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TALL VON-MISES TRUSSES' SKEW-SYMMETRIC DEFORMATION

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The work's aim is to investigate the tall two-rods three-hinged von-Mises trusses' deformation regularities at the sloped load that applied to the ridge joint.The horizontal elastic support influence in the ridge joint when changing the force's inclination angle in a wide range is also investigated Particular attention is paid to the tall two-rod trusses' skew-symmetric stability loss possibility. The possibility of the skew-symmetric shape of a loss of stability of high trusses with at a very small angle of inclination of the force from the vertical axis was confirmed. The horizontal elastic support's influence on increasing the stability against skew-symmetric deformation was shown.It was found that skew-symmetry deformation is essentially non-linear, but under certain conditions it is not catastrophic.It is also noticed that asymmetric deformation depends on vertical deformation.Scientific novelty lies in a detailed study of the tall two-rod three-hinged trusses' deformation, and the establishment of the tendency of such structures to skew-symmetric buckling.The tall von-Mises trusses' new detailed deformation regularities character at skew-symmetric deformation at small inclination angles of force that applied in the ridge joint has been established. Also, the two-rod structures' new deformation regularities has been revealed with a wide inclination angles range of the concentrated force applied in the ridge joint. It is shown that on increasing the loading's inclination angles, which coincide with the rod's inclination angles , the stability loss of the individual rods is possible, since there is a significant increase in the truss' carrying capacity. The research results can be used in the structure design of large general dimensions, modeling of which gives the real structure work under various loads.
Keywords: buckling; skew-symmetric deformation; tall von-Mises truss; horizontl elastic supports; sloped load; three-hinged truss; ridge joint.

1. Introduction. Topicality. Simple structural systems, which contain two three or four tall support legs, are in use in special metal constructions. The structures with two legs additionally could be fixed by the cable-stayed system. There are not enough researches, which consists to the deformation patterns of such systems in the literature. The behavior of such systems on reach maximum carrying capacity could be non-linear by the character. It is important because the support legs' dimensions could reach 10-50 m for such structures as Ferris wheels and other attractions, bridge and overpass supports

[1]. The design scheme of such systems is hinge-connected: support legs are hinged to the base and hinge-connected in the ridge. (Fig. 1).

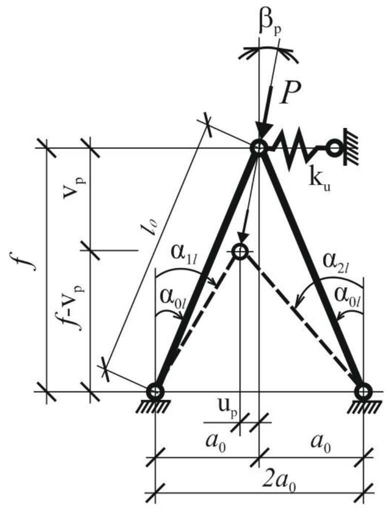

Fig. 1. The tall three-hinged system's design scheme

Thus, there are a number of tasks of the tall elastic three-hinged systems stress-strain state.

Historically, the elastic shallow von-Mises trusses' stability loss of has captured the researchers attention primarily by nonlinear snapping-through collapsing processes [25, 26]. Subsequently, such effects as snapping-through and nonlinear deformation were found as a thin-walled shells' stability loss characteristic sign [1, 2, 3, 4, 20], various structural systems' nonlinear deformation's aspects, including the stability loss, are described in [5, 11, 14, 22, 23]. The efficiency of using spatial rod systems is due to construction volume approximation's possibility to the functional one on well-known criterion [6]. Therefore, the stability problems study of rod spatial domes and shells, frame systems with constant and variable cross-section is aimed at creating economic structural systems [7, 8, 9, 10, 11, 22]. The revealed aspects of calopstic nonlinear deformation of systems found a new direction for complex systems under the influence of destructive loads [12].

In general, the three-hinged systems, which type is shown at Fig. 1, could be assigned name the tall von-Mises trusses (Mises R. (Richard Edler von Mises) [10, 15, 25, 26].

