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ALGORITHM FOR COMPUTATIONAL COSTS REDUCING IN PROBLEMS OF CALCULATION OF ASYMMETRICALLY LOADED SHELLS OF ROTATION

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The problem of calculating shells of rotation of a variable along the stiffness meridian under asymmetric loading is reduced to a set of systems of one-dimensional boundary value problems with respect to the amplitudes of the expansion of the desired functions in trigonometric Fourier series. An approach to reduce the required number of solutions such one-dimensional problems is proposed. The approach is based on predicting the values of variables along the meridian of expansion coefficients. This allows reducing the computational steps for finding a solution. The results of calculating the stress-strain state of a steel ring plate under asymmetric transverse loading are given as an example.
Keywords: rotation shells, variable stiffness, asymmetric loading, coefficient prediction in the Fourier method, reduction in computing costs.

Introduction

Shell structures are used in the creation of structures of modern engineering, in the oil and gas, chemical and other industries widely. By this way requires not only weight indicators, but also various characteristics that lead to the need to build more reliable models and methods for calculating shell structures with inhomogeneous (in particular, with variable stiffness) parameters. [1 – 6].

In problems of determining the optimal distribution of material [7] or calculating the durability of shells, taking into account the degradation of their surface in an aggressive environment [8], the stiffness parameters change at each step of successive approximations. This leads to difficulties in rebuilding the grid with using well-known finite element analysis packages [9, 10] for each step of the iterative computational (search) algorithm.

A fairly common approach to studying the behavior of such structural elements is to directly solve boundary value problems for systems of partial differential equations describing their state, where the components of the stress-strain state are unknown [1, 5, 11].

In the case of shells of rotation with a variable wall thickness along the meridian, the method of separation of variables is used using decompositions of the components of the stress-strain state and load in trigonometric Fourier series in a circular coordinate [1, 2, 5, 11 – 14]. As a result, the problem of solving a system of partial differential equations is reduced in the general case to solving $n + 1$ systems of ordinary differential equations to find n harmonics of the expansion of the desired functions in Fourier series. Here, the inhomogeneity parameters (changing the shell wall thickness) are taken into account quite simply, since they turn out to be components of the coefficients of these systems. Further, the main computational steps of this approach are mainly associated with the need to solve a large number of such one-dimensional boundary value problems only. Therefore, reducing the computational cost of finding their solution is quite important.

1. Mathematical model description

The equations of the moment theory of shells under asymmetric loading are accepted under the assumption that shells of revolution (with a generally arbitrary shape of the meridian), round (ring) plates in particular, are isotropic, elastic and thin-walled. The validity of Kirchhoff hypotheses, small deformations and rotation angles in comparison with unity is accepted. The shell wall thickness is taken variable along the meridian $h = h(s)$. It should be noted that the change in the thickness $h(s)$ of the shells of rotation should be sufficiently smooth dh/ds much less, then one [1, 2, 15]. Otherwise, the accepted initial hypotheses will be violated, and the results obtained in the calculations may turn out to be unreliable.

In this case, the equations of moment theory for thin elastic shells of rotation of variable stiffness under asymmetric loading can be, as is known [1, 2, 5, 11], reduced to a system of eight partial differential equations. For power factors, generally accepted designations are introduced N_1, N_2, S, M_1, M_2, M, Q_1, Q_2; for appropriate movements and rotation angles – $u, v, w, \vartheta_1, \vartheta_2, \vartheta$, and for the meridional, circumferential and normal components of the external loading intensity – q_1, q_2, q_3.

These well-known equations of the theory of shells, in order to reduce the bulkiness of the presentation, are not given in the article, and in the future, for the sake of definiteness, the statements are taken in the form [1], where four variables that characterize the displacements are taken as the main variables u, v, w, ϑ_1 and four corresponding force factors N_1, S^*, Q_1^*, M_1, where $S^* = S + \dfrac{2M}{R_2}$; $Q_1^* = Q_1 + \dfrac{1}{r}\dfrac{\partial M}{\partial \varphi}$ – reduced efforts.

We will further use the decompositions of the load, displacements, and forces acting in the shell into Fourier series [16] along the circumferential coordinate φ in the form:

$$f = \sum_{k=0}^{\infty} f_k^c \cos k\varphi + \sum_{k=1}^{\infty} f_k^t \sin k\varphi\,; \quad \psi = \sum_{k=1}^{\infty} \psi_k^c \sin k\varphi - \sum_{k=0}^{\infty} \psi_k^t \cos k\varphi, \qquad (1)$$

where the functions f in the common notation [1] mean functions $u, w,$ ε_1, ε_2, ϑ_1, χ_1, χ_2, $N_1, N_2, Q_1, M_1, M_2, q_1, q_3$; functions ψ – functions v, γ_{12},

ϑ_2, χ_{12}, S, Q_2, M, q_2, and f^c_k, f^t_k, ψ^c_k, ψ^t_k – coefficients of their expansion in trigonometric series.

