.0	0.6	1.2	2.2	5.7	1.2	1.2	2.2	1.1	3.1	1.2	1.9	2.0
6390	1.9	1.8	1.8	1.2	1.8	0.9	3.4	1.9	1.5	2.4	1.6	1.8	1.8
6400	1.4	1.0	2.0	1.4	1.1	0.9	2.0	0.1	1.4	2.41	0.8	1.5	1.3

This Table also shows large changes in the transient chaos lifetime for the same initial conditions, but for different values of the control parameter (technological mass of the mold with concrete m_2). This change is clearly visible in every column of the Table 3.

We would like to draw attention to the average lifetime. The average transient lifetime is a quantitative measure of how long the transient chaos exists. This is common characteristic of transient chaos [11, 17, 19]. It is often calculated by averaging a large ensemble of realizations from 100 [11, 19], 100 and 10000 [11] to 3 million [17].

However, the difference in values in a Table 3 row for the same control parameter value and different initial conditions is often very large. That is why averaging should be carried out over a large ensemble of realizations. Fig. 13 shows the dependence of the average chaotic transient lifetime $\langle T \rangle$ on the technological mass of the mold with concrete m_2. As shown in Fig. 13 and in Table 3, as m_2 is decreased, $\langle T \rangle$ increases dramatically.

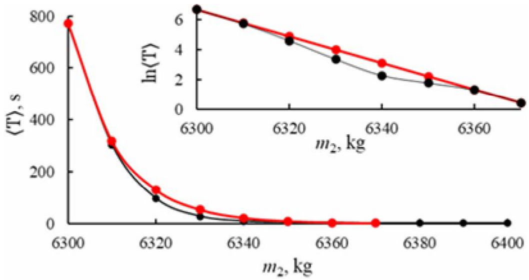


Fig. 13. Dependence of average chaotic transient lifetime $\langle \mathrm{T} \rangle$ on m_2 on a linear-linear plot and on a logarithmic versus linear scale (inset). All points are result of averaging 12 realizations

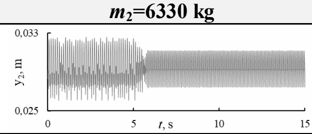
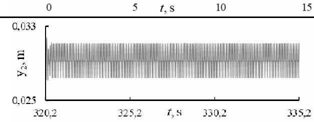
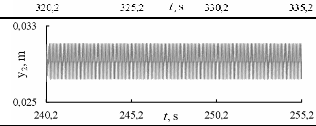
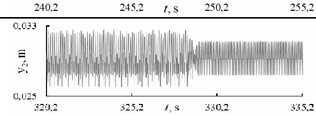
The average transient lifetime obeysan exponential law $\langle \mathrm{T} \rangle \approx Ce^{-\kappa m_2}$, where $\kappa > 0$ .Then in a logarithmic versus linear scale we have a straight line with slope - κ, κ=0.089. The red curve on linear-linear plot and the red straight line on a log-linear plot were plotted according to the exponential law and the equation of the straight line, respectively.

The slope κ is called the escape rate. It is a quantity measuring how quickly the trajectories initiated from random initial conditions escape any neighborhood of the nonattracting chaotic set. In other words, how long the transient chaos exists. Since the average lifetime depends on many details, the escape rate κ is a more appropriate characteristic of the decay process than $\langle \mathrm{T} \rangle$. The escape rate is a unique property of the underlying nonattracting chaotic set, in contrast to the average lifetime [21].

The initial conditions for Table 3 were chosen in different points of one vibro-impact state. Now, Table 4 shows four different motions for two values of the technological mass of the upper body (mold with concrete) when choosing the initial conditions in different states of vibro-impact system.

Table 4

Lifetime of transient chaos τ for two values of the control parameter when choosing the initial conditions in different states of vibro-impact system

Initial conditions	**τ, s for m_2=6330 kg**	**τ, s for m_2=6400 kg**	**Time series for the upper body at m_2=6330 kg**
In a quiescent state	6.15	1.83	
In a chaotic stateat m_2= 3200 kg	0.57	0.56	
In a periodic state at m_2= 5700 kg	0.28	0.26	
In a chaotic state at m_2= 5700 kg	8.45	1.58	

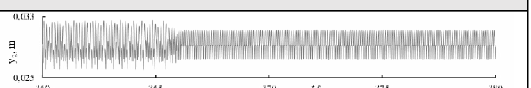
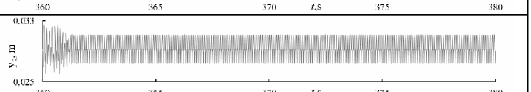
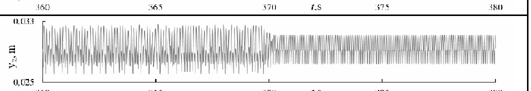
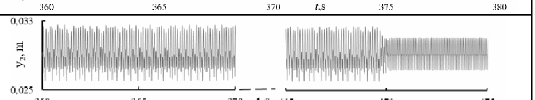
One can see the substantially different motion regimes. In the first and last cases, the modes are similar, only transient chaos lifetime differs. The second and especially the third cases are different. In the second case, the lifetime is very short. In the third case, the transient chaos, even the transitional process, is very, very short, almost nonexistent. Let's repeat once more that transient chaos of short lifetime may be difficult to identify. The motion pictures for these two values of control parameter are similar, but the lifetimes are different.

Note. As can be seen from Fig. 4 there are two coexisting regimes at m_2 =5700 kg, which arise under different initial conditions, – a periodic regime with a negative sign of the Lyapunov exponent and a chaotic regime with a positive sign of the Lyapunov exponent. The initial conditions for regimes in the third and fourth cases in Table 4 are chosen in these states.