In recent years, interesting new studies [10, 13, 15, 16-19, 21, 22, 24] have been devoted to the rod systems' like von-Mises trusses nonlinear deformation study. But the tall trusses' systems at a sloped loading demand additional research. Particular attention should be paid to the tall two-rod systems' tendency to skew-symmetric deformation and stability loss with skew-symmetric shape [10, 13, 15, 16-19, 21, 22, 27]. In specified works, the von-Mises shallow trusses studies' overview under various conditions of fixing and loading is provided.

In studies [17, 19, 27], it is shown that tall three-hinged trusses have a nonlinear deformation character under asymmetric load, and, accordingly, are subject to the skew-symmetric buckling. But more detailed von-Mises trusses' studies on the tendency to manifest skew-symmetric stability loss have not been carried out enough.

2. The work aim is to investigate the influence of the horizontal elastic support and inclined loading in the truss ridge joint to the tall von-Mises trusses deformational state.

3. Research. The tall three-hinged truss deformational scheme was reviewed (Fig. 1) under applying the loading force P with an angle β_P in the ridge joint. To investigate the tall three-hinged truss deformational state the classic, static and energetic approaches used [3, 6]. In these researches, the structure stress-strain is determining through the trusses deformational calculations taking into account the rods reduction and potential energy change.

The three-hinged tall truss, which consists of two rods with a length l_0 was considered (Fig. 1). The truss has a span $2*a_0$ and height of structure f. The support's leg inclination angle out of the vertical axis is α_{0l}. The horizontal support rigidity characteristic marked as k_u.

After loading the considered system by force P inclined with an angle β_P at the structure's top joint there will be rods compression with their length reduction: $\Delta l_{01} = l_0 - l_1$; $\Delta l_{02} = l_0 - l_2$. At the deformation state, the rods length will have the values l_1 and l_2. The relative deformations of each inclined rack will be:

$$\varepsilon_1 = \Delta l_{01} / l_0 = 1 - l_1 / l_0, \tag{1.a}$$

$$\varepsilon_2 = \Delta l_{02} / l_0 = 1 - l_2 / l_0. \tag{1.b}$$

The forces in the rods due to force balance in the joint:

$$P\sin\beta_P - u_P k_u = N_1 \sin(\alpha_{1l}) - N_2 \sin(\alpha_{2l}),$$
$$P\cos\beta_P = N_1 \cos(\alpha_{1l}) + N_2 \cos(\alpha_{2l}). \tag{2}$$

The relations between the three-hinged truss' dimensions and the topjoint's displacements could be written as:

$$\begin{gathered}
\mathrm{tg}\alpha_{0l} = \frac{a_0}{f} \rightarrow f = \frac{a_0}{tg\alpha_{0l}};\, u_p = v_p \mathrm{tg}\beta_P, \\
a_0 - u_P = a_0\left(1 - \frac{u_P}{a_0}\right);\, a_0 - u_P = a_0\left(1 - \frac{v_P}{a_0}\mathrm{tg}\beta_P\right), \\
a_0 + u_P = a_0\left(1 + \frac{u_P}{a_0}\right);\, a_0 + u_P = a_0\left(1 + \frac{v_P}{a_0}\mathrm{tg}\beta_P\right), \\
(f - v_P) = \left(\frac{a_0}{\mathrm{tg}\alpha_{0l}} - v_P\right) = a_0\left(\frac{1}{\mathrm{tg}\alpha_{01}} - \frac{v_P}{a_0}\right).
\end{gathered} \tag{3}$$

Geometrically the relation through the top-joint's displacements and structure's initial dimensions to the deformations links the rod's angles trigonometric functions after the deformations each to other.

$$\sin(\alpha_{1l}) = \frac{a_0 - u_P}{\sqrt{(a_0 - u_P)^2 + (f - v_P)^2}},\quad \cos(\alpha_{1l}) = \frac{f - v_P}{\sqrt{(a_0 - u_P)^2 + (f - v_P)^2}}, \tag{4}$$

$$\cos(\alpha_{2l}) = \frac{f - v_P}{\sqrt{(a_0 + u_P)^2 + (f - v_P)^2}},\quad \sin(\alpha_{2l}) = \frac{a_0 + u_P}{\sqrt{(a_0 + u_P)^2 + (f - v_P)^2}}. \tag{5}$$