With this choice of functions, the expansion coefficients with the superscript "t", which correspond to a skew-symmetric deformation of the shell meridian, are determined by exactly the same system of equations as the coefficients with the index "c", which correspond to the symmetric deformation. Therefore, the results of further transformations for these coefficients coincide, which allows them to be carried out only for functions with index "c", omitting this sign.

Moreover, the displacements and forces that correspond to the "k"-th term of the expansion are determined by the formulas [1]:

$$\begin{aligned} &u = u_k \cos k\varphi, && v = v_k \sin k\varphi, && w = w_k \cos k\varphi, \\ &\vartheta_1 = \vartheta_{1k} \cos k\varphi, && \vartheta_2 = \vartheta_{2k} \sin k\varphi, && N_1 = N_{1k} \cos k\varphi, \\ &S^* = S_k^* \sin k\varphi, && Q_1^* = Q_{1k}^* \cos k\varphi, && N_2 = N_{2k} \cos k\varphi, \\ &M_1 = M_{1k} \cos k\varphi, && M_2 = M_{2k} \cos k\varphi, && M = M_k \sin k\varphi. \end{aligned} \tag{2}$$

The use of the Fourier method (this is possible in the case when the shell wall thickness in the circumferential direction is constant, but changes only in the meridional $h = h(s)$ allows reducing the adopted system of partial differential equations of state of the shell to a system of ordinary differential equations with respect to the expansion coefficients of the corresponding functions in trigonometric series in the form:

$$\frac{du_k}{ds} = -\mu \frac{\cos\theta}{r} u_k - \mu \frac{k}{r} v_k - \left(\frac{1}{R_1} + \mu \frac{\sin\theta}{r} \right) w_k + \frac{1-\mu^2}{Ehr} (N_{1k} r),$$

$$\frac{dv_k}{ds} = \frac{k}{r} u_k + \frac{\cos\theta}{r} v_k + \frac{2(1+\mu)}{Ehr} (S_k^* r),$$

$$\frac{dw_k}{ds} = \frac{1}{R_1} u_k - \vartheta_{1k},$$

$$\frac{d\vartheta_{1k}}{ds} = -\mu \frac{k}{r^2} \sin\theta v_k - \mu \frac{k^2}{r^2} w_k - \mu \frac{\cos\theta}{r} \vartheta_{1k} + \frac{12(1-\mu^2)}{Eh^3 r} (M_{1k} r),$$

$$\begin{aligned} \frac{d(N_{1k} r)}{ds} = &-\frac{Eh}{r} \left[\cos^2\theta + \frac{k^2 h^2 \sin^2\theta}{6(1+\mu) r^2} \right] u_k + k \frac{Eh}{r} \cos\theta v_k + \\ &+ \frac{Eh}{r} \sin\theta \cos\theta \left[1 - \frac{k^2 h^2}{6(1+\mu) r^2} \right] w_k - \frac{k^2 E h^3}{6(1+\mu) r^2} \sin\theta \cdot \vartheta_{1k} + \\ &+ \frac{\mu}{r} \cos\theta (N_{1k} r) - \frac{k}{r} (S_k^* r) - \frac{1}{R_1} (Q_{1k}^* r) - q_{1k} r, \end{aligned}$$

$$\frac{d\left(S_k^* r\right)}{ds}=\frac{Eh}{r}k\cos\theta u_k+\frac{Eh}{r}k^2 v_k+\frac{Eh}{r}k\sin\theta\left(1+\frac{k^2h^2}{12r^2}\right)w_k+$$

$$+\frac{Eh^3}{12r^2}k\sin\theta\cos\theta\cdot\vartheta_{1k}+\mu\frac{k}{r}(N_{1k}r)-\frac{\cos\theta}{r}(S_k^* r)+\mu\frac{k}{r^2}\sin\theta(M_{1k}r)-q_{2k}r,$$

$$\frac{d\left(Q_{1k}^* r\right)}{ds}=\frac{Eh}{r}\sin\theta\cos\theta\left[1-\frac{k^2h^2}{6(1+\mu)r^2}\right]u_k+\frac{Eh}{r}k\sin\theta\left(1+\frac{k^2h^2}{12r^2}\right)v_k+$$

$$+\frac{Eh}{r}\left[\sin^2\theta+\frac{k^4h^2}{12r^2}+\frac{k^2h^2\cos^2\theta}{6(1+\mu)r^2}\right]w_k+\frac{3+\mu}{1+\mu}\cdot\frac{Eh^3}{12r^2}k^2\cos\theta\cdot\vartheta_{1k}+$$

$$+\left(\frac{1}{R_1}+\mu\frac{\sin\theta}{r}\right)\left(N_{1k}r\right)+\mu\frac{k^2}{r^2}\left(M_{1k}r\right)-q_{3k}r,$$

$$\frac{d\left(M_{1k}\cdot r\right)}{ds}=-\frac{k^2Eh^3}{6(1+\mu)r^2}\sin\theta\cdot u_k+\frac{Eh^3}{12r^2}k\sin\theta\cos\theta\cdot v_k+\frac{3+\mu}{1+\mu}\frac{Eh^3}{12r^2}k^2\cos\theta\cdot w_k+$$

$$+\frac{Eh^3}{12r}\left(\cos^2\theta+\frac{12k^2}{1+\mu}\right)\vartheta_{1k}+(Q_{1k}^* r)+\mu\frac{\cos\theta}{r}\left(M_{1k}\cdot r\right). \qquad (3)$$

Here, the components of the vector of the main variables of the stress-strain state are the expansion coefficients of displacements $u_k, v_k, w_k, \vartheta_{1k}$. As for the coefficients of decomposition of force factors, it is convenient to take their product by the radius r of the parallel circle as the main unknowns $N_{1k}r$, $S_k^* r$, $Q_{1k}^* r$, $M_{1k}r$.