It should be noted that the initial conditions in the rows of Table 4 are substantially different. However, if we change the initial conditions very little, then the transient chaos lifetime will still be different, despite the slight change in the initial conditions. Table 5 shows this change for m_2 =6330 kg, when only one variable in the initial conditions changes by a very small amount. Of the five variables in the initial conditions, namely, t_0, y_1, y_2, $\dot{y}_1$, $\dot{y}_2$, we change only y_1. In the 1st row y_1 is not changed, in the 2nd row it changes in such manner $y_1 = y_1 + 10^{-10}$, in the 3rd row it changes as $y_1 = y_1 + 10^{-9}$, in the 4th row $y_1 = y_1 + 10^{-7}$.

Table 5

Lifetime of transient chaos τ with a very small change in initial conditions for m_2=6330 kg

Initial condition for y_1, m	**Transient lifetime τ, s**	**Time series for the upper body at m_2=6330 kg**
0.00038911750	6.09	
0.00038911760	1.39	
0.00038911850	10.44	
0.00038921750	110.23	

We see how the slightest difference in the initial conditions leads to a big difference in the life of transient chaos. In the fourth case, a huge increase in the lifetime is observed, again we see a very long chaotic transient.

Exactly this circumstance is unsafe and alarming. Small imperfections and small deviations in the initial state of the nonlinear dynamical system can lead to unwanted unpredictable results later. In particular, long-term weather forecasts are often incorrect because of this.

Thus, Table 2, 3, 4, 5 clearly demonstrate the "waywardness" of a transient chaos, that is, its strong dependence on both the values of the control parameter and the initial conditions.

5. Conclusions

The model of platform-vibrator with shock corresponds to unusual 2-DOF two-body nonlinear non-smooth discontinuous vibro-impact system with soft impact. It exhibits transient chaos – a "wayward", not fully understood phenomenon that occurs in chaotic dynamical systems with varying the control parameter. The technological mass of the mold with concrete was chosen as a control parameter. We visibly showed the chaotic and periodic parts of the signal and confirm the chaoticity of the former and the periodicity of the latter, using their generally accepted characteristics, namely, phase trajectories, Poincaré maps, Fourier spectra, the largest Lyapunov exponent, and surfaces of wavelet coefficients. The dependence of the transient chaos on control parameter value was demonstrated. We focused on the sensitive dependence of the transient chaos on the initial conditions, that is, the basic feature of chaotic dynamics. We have shown that the average transient lifetime obeys an exponential law, which is typical to many chaotic systems. Both permanent and transient chaos may often be dangerous and unwanted states. Therefore, when operating the equipment, it is desirable to avoid the control parameter range in which these states can occur.

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ПЕРЕХІДНИЙ ХАОС В УДАРНО-ВІБРАЦІЙНОМУ МАЙДАНЧИКУ

Ударно-вібраційний майданчик широко застосовується у будівельній галузі для ущільнення та формування великогабаритних бетонних виробів. Його математична модель, яка створена у нашій попередній роботі, відповідає всім основним вимогам ударно-вібраційної технології для виробництва збірного залізобетону на низькочастотних резонансних платформах-вібраторах. Модель відповідає двох масовій віброударній системі з двома ступнями вільності з м'яким ударом. Це сильно нелінійна негладка розривна система є незвичайною віброударною системою завдяки своїм специфічним властивостям. Верхнє тіло з дуже великою масою відривається від нижнього на дуже маленьку відстань, а потім падає на м'який обмежник, що й викликає м'який удар. Потім воно знову відскакує та падає і так далі. М'який удар моделюється нелінійною контактною силою Герца. Модель продемонструвала багато унікальних явищ, властивих нелінійним негладким динамічним системам при зміні керуючих параметрів. У цій роботі ми демонструємо перехідний хаос у віброударній системі. Наявність перехідного хаосув ударно-вібраційному майданчику є чудовим явищем само по собі, крім того допомагаєзрозуміти динамічні процеси, що відбуваються в ударно-вібраційному майданчику при зміні технологічної маси форми з бетоном. Фазові траєкторії, перерізи Пуанкаре, графіки переміщень та контактних сил, спектри Фур'є, показники Ляпунова та вейвлет-характеристики використовуються в чисельних дослідженнях для визначення хаотичних та періодичних фаз реалізації. Показано як залежність перехідного хаосу від значення керуючого параметра, так і чутливу залежність від початкових умов. Ми сподіваємось, що цей аналіз може допомогти уникнути небажаної поведінки ударно-вібраційного майданчика під час експлуатації та підібрати відповідні параметри при проєктуванні, оскільки перехідний хаос може бути небезпечним та небажаним станом віброударної системи.

Ключові слова: ударно-вібраційний майданчик, вібро-ударний, технологічна маса, форма з бетоном, перехідний хаос, залежність від початкових умов.

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Математична модель ударно-вібраційного майданчика, що широко застосовуєтьсяу будівельній галузі для ущільнення та формування бетонних виробів,при зміні контрольних параметрів демонструє низку нелінійних явищ,зокрема таке цікаве та “примхливе” явище, як перехідний хаос.
Табл 5. Рис. 13. Бібліогр. 30 назв.

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The mathematical model of platform-vibrator with shock, which is widely used in the construction industry for compacting and molding concrete products, exhibits many nonlinear phenomena when varying the control parameters. In particular, there is transient chaos, which is an interesting and “capricious” phenomenon.
Table 5. Fig. 13. Ref. 30

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