From the equations (3…5) the truss deformational state consists of the deformations continuity equation trough the equal vertical displacements at the top joint with the inclined loading.

$$f - v_P = l_{1l}\cos(\alpha_{1l}) = l_{2l}\cos(\alpha_{2l});$$
$$2a_0 = l_{1l}\sin(\alpha_{1l}) + l_{2l}\sin(\alpha_{2l}). \quad (6)$$

The rod's relative deformations is (Fig. 1):

$$\boldsymbol{\varepsilon_1 = 1 - \left(1 - \frac{v_P}{a_0}\mathrm{tg}\beta_p\right)\frac{\sin(\alpha_{0l})}{\sin(\alpha_{1l})}},$$
$$\varepsilon_2 = 1 - \left(1 + \frac{v_P}{a_0}\mathrm{tg}\beta_P\right)\frac{\sin(\alpha_{0l})}{\sin(\alpha_{2l})}. \quad (7)$$

The rod's compressive internal forces at the deformational state with the inclined loading is:

$$\boldsymbol{N_1 = \varepsilon_1 EA_{cal}; \rightarrow N_1 = \left[1 - \left(1 - \frac{v_P}{a_0}\mathrm{tg}\beta_P\right)\frac{\sin(\alpha_{0l})}{\sin(\alpha_{1l})}\right]EA_{cal}},$$
$$N_2 = \varepsilon_2 EA_{cal}; \rightarrow N_2 = \left[1 - \left(1 + \frac{v_P}{a_0}\mathrm{tg}\beta_P\right)\frac{\sin(\alpha_{0l})}{\sin(\alpha_{2l})}\right]EA_{cal}. \quad (8)$$

The external forces balance with the elastic support's internal forces ($r_u = - u_p k_u$) gives an equation:

$$P\sin\beta_P - u_P k_u = N_1\sin(\alpha_{1l}) - N_2\sin(\alpha_{2l}),$$
$$P\cos\beta_P = N_1\cos(\alpha_{1l}) + N_2\cos(\alpha_{2l}). \quad (9)$$

Combining the ratios (3 ... 8) with equation 9 gives the criterion for von-Mises truss' operation at an inclined load and the horizontal elastic support in the top joint.

$$\frac{P\cos\beta_P}{EA_{cal}} =$$
$$= \frac{\frac{1}{\mathrm{tg}\alpha_{0l}} - \frac{v_P}{a_0}}{\sqrt{\left(1 - \frac{v_P}{a_0}\mathrm{tg}\beta_P\right)^2 + \left(\frac{1}{\mathrm{tg}\alpha_{0l}} - \frac{v_P}{a_0}\right)^2}} + \frac{\frac{1}{\mathrm{tg}\alpha_{0l}} - \frac{v_P}{a_0}}{\sqrt{\left(1 + \frac{v_P}{a_0}\mathrm{tg}\beta_P\right)^2 + \left(\frac{1}{\mathrm{tg}\alpha_{0l}} - \frac{v_P}{a_0}\right)^2}} -$$
$$- 2\sin(\alpha_{0l}),$$
$$\frac{P\sin\beta_P}{EA_{cal}} - \frac{u_P}{a_0}\frac{k_u a_0}{EA_{cal}} =$$
$$= \frac{1 - \frac{v_P}{a_0}\mathrm{tg}\beta_P}{\sqrt{\left(1 - \frac{v_P}{a_0}\mathrm{tg}\beta_P\right)^2 + \left(\frac{1}{\mathrm{tg}\alpha_{0l}} - \frac{v_P}{a_0}\right)^2}} - \frac{1 + \frac{v_P}{a_0}\mathrm{tg}\beta_P}{\sqrt{\left(1 + \frac{v_P}{a_0}\mathrm{tg}\beta_P\right)^2 + \left(\frac{1}{\mathrm{tg}\alpha_{0l}} - \frac{v_P}{a_0}\right)^2}} +$$
$$+ 2\frac{v_P}{a_0}\mathrm{tg}\beta_P\sin(\alpha_{0l}).$$