The coefficients of decomposition of displacements and forces, which are not the main variables, using the relations of the theory of elasticity and the dependencies between displacements and deformations are expressed in terms of the main variables as follows:

$$\vartheta_{2k}=\left(\frac{\sin\theta}{r}v_k+\frac{k}{r}w_k\right)\sin k\varphi,$$

$$N_{2k}=\left[\mu N_{1k}+Eh\left(\frac{k}{r}v_k+\frac{\cos\theta}{r}u_k+\frac{\sin\theta}{r}w_k\right)\right]\cos k\varphi,$$

$$M_{2k}=\left[\mu M_{1k}+\frac{Eh^3}{12}\left(\frac{\cos\theta}{r}\vartheta_{1k}+\frac{k}{r^2}\sin\theta v_k+\frac{k^2}{r^2}w_k\right)\right]\cos k\varphi,$$

$$M_k=D\left(-\frac{k}{r}\vartheta_{1k}-\frac{k\cos\theta}{r^2}w_k+\frac{k\sin\theta}{r^2}u_k\right)\sin k\varphi. \qquad (4)$$

Here k – is the harmonic number of the decomposition; $R_1, R_2, r(s)$ – the radii of curvature of the shell surface and the parallel circle; $\theta(s)$ – angle between normal and axis of rotation shell; $D = Eh^3 / (12(1-\mu^2))$ – cylindrical stiffness; E, μ – elastic modulus and Poisson's ratio, respectively.

A disadvantage of the system of equations (3) is that the forces and displacements are related to the local coordinate system associated with the normal and tangent to the meridian of the shell. Therefore, the coefficients of the system have discontinuities when the meridian of the shell consists of several sections with corner points between them. In this case, it is necessary to draw up the compatibility equation for different sections.

According to [1], these difficulties can be circumvented if we pass to global coordinates. For this, forces and displacements are projected not on the tangent and normal to the meridian, but on the normal to the axis of symmetry of the shell and to the axis itself. In this case, instead of displacements u, w, displacements ξ, ζ are introduced, and instead of forces N_1, Q_1^*, forces X, Z are introduced as follows:

$$\xi = u\cos\theta + w\sin\theta \qquad \zeta = u\sin\theta - w\cos\theta,$$

$$X = N_1\cos\theta + Q_1^*\sin\theta \qquad Z = N_1\sin\theta - Q_1^*\cos\theta. \tag{5}$$

The same dependencies are related to each other and the coefficients of the expansion in the Fourier series of the corresponding functions. Substituting $u_k, w_k, N_{1k}, Q_{1k}^*$ and their derivatives through ξ_k, ζ_k, X_k, Z_k into system (3) brings it to the form:

$$\frac{d\xi_k}{ds} = -\mu\frac{\cos\theta}{r}\xi_k - \mu\frac{k\cos\theta}{r}v_k - \sin\theta\cdot\vartheta_{1k} + \frac{1-\mu^2}{Eh}\frac{\cos^2\theta}{r}\left(X_k r\right) + \frac{1-\mu^2}{Eh}\frac{\sin\theta\cos\theta}{r}\left(Z_k r\right),$$

$$\frac{d\zeta_k}{ds} = -\mu\frac{\sin\theta}{r}\xi_k - \mu\frac{k\sin\theta}{r}v_k + \cos\theta\cdot\vartheta_{1k} + \frac{1-\mu^2}{Eh}\frac{\sin\theta\cdot\cos\theta}{r}\left(X_k r\right) + \frac{1-\mu^2}{Eh}\frac{\sin^2\theta}{r}\left(Z_k r\right),$$

$$\frac{dv_k}{ds} = k\frac{\cos\theta}{r}\xi_k + k\frac{\sin\theta}{r}\zeta_k + \frac{\cos\theta}{r}v_k + \frac{2(1+\mu)}{Ehr}\left(S_k^*\cdot r\right),$$

$$\frac{d\vartheta_{1k}}{ds} = -\mu k^2\frac{\sin\theta}{r^2}\xi_k + \mu k^2\frac{\cos\theta}{r^2}\zeta_k - \mu k\frac{\sin\theta}{r^2}v_k - \mu\frac{\cos\theta}{r}\vartheta_{1k} + \frac{12(1-\mu^2)}{Eh^3 r}\left(M_{1k}\cdot r\right),$$