Finally:

$$\frac{P\cos\beta_P}{EA_{cal}} = \\ = \frac{1}{\sqrt{1+\frac{\left(1-\frac{v_P}{a_0}\mathrm{tg}\beta_P\right)^2}{\left(\frac{1}{\mathrm{tg}\alpha_{0l}}-\frac{v_P}{a_0}\right)^2}}} + \frac{1}{\sqrt{1+\frac{\left(1+\frac{v_P}{a_0}\mathrm{tg}\beta_P\right)^2}{\left(\frac{1}{\mathrm{tg}\alpha_{0l}}-\frac{v_P}{a_0}\right)^2}}} - 2\sin(\alpha_{0l}), \tag{10.a}$$

$$\frac{\boldsymbol{P\sin\beta_P - u_P k_u}}{\boldsymbol{EA_{cal}}} = \\ = \frac{1}{\sqrt{1+\frac{\left(\frac{1}{\mathrm{tg}\alpha_{0l}}-\frac{v_P}{a_0}\right)^2}{\left(1-\frac{v_P}{a_0}\mathrm{tg}\beta_P\right)}}} - \frac{1}{\sqrt{1+\frac{\left(\frac{1}{\mathrm{tg}\alpha_{0l}}-\frac{v_P}{a_0}\right)^2}{\left(1+\frac{v_P}{a_0}\mathrm{tg}\beta_P\right)^2}}} + \\ + 2\frac{v_P}{a_0}\mathrm{tg}\beta_P\sin(\alpha_{0l}). \tag{10.b}$$

The equation (10.a) is a regularity that describes the truss' deformation in the vertical direction. Equation (10.a) is similar to the von-Mises truss' level of stability, which was obtained in [10]. The equation (10.b) is a regularity that describes the truss’ deformation in the horizontal direction with the inclined loading taking into account the elastic support’s horizontal reaction.

Equation (10.a) was well studied for sloped von-Mises trusses with the support racks’ angle to the vertical axis $\alpha_{0l} \geq 70°$earlier at the scientific researches [10, 15].

Equation (10.a) will became to the well-known equation [] when force’s angle is absent β_P =0, $\cos\beta_P = 1$ (formula 11).

$$\frac{P}{EA_{cal}} = 2\left(\frac{1}{\mathrm{tg}\alpha_{0l}}-\frac{v_P}{a_0}\right)\left[\frac{1}{\sqrt{1+\left(\frac{1}{\mathrm{tg}\alpha_{0l}}-\frac{v_P}{a_0}\right)^2}} - \sin(\alpha_{0l})\right]. \tag{11}$$

4. The numerical studies results. The deformed state numerical researches of the tall three-hinged trusses with sloped load. The tall two-rod trusses’ numerical studies with an inclined loading have done with support's rack sloping angle values from the vertical axis $15^\circ \leq \alpha_{0l} \geq 70^\circ$. The research aim is the tall von-Mises trusses tendency influence to nonlinear deformation under inclined load.

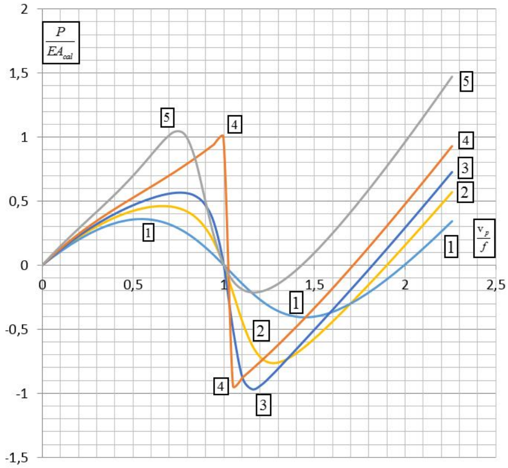

Fig. 2. The von-Mises trusses deformations numerical studies using an equation (10.a) with the relative vertical displacements v_P/f with α_{0l} = 35°. Where: 1 - β_P=10°; 2 - β_P=25°; 3 - β_P=30°; 4 - β_P=35°; 5 - β_P=45°