$$\frac{d\left(X_k\cdot r\right)}{ds} = \frac{Eh}{r}\left(1 + \frac{h^2k^4}{12r^2}\sin^2\theta\right)\xi_k - \frac{Eh^3}{12}\cdot\frac{k^4\sin\theta\cdot\cos\theta}{r^3}\zeta_k +$$

$$+\frac{Ehk}{r}\left(1+\frac{h^2k^2}{12r^2}sin^2\,\theta\right)v_k+\frac{Eh^3k^2}{12}\cdot\frac{sin\,\theta\cdot cos\,\theta}{r^2}\vartheta_{1k}+\mu\frac{cos\,\theta}{r}\left(X_k\cdot r\right)+$$

$$+\mu\frac{sin\theta}{r}\left(Z_k\cdot r\right)-k\frac{cos\theta}{r}\left(S_k^*\cdot r\right)+\mu k^2\frac{sin\theta}{r^2}\left(M_{1k}\cdot r\right)-rq_{xk},$$

$$\frac{d\left(Z_k\cdot r\right)}{ds}=-k^4\frac{Eh^3}{12}\cdot\frac{sin\,\theta\cdot cos\,\theta}{r^3}\xi_k+\frac{Eh^3}{12r^3}\left(\frac{2k^2}{1+\mu}+k^4\,cos^2\,\theta\right)\zeta_k-$$

$$-\frac{Eh^3k^3}{12}\cdot\frac{sin\,\theta\cdot cos\,\theta}{r^3}v_k-\frac{Eh^3k^2}{12}\cdot\frac{2+(1+\mu)cos^2\,\theta}{(1+\mu)r^2}\vartheta_{1k}-k\frac{sin\,\theta}{r}\left(S_k^*\cdot r\right)-$$

$$-\mu k^2\frac{cos\theta}{r^2}\left(M_{1k}\cdot r\right)-rq_{zk},$$

$$\frac{d\left(S_k^*\cdot r\right)}{ds}=\frac{Ehk}{r}\left(1+\frac{h^2k^2}{12r^2}sin^2\,\theta\right)\xi_k-\frac{Eh^3k^3}{12}\cdot\frac{sin\,\theta\cdot cos\,\theta}{r^3}\zeta_k+$$

$$+\frac{Ehk^2}{r}v_k+\frac{Eh^3k}{12r^2}\cdot sin\,\theta\cdot cos\,\theta\cdot\vartheta_{1k}+\mu k\frac{cos\,\theta}{r}\left(X_kr\right)+$$

$$+\mu\frac{k\,sin\,\theta}{r}(Z_k\cdot r)-\frac{cos\theta}{r}\left(S_k\cdot r\right)+\mu\frac{k\,sin\theta}{r^2}\left(M_{1k}\cdot r\right)-rq_{2k},$$

$$\frac{d\left(M_{1k}\cdot r\right)}{ds}=\frac{Eh^3k^2}{12}\cdot\frac{sin\,\theta\cdot cos\,\theta}{r^2}\xi_k-\frac{Eh^3k^2}{12}\frac{2+(1+\mu)\,cos^2\,\theta}{(1+\mu)r^2}\zeta_k+$$

$$+\frac{Eh^3k}{12}\cdot\frac{sin\,\theta\cdot cos\,\theta}{r^2}v_k+\frac{Eh^3}{12r}\left(cos^2\,\theta+\frac{2k^2}{1+\mu}\right)\vartheta_{1k}+\qquad(6)$$

$$+sin\theta\left(X_kr\right)-cos\,\theta\left(Z_kr\right)-\mu\frac{cos\theta}{r}\left(M_{1k}\cdot r\right).$$

where for the radial and axial components of the loading such designations are introduced:

$$q_{xk}=q_{1k}\,cos\,\theta+q_{3k}\,sin\,\theta\,;\qquad q_{zk}=q_{1k}\,sin\,\theta-q_{3k}cos\,\theta.\qquad(7)$$

Since the coefficients of the obtained system of equations do not contain the curvature of the meridian $1/R_1$, they remain continuous even for a shell whose curvature is discontinuous. As a result, the main unknowns assigned to the fixed coordinate system remain continuous with an arbitrary meridian shape, including for composite shells, which allows us not to compose docking equations for such cases. As for power unknowns, $X_kr, Z_kr, S_k^*r, M_{1k}r$ experience discontinuities of a previously known magnitude only where concentrated efforts are applied to the shells on a specific parallel of the load.

For arbitrary *k*, the system of equations (6) is of the eighth order. For $k=0$ the system breaks down into two: a system that describes axisymmetric torsion, and a system that describes axisymmetric bending of the shell. System order can also be reduced in case of wind load ($k=1$). For the case $\theta=0$ (6) also splits into two systems of four equations, which describe, respectively, asymmetric transverse bending and tensile tension in its plane of an annular plate along the radius of thickness.

It is convenient to represent the resulting closed-loop system (6) of linear differential equations in matrix form:

$$\frac{d\overline{Y}_k}{ds}=\overline{A}_k\overline{Y}_k+\overline{B}_k\,, \tag{8}$$

where the components of the state vector $\overline{Y}_k=\{y_{ik}\}$ for the shells are quantities ξ_k, ζ_k, v_k, ϑ_{1k}, $X_k r$, $Z_k r$, $S_k^* r$, $M_{1k} r$, and matrix elements $\overline{A}_k$ and column vector loads $\overline{B}_k$ – corresponding variable system coefficients (6).

When solving specific problems, the system of equations of state (6) is supplemented by an appropriate number of boundary conditions characterizing the method of fixing the shell contour at the start s_0 and end s_L points:

$$\overline{F}\left(\overline{Y}(s_p)\right)=0,\quad (s_p=s_0\vee s_L)\,. \tag{9}$$

Expanding the elements of matrix $\overline{F}$ (9) in Fourier series, we can obtain the corresponding boundary conditions for all harmonics. For asymmetric loading of shells, system (6) consists of $8k$ equations, and for round (ring) plates, it consists of $4k$ equations. The solution of problems (8), (9) is carried out by the sweep method with orthogonalization according to S. K. Godunov [17].

Thus, the problem of calculating shells of rotation of a variable along the stiffness meridian under asymmetric loading reduces to solving a set of boundary value problems for systems of ordinary differential equations (8) $(k=1,2,...,\infty)$ with respect to variables along the meridian of the coefficients of expansion of the main variables in Fourier series with boundary conditions (9).

2. An algorithm for predicting the values of expansion coefficients in trigonometric Fourier series

In this section, we propose a technique for reducing the number of one-dimensional boundary value problems (8), (9) necessary to achieve the given accuracy of solving the problem in the form (1) to determine the stress-strain state of shells of rotation of a variable along the meridian of the wall thickness.

It is known [16] that the approximation of a certain function $g(z)$, $z\in[-\pi,\pi]$ can be represented in the form of a trigonometric Fourier series:

$$\mathrm{g}(z)\approx a_0/2+\sum_{k=1}^{m}\left(a_k\,cos\,(kz)+b_k\,sin\,(kz)\right), \tag{10}$$

where the coefficients a_0, a_k, b_k (the sequence of values of which converge with increasing harmonic numbers, not necessarily monotonously, to zero) are determined by the well-known Euler – Fourier formulas:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} g(z)dz, \quad a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} g(z)\cos(kz)dz,$$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} g(z)\sin(kz)dz, \quad k = 1,2,...,\infty. \tag{11}$$

The idea of the proposed approach is to reduce the number of calculations of coefficients a_k, b_k in accordance with (11) by periodically extrapolating their values using the results of calculations of the previous coefficients of this series, thus replacing them with some forecast values calculated using simple formulas.

To construct such forecast values in optimization problems in [18], the simulation forecasting algorithm was used, one of the difficulties of its application was the need to determine the weight coefficients of the forecast formula using the results of a special numerical experiment, and interpolation with cubic splines when solving shell calculation problems in [19].

In this paper, to solve this problem, we propose the joint use of the Aitken – Steffensen extrapolation dependences [20] and in the form of an increment of the Adams method [7], which is quite effective in solving the Cauchy problem for systems of ordinary differential equations and is based on extrapolation dependences -compasses of Lagrange and Newton.

Let, as a result of three successive calculations of the coefficients of the Fourier series a_k, b_k (hereinafter, denoted $c_k = a_k \vee b_k$), we obtain the values c_{k-2}, c_{k-1}, c_k .

In this case, the following cases of sequences of changes in the coefficient values c_k depending on the number are possible k (Fig. 1).

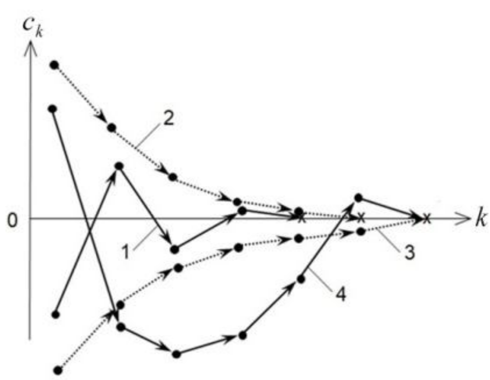

Fig. 1. Possible trajectories of changes in values Fourier series coefficients depending on the harmonic number

Here, line 1 corresponds to the case of a nonmonotonic sequence:

$$(c_{k-1}-c_{k-2})(c_k-c_{k-1})<0 \tag{12}$$

lines 2, 3 – respectively, monotonically decreasing and increasing sequences:

$$(c_{k-1}-c_{k-2})(c_k-c_{k-1})\geq 0 \tag{13}$$

line 4 – mixed.