The tall three-hinged truss structure' deformations (Fig. 2) corresponds to the low structures' deformation regularities at the angles of inclination of the support racks α_{0l} ≥ 70°. However, such deformation is possible only with significant deformations and absolutely elastic material. The maximum critical loading was determined with a top joint's serial vertical displacements. It is shown that an increasing the force P inclination angle β_P increases the critical loading value. But the deformation's character significantly changes if an angle value β_P is equals or bigger than support rack's angle from vertical $\beta_P \geq 35^o$.

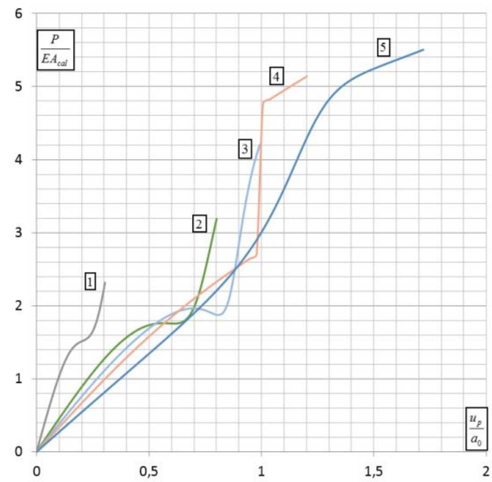

Fig. 3. The von-Mises truss numerical studies results using the equation (10.b) with the relative horizontal displacements v_P/f with α_{0l} = 35° (k_u =0). Where: 1 - β_P = 10°; 2 - β_P = 25°; 3 - β_P = 30°; 4 - β_P = 35°; 5 - β_P = 45°

The von-Mises truss deformation analysis using an equation (10.b) with the relative horizontal displacements v_P/f and $\alpha_{0l} = 35^{\circ}$ reveals the nonlinear local effects with big loading values, which are relative to reaching a structure a critical loading obtained with the equation (10.a).

1. Elastic horizontal support's impact to the structure deformation with an inclined loading. The Numerical studies of the horizontal elastic support's effect on the tall trusses' deformation were carried out (Fig. 4).

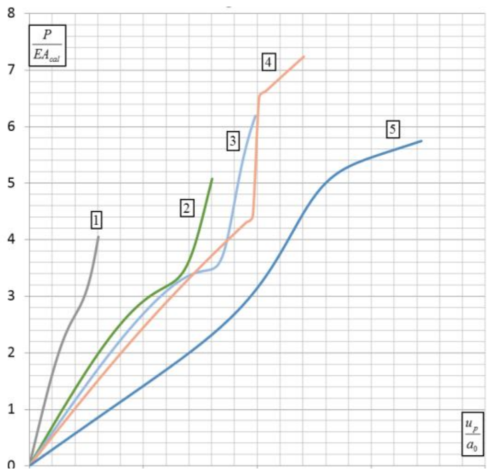

Fig. 4. The von-Mises truss' numerical studies results by equation (10.b) with the relative horizontal displacements v_P/f and α_{0l}=35°, and with relative horizontal elastic support's stiffness($(k_u a_0)/(EA_{cal})$=1,0). Where: 1 - β_P=10°; 2 - β_P=25°; 3 - β_P=30°; 4 - β_P=35°; 5 - β_P=45°

The tall trusses' strain-state analysis and comparison using an equation (10.b) without horizontal elastic support (Fig. 3, k_u =0 and with horizontal elastic support shows that horizontal elastic support (Fig. 4, ($k_u a_0$ =1,0)) ($k_u a_0$ - an elastic horizontal support relative stiffness) reduces the nonlinear effects influence and increases the structure's carrying capacity. The structure's deformation nature along the axis u_P/a_0 is determined by the structure's riged joint deformation along the axis, depending on the values (v_P/f).

It is noticed that the influence of the elastic horizontal support increases at $\beta_P \geq \alpha_{0l}$, comparison of graph 5 to graphs 2, 3, 4 in Fig. 3 and in Fig. 4.