The predicted values of the next member c_{k+1} of the sequence of these coefficients are determined by the results of the values of the three previous members of this series c_{k-2}, c_{k-1}, c_k, calculated using (11) depending on the type of iterative process. In the case when the sequence c_k is nonmonotonic (12), the forecast is proposed to be carried out according to the Aitken – Steffensen formula [20]:

$$c_{k+1} = \frac{c_k c_{k-2} - (c_{k-1})^2}{c_{k-2} - 2c_{k-1} + c_k}, \quad k \geq 2, \tag{14}$$

and in the case of a monotonic process (13) – in the form of an increment in the second order Adams formula:

$$c_{k+1} = \frac{23c_k - 16c_{k-1} + 5c_{k-2}}{12}, \quad k \geq 2, \tag{15}$$

Further, using coefficients (11), c_{k+2}, c_{k+3} are calculated and the process of forecasting the next value c_{k+4} is continued taking into account three new points, starting from the forecast point.

Thus, this approach avoids the need to calculate the coefficients of the Fourier series at every third step.

3. Numerical results

The developed approach was tested for the case of acceleration of convergence of the iterative algorithm for solving physically nonlinear problems of the mechanics of shells of rotation in [7].

Here, the reliability of the proposed approach was verified using the results of a system numerical experiment by predicting the values of the expansion coefficients of Fourier series of known functions, typical sequences of which are shown in Fig. 1.

As an example in Fig. 2(a), a sequence of values of coefficients (11) for $k = \overline{0,20}$ is shown where «•» is the predicted values (14), (15) of the coefficients a_k of the function:

$$f(z) = \begin{cases} 1, & 0 \leq z \leq d, \quad d = \pi/10, \\ 0, & d < z \leq \pi, \end{cases}$$

the coefficients of the expansion of which in a Fourier series in cosines are: $a_0 = 2d/\pi$; $a_k = 2\,sin(kd)/(k\pi)$; $k = \overline{1,\infty}$.

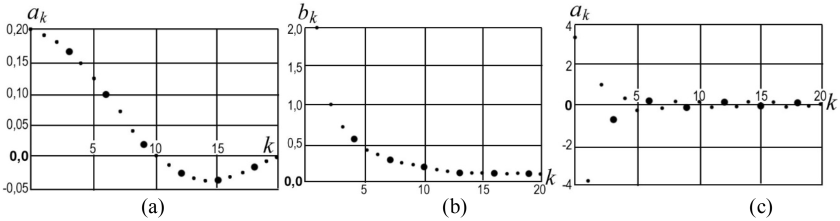

Fig. 2. The calculated (·) and forecast (•) values
the coefficients Fourier expansion of given functions

Fig. 2(b) shows a graph of the monotonically decreasing values of the calculated and predicted expansion coefficients in the Fourier series with respect to the sines of the function $f(z)=\pi-z,\ \ z\in\left[0,2\pi\right]$, which have the form $b_k=2/k,\ \ k=\overline{1,\infty}$.

Changing the expansion coefficients in the Fourier series with respect to the cosines of a function $f(z)=z,\ z\in[-\pi,\pi]$, for which $a_0=\pi^2/3$; $a_k=\left(-1\right)^k\ 4/k^2$, $k=\overline{1,\infty}$ are given in Fig. 2(c).

The relative error in the calculation, for example, of the value of the function $f(z)=1$ (for $z=0$) (Fig. 2(a)) when using the first 20 harmonics of decomposition, of which 7 are determined by the forecast results, was 1,4%, and when using the exact values of the coefficients – 0,3%.

It should be noted that for higher harmonics, the relative error in the predictive determination of the values of the coefficients of the Fourier series can increase. However, taking into account that with increasing k the coefficients of the Fourier series decrease to zero, their contribution to the final value of the function decreases, which, on the whole, practically does not affect the accuracy of the solution to the problem. From the analysis of the results shown in Fig. 2, it follows that, if necessary, some of the Fourier coefficients can be determined approximately, that is, not by the usual way of calculating the integrals (11), but as a result of interpolation at some reference points.

It should be noted that the examples of forecasting the Fourier coefficients considered above to illustrate the main idea of the proposed approach do not have independent practical value, since the process of interpolating the coefficients for the considered functions of one variable requires no less computational costs than the usual method of obtaining the coefficients in the form (10), (11).

At the same time, this approach is very effective in problems of calculating asymmetrically loaded shells of revolution with a variable along the meridian thickness, when the Fourier coefficients (1) are functions of the longitudinal coordinate, and are calculated as a result of solving the boundary value problem (8), (9). In this case, the systems of differential equations (6) with respect to the amplitudes of expansion into trigonometric series are solved only for individual “reference” harmonics, and the amplitudes for each third harmonic are calculated as a result of interpolation of their values $(s_0\le s^*\le s_L)$ for all nodal integration points (8), (9). This can significantly reduce the computational cost of obtaining a solution as a whole.

The effectiveness of the approach is illustrated in Fig. 3(a) by the calculation of a thin elastic steel ring plate under the action of an asymmetric load:

$$q\left(r,\varphi\right)=\begin{cases}q(r), & -\pi/5\le\varphi\le\pi/5,\\ 0, & \varphi\notin\left[-\pi/5;\pi/5\right].\end{cases}$$

represented by its expansion in Fourier series in cosines:

$$q(r,\varphi) = q_0(r)/2 + \sum_{k=1}^{N} q_k(r)\cos(k\varphi), \quad -\pi \le \varphi \le \pi .$$

The solution of the arising systems of ordinary differential equations (8), (9) with respect to the expansion coefficients was carried out by the method of sweeping with orthogonalization according to Godunov for various values of geometric parameters and options for fixing the contour. The dependence on the number k of the coefficients of the expansion of the radial deflection parameter $w_k E / q_k$ at the point $r = r_2$ in the Fourier series in cosines is shown in Fig. 3(b), and in Fig. 3(c) is the parameter $M_{1(k)}^{(k)} / q_k$ of the longitudinal moment at the point $r = r_1$ for the case of an annular plate (Fig. 3(a)) with a thickness $h = 0,05\,\text{m}$ with a clamped internal $r_1 = 0,2\text{m}$ and free external contour $r_2 = 1\,\text{m}$; $E = 200\,\text{GPa}$; $\mu = 0,3$.

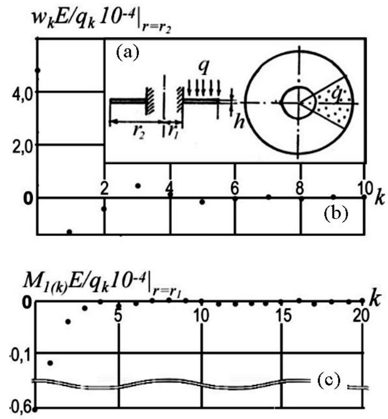

Fig. 3. The values of the expansion coefficients in the Fourier series of the deflection (b) and longitudinal moment (c) for the annular plate (a)

In this case, to compute the terms of the Fourier series $w_k(s)$, $M_{1k}(s)$, $(k = \overline{0,20})$, 14 solutions of the boundary value problems (8), (9) were needed without a noticeable loss of accuracy (about 0,07%) compared to using the Fourier coefficients obtained as a result of solving 21 such boundary value problems.

The computational efficiency of the proposed approach is most clearly manifested when it is used in problems of optimal design of structures [7], since multi-step iterative optimization algorithms provide for the solution of direct problems of calculating the stress-strain state of a structure at each search step, which leads to rather large computational costs.

Conclusions

Thus, the presented article proposes a fairly general and effective way to reduce the computational costs arising in the problems of calculating asymmetrically loaded elements of shell structures using the Fourier method by reducing the number of solutions to the corresponding one-dimensional boundary value problems. Such an approach can be useful in solving a fairly wide range of problems in the mechanics of shells, as well as the mechanics of liquid, gas, and plasma, and in other fields.

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АЛГОРИТМ ЗМЕНШЕННЯ ОБЧИСЛЮВАЛЬНИХ ВИТРАТ В ЗАДАЧАХ РОЗРАХУНКУ НЕСИМЕТРИЧНО НАВАНТАЖЕНИХ ОБОЛОНОК ОБЕРТАННЯ

Задача розрахунку оболонок обертання змінної уздовж меридіана жорсткості при несиметричному навантаженні зводиться до сукупності систем одновимірних крайових задач щодо амплітуд розкладання шуканих функцій в тригонометричні ряди Фур'є.

Пропонується методика зменшення кількості одновимірних крайових задач, необхідних для досягнення заданої точності визначення напружено-деформованого стану оболонок обертання зі змінною уздовж меридіана товщиною стінки при несиметричному навантаженні. Ідея запропонованого підходу полягає в застосуванні періодичного екстраполювання (прогнозування) значень коефіцієнтів розкладання шуканих функцій з використанням результатів обчислень попередніх коефіцієнтів відповідного тригонометричного ряду, замінюючи їх, таким чином, деякими прогноз-значеннями, обчисленими за простими формулами.

Для вирішення цієї задачі пропонується сумісне використання екстраполяційних залежностей Ейткена – Стеффенса і в формі складової приросту в методі Адамса, який є досить ефективним при розв'язанні задачі Коші для систем звичайних диференціальних рівнянь і базується на екстраполяційних залежностях Лагранжа і Ньютона.

Перевірка достовірності запропонованого підходу здійснювалася за результатами системного числового експерименту шляхом прогнозування значень коефіцієнтів розкладень в ряди Фур'є відомих функцій однієї змінної.

Підхід виявляється досить ефективним в задачах розрахунку несиметрично навантажених оболонок обертання зі змінною вздовж меридіана товщиною, коли коефіцієнти розкладання шуканих функцій в ряди Фур'є є функціями поздовжньої координати і обчислюються в результаті розв'язання відповідної крайової задачі. В цьому випадку підхід дозволяє вирішувати системи диференціальних рівнянь щодо амплітуд розкладання в тригонометричні ряди тільки для окремих «опорних» гармонік, а амплітуди для кожної третьої гармоніки можуть бути обчислені в результаті інтерполяції їх значень для всіх вузлових точок інтегрування відповідної крайової задачі. Це дозволяє істотно скоротити обчислювальні витрати на отримання розв'язку в цілому.