2. An asymmetrical shape stability loss' possibility analysis under symmetrical load.

The skew-symmetric shape buckling possibility for the tall von-Mises trusses were investigated in works [10]. The tall von-Mises trusses' deformation with small vertical force's angles (Fig. 5) which is applied at the ridge joint by equation (10.b) and β_P=0,01° and with horizontal elastic support absense: k_u=0 is showing (Fig. 5). Based on the numerical studies results has built the dependencies' diagrams of the ridged force's projection on the horizontal axis ($P\sin\beta_P$ with β_P=0,01°) to the relative horizontal displacements(u_P/a_0) with the angles' range from the vertical axis α_{0l}=15°… 33°. It has been confirmed that, at angle α_{0l}<22,637 °, the tall trusses are subject to a skew-symmetric stability loss (Fig. 5, diagrams 1, 2, 3). This effect manifests itself at the very beginning of loading and is not catastrophic, and is associated with deformation with the symmetrical load.

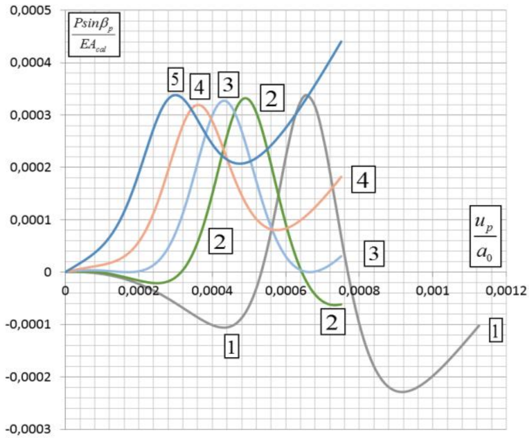

Fig. 5. The von-Mises truss' deformation numerical studies results by equation (10.b) with the relative horizontal displacements u_P/a_0 with β_P=0,01°, and with the relative horizontal elastic support's stiffnes (k_u=0).Where: 1 – α_{0l}=15°; 2- α_{0l}=20°; 3 - α_{0l}=22,637°; 4 - α_{0l}=27°; 5- α_{0l}=33°

An Influence of the horizontal elastic support that placed at the ridge joint to the skew-symmetric stability loss tendency was investigated (Fig. 6, 7, 8).

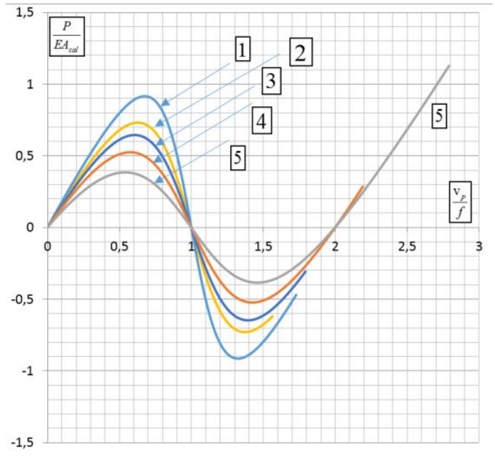

Fig. 6. The von-Mises truss' deformations numerical studies by the equation (10.a) with the vertical relative displacements v_P/f and with $((k_u a_0)/(EA_{cal})$=0,25), β_P=0,01°. Where:1 - α_{0l}=15 °; 2 - α_{0l} =20°; 3 - α_{0l}=22,637 °; 4 - α_{0l}=27°; 5- α_{0l}=33 °.

It has been confirmed that the horizontal elastic support has no effect on the truss' stability in the vertical direction (Fig. 1, Fig. 6). The skew-symmetric buckling study was carried out at a small load angle β_P =0,01° (β_P =0,00017) applied in the ridge joint. The results of such studies are presented in (Fig. 7, 8).

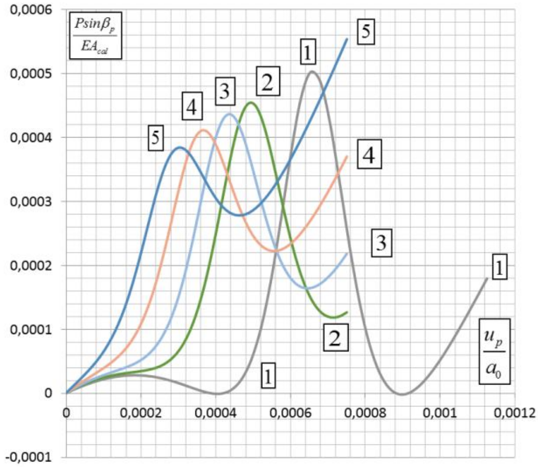

Fig. 7. The force's projection altereations numerical studies results $(P\sin\beta_P)/(EA_{cal})$ by the equation (10.b) with the horizontal relative displacements u_P/a_0 and β_P =0,01° and with the horizontal support's relative stiffnes $((k_u a_0)/(EA_{cal}) = 0{,}25)$. Where: 1 - α_{0l}=15°; 2 - α_{0l}=20°; 3 - α_{0l}=22,637°; 4 - α_{0l}=27°; 5- α_{0l}=33°

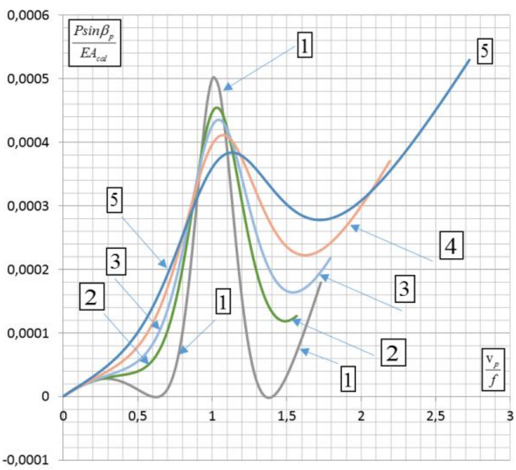

Fig. 8. The force's projection altereations numerical studies results $(P\sin\beta_P)/(EA_{cal})$ by the equation (10.b) with the vertical relative displacements v_P/f and β_P =0,01° and with the horizontal support's relative stiffnes $((k_u a_0)/(EA_{cal}) = 0{,}25)$. Where: 1 - α_{0l}=15°; 2 - α_{0l}=20°; 3 - α_{0l}=22,637°; 4 - α_{0l}=27°; 5 - α_{0l}=33°

Studies have confirmed (Fig. 5, 7, 8) that at small inclination angles the two-rod trusses could be subjected to a buckling with a skew-symmetric shape α_{0l}<22,637°, but with a horizontal elastic support the value of the critical angle from the vertical decreases. This behavior is observed at $\frac{k_u a_0}{EA_{cal}}$ =0.25 and with the angle values α_{0l}<15,0°.

5. Conclusions. Scientific novelty and practical significance. It is shown that for tall trusses a load inclination angles dohave avalue. Increasing an inclination angle the trusses' load capacity also increasing but it changes the deformation pattern. With force's inclination angle, approaching to racks angle ($\boldsymbol{\beta_P \to \alpha_{0l}}$) the stability loss will occur faster since reaching the critical loading (Fig. 2). It was shown that with an increasing the load's inclination angles, which coincide with the rods' inclination angles, the individual rods stability loss is possiblesince there is a significant increasing the truss carrying capacity (Fig. 2, 3, 4, diagrams 4, 5).

It was confirmed three-hinged trusses' deformation nonlinear nature. The horizontal support's stiffness reduces the skew-symmetric effects (Fig. 4, 6, 7, 8). The initial imperfections' influence on the tall trusses' nonlinear skew-symmetric deformations (Fig. 5, 7, 8) happens with the rod's angles from the vertical axis α_{0l}< 22,637°. However, it was shown that the horizontal elastic support's influence on increasing the stability from skew-symmetric deformations and corresponds to decreasing the truss' rod critical angle values from the vertical axis (Fig. 7, 8). It was found that skew-symmetric deformation has essentially nonlinear, but under certain conditions, it is not catastrophic.