Як приклад наведено результати розрахунку напружено-деформованого стану сталевої кільцевої пластини при несиметричному поперечному навантаженні.

Ключові слова: оболонки обертання, змінна жорсткість; несиметричне навантаження, прогнозування коефіцієнтів в методі Фур'є; зниження обчислювальних витрат.

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ALGORITHM FOR REDUCING THE CALCULATIONAL COSTS IN THE PROBLEM OF CALCULATION ASYMMETRIC LOADING ROTATION SHELLS

The problem of calculating the shells of rotation of a variable along the meridian of rigidity under asymmetric loading is reduced o a set of systems of one-dimensional boundary value problems with respect to the amplitudes of decomposition of the required functions into trigonometric Fourier series.

A method for reducing the number of one-dimensional boundary value problems required to achieve a given accuracy in determining the stress-strain state of the shells of rotation with a variable along the meridian wall thickness under asymmetric load. The idea of the proposed approach is to apply periodic extrapolation (prediction) of the values of the decomposition coefficients of the required functions using the results of calculations of previous coefficients of the corresponding trigonometric series, thus replacing them with some prediction values calculated by simple formulas.

To solve this problem, we propose the joint use of Aitken-Steffens extrapolation dependences and Adams method in the form of incremental component, which is quite effective in solving the Cauchy problem for systems of ordinary differential equations and is based on Lagrange and Newton extrapolation dependences.

The validity of the proposed approach was verified b the results of a systematic numerical experiment by predicting the values of the expansion coefficients in the Fourier series of known functions of one variable.

The approach is quite effective in the calculation of asymmetrically loaded shells of rotation with variable along the meridian thickness, when the coefficients of decomposition of the required functions into Fourier series are functions of the longitudina lcoordinate and are calculated by solving the corresponding boundary value problem. In this case, the approach allows solving solutions of differential equations for the amplitudes of decompositionin to trigonometric series only for individual "reference" harmonics, and the amplitudes for every third harmonic can be calculated by interpolating their values for all node integration points of the corresponding boundary value problem. This significantly reduces the computational cost of obtaining the solution as a whole.

As an example, the results of the calculation of the stress-strain state of a steel annular plate under asymmetric transverse loading are given.

Keywords: rotation shells**,** variable stiffness, asymmetric loading, coefficient prediction in the Fourier method, reduction in computing costs.

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АЛГОРИТМ СНИЖЕНИЯ ВЫЧИСЛИТЕЛЬНЫХ ЗАТРАТ В ЗАДАЧАХ РАСЧЕТА НЕСИММЕТРИЧНО НАГРУЖЕННЫХ ОБОЛОЧЕК ВРАЩЕНИЯ

Задача расчета оболочек вращения переменной вдоль меридиана жесткости при несимметричном нагружении сводится к совокупности систем одномерных краевых задач относительно амплитуд разложения искомых функций в тригонометрические ряды Фурье. Предложен подход, основанный на прогнозировании значений переменных вдоль меридиана коэффициентов разложений для сокращения необходимого количества решений таких одномерных задач. Это позволяет снизить вычислительные затраты на поиск решения. В качестве примера приведены результаты расчёта напряженно-деформированного состояния стальной кольцевой пластины при несимметричном поперечном нагружении.

Ключевые слова: оболочки вращения, переменная жесткость, несимметричное нагружение, прогнозирования коэффициентов в методе Фурье; снижение вычислительных затрат.

УДК 539.3
Дзюба А.П., Сафронова І.А., Левитіна Л.Д. **Алгоритм зменшення обчислювальних витрат в задачах розрахунку несиметрично навантажених оболонок обертання** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2020. – Вип. 105. – С. 99-113. – Англ.

Запропоновано алгоритм прогнозування значень змінних уздовж меридіана коефіцієнтів розкладання в ряди Фур'є для зниження обчислювальних витрат в задачах розрахунку несиметрично навантажених оболонок обертання змінної жорсткості.
Іл. 3. Бібліогр. 20 назв.

UDC 539.3
Dzyuba A. P., Safronova I. A., Levitina L. D. **Algorithm for reducing computational costs in problems of calculation of asymmetrically loaded shells of rotation** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – K.: KNUBA, 2020. – Issue 105. – P. 99-113.

An algorithm for predicting the values of variables along the meridian of decomposition coefficients into Fourier series to reduce computational costs in the calculation of asymmetrically loaded shells of rotation of variable stiffness is proposed.
Figs. 3. Refs. 20.

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Дзюба А.П., Сафронова И.А., Левитина Л.Д. **Алгоритм снижения вычислительных затрат в задачах расчета несимметрично нагруженных оболочек вращения** // Сопротивление материалов и теория сооружений: науч.-техн. сборник. – К.: КНУБА, 2020. – Вып. 105. – С. 99-113. – Англ.

Предложен алгоритм прогнозирования значений переменных вдоль меридиана коэффициентов разложения в ряды Фурье для снижения вычислительных затрат в задачах расчета несимметрично нагруженных оболочек вращения переменной жесткости.
Ил. 3. Библиогр. 20 назв.

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