It found that asymmetric deformations essentially depend on vertical deformations. The scientific novelty is laying on the tall two-rod three-hinged truss' detailed investigation and on such structures' skew-symmetric buckling tendency determination. The study resultsused for designing the large general dimensions structures which modeling gives us the structures real work under the different loads.

It is important to investigate refined models and the structures' buckling tendency when designing various structures with large overall dimensions in which testing cannot be fully performed but when their initial imperfections affect the structures' operation as a whole.

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КОСОСИМЕТРИЧНА ДЕФОРМАЦІЯ ВИСОКИХ ФЕРМ МИЗЕСА

Мета роботи - дослідити закономірності деформації високих двоштангових три шарнірних ферм фон-Мізеса при навантаженні з нахилом, що прикладається до конькового з'єднання. Також досліджується вплив горизонтальної пружної опори на конькове з'єднання при зміні кута нахилу сили в широкому діапазоні. Особливу увагу приділено можливості втрати симметричної стійкості високих дво ступінчастих ферм. Підтверджено можливість кососиметричної форми втрати стійкості високих ферм при дуже малому куті нахилу сили від вертикальної осі. Показано вплив горизонтальної пружної опори на підвищення стійкості проти кососиметричної деформації, встановлено, що кососиметртчна деформація по суті є нелінійною, але за певних умов вона не катастрофічна. Також зазначається, що асиметрична деформація залежить від вертикальної деформації. Наукова новизна полягає в детальному вивченні деформації високих двоштангових три шарнірних ферм та встановленні схильності таких конструкцій до кососиметричного вигину. Встановлено нові деталізовані закономірності деформації високих ферм фон Мізеса при кососиметричній деформації при малих кутах нахилу сили, що застосовується в коньковому з'єднанні. Також виявлено нові закономірності деформації двострижневих конструкцій із широким діапазоном кутів нахилу концентрованої сили, прикладеної в коньковому з'єднанні. Показано, що при збільшенні кутів нахилу навантаження, які збігаються з кутами нахилу стрижня, можлива втрата стійкості окремих стрижнів, оскільки відбувається значне збільшення несучої здатності ферми. Результати досліджень можуть бути використані при проектуванні конструкцій великих загальних розмірів, моделювання яких дає реальну роботу конструкції під різними навантаженнями.

Ключові слова: згин; кососиметрична деформація; висока ферма Мізеса; горизонтальна пружна опора; навантаження з нахилом; три шарнірна ферма; конькове з'єднання.

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TALL VON-MISES TRUSSES' SKEW-SYMMETRIC DEFORMATION

The work's aim is to investigate the tall two-rods three-hinged von-Mises trusses' deformation regularities at the sloped load that applied to the ridge joint.The horizontal elastic support influence in the ridge joint when changing the force's inclination angle in a wide range is also investigated Particular attention is paid to the tall two-rod trusses' skew-symmetric stability loss possibility. The possibility of the skew-symmetric shape of a loss of stability of high trusses with at a very small angle of inclination of the force from the vertical axis was confirmed. The horizontal elastic support's influence on increasing the stability against skew-symmetric deformation was shown.It was found that skew-symmetry deformation is essentially non-linear, but under certain conditions it is not catastrophic.It is also noticed that asymmetric deformation depends on vertical deformation.Scientific novelty lies in a detailed study of the tall two-rod three-hinged trusses' deformation, and the establishment of the tendency of such structures to skew-symmetric buckling.The tall von-Mises trusses' new detailed deformation regularities character at skew-symmetric deformation at small inclination angles of force that applied in the ridge joint has been established. Also, the two-rod structures' new deformation regularities has been revealed with a wide inclination angles range of the concentrated force applied in the ridge joint. It is shown that on increasing the loading's inclination angles, which coincide with the rod's inclination angles, the stability loss of the individual rods is possible, since there is a significant increase in the truss' carrying capacity. The research results can be used in the structure design of large general dimensions, modeling of which gives the real structure work under various loads.

Keywords: buckling; skew-symmetric deformation; tall von-Mises truss; horizontl elastic supports; sloped load; three-hinged truss; ridge joint.

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Fig. 8. Ref. 27.

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