UDC 539.375

BASIC RELATIONSHIPS FOR PHYSICALLY AND GEOMETRICALLY NONLINEAR PROBLEMS OF DEFORMATION OF PRIMATIC BODIES

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DOI: 10.32347/2410-2547.2020.104.255-264

The initial relations of thermo elastic-plastic deformation of prismatic bodies are given in the paper. The basic concepts, indifference of deformation tensors, with the condition of energy conjunction in description of the shaping process are laid out on the basis of classical works.

Keywords: prismatic bodies, physical and geometric nonlinearity, thermo elasticplastic deformation, shaping process, Finger measure, Aldroid derivative.

Introduction. A number of responsible structures elements, which are prismatic bodies, are undergoing a significant shaping in the process of manufacturing and operation, which often take place at high temperatures, which leads to changes in the physical and mechanical characteristics of the material and the development of various types of deformations. Due to the possibility of simultaneous occurrence of plasticity and creep deformations caused both of the presence of force load and external temperature influences, determining the bearing capacity of these objects requires the solution of the problems of thermo elastoplasticity. The solution authenticity of such problems of the deformable body mechanics depends essentially on the adequacy of the physical relations used to the considered processes of the material deformation, in particular taking into account the presence of large deformations.

The purpose of this work is to select adequately the basic relations of geometrically nonlinear problems of thermo elasto-plasticity for prismatic bodies.

Initial relations for the problems of the theory of elasticity, plasticity and creep. Consider a curvilinear prismatic body of complex shape (Fig. 1) with variable geometric and physical characteristics in the basic coordinate system $z^{i'}$. It is used to describe boundary conditions, external influences, and object configuration. Fig. 1 shows also a local curvilinear coordinate system x^i that is related to its geometry.

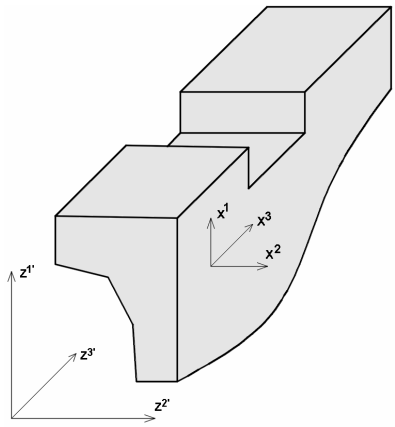

Fig. 1. Curvilinear prismatic body of complex shape

The transformation tensor that determines the relationship between the local and basic coordinate systems is known at each point in the body:

$$z^{i'}_{,j} = \frac{\partial z^{i'}}{\partial x^j}. \qquad (1)$$

The indexes indices by Latin letters taking values 1, 2, 3, and taking the values 1, 2 when indices in Greek letters hereinafter.

The covariant components of the metric tensor of the local coordinate system are represented by the covariant components of the metric tensor of the basic coordinate system according to formula:

$$g_{ij} = z^{m'}_{,i} z^{n'}_{,j} g_{mn}. \qquad (2)$$

It is most advisable to use a Cartesian coordinate system as a basis for the study of prismatic bodies. Three components of the metric tensor are non-zero in this case:

$$g_{1'1'} = 1, \quad g_{2'2'} = 1, \quad g_{3'3'} = 1. \qquad (3)$$

Then the covariance components of the metric tensor of the local coordinate system are determined by the formula:

$$g_{ij} = z^{m'}_{,i} z^{n'}_{,j}. \qquad (4)$$

We find the covariance components of the metric tensor of the local coordinate system using the following relation:

$$g^{ij} = \frac{A(g^{ij})}{g}. \qquad (5)$$

where $A(g^{ij})$ is the algebraic complement of the each element in a matrix composed of the covariance components of the metric tensor, $g = \det(g_{ij})$ - the determinant of that matrix.

The relation for determining the deformation components due to the displacements in the local coordinate system have the form [20]:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x^j} + \frac{\partial u_j}{\partial x^i}\right) - u_k \Gamma^k_{ij}, \qquad (6)$$

where Γ^k_{ij} - the second kind Christoffel symbols.

In the basis Cartesian coordinate system all the Christoffel symbols are equal to zero and the displacements in the local and base coordinate systems are related by the ratios:

$$u_k = u_{m'} z^{m'}_{,k} . \qquad (7)$$

On the basis of formulas (6) and (7) we obtain the expression of the components of the strain tensor in the local coordinate system by displacements in the basic one:

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{m',i} z^{m'}_{,j} + u_{m',j} z^{m'}_{,i} \right) . \qquad (8)$$

In problems of thermoelasticity the components of the complete deformation tensor are equal to amount of elastic and temperature components:

$$d\varepsilon_{ij} = d\varepsilon^{e}_{ij} + d\varepsilon^{T}_{ij} , \qquad (9)$$

where $\varepsilon^{T}_{ij} = \alpha_T T g_{ij}$, α_T - coefficient of linear expansion of material, T – an increase of temperature in the investigated point of the body relative to its original state.

Components of the stress tensor under elastic loading connected through the components of the strain tensor in accordance with Hooke's law:

$$\sigma^{ij} = C^{ijmn} \varepsilon^{e}_{mn} , \qquad (10)$$

or subject to (7)

$$\sigma^{ij} = C^{ijmn} (\varepsilon_{mn} - \varepsilon^{T}_{mn}) . \qquad (11)$$

The components of the elasticity tensor constant for isotropic bodies are found from the relations:

$$C^{ijmn} = \lambda g^{mn} g^{ij} + \mu (g^{mi} g^{nj} + g^{mj} g^{ni}) , \qquad (12)$$

where are the Lame coefficients λ and μ are determined by the Poisson's ratio $\nu = \nu(z^{i'}, T)$ and material elasticity modulus (Young's modulus) $E = E(z^{i'}, T)$, that depend on the temperature *T*:

$$\lambda = \frac{E\nu}{(1-2\nu)(1+\nu)}, \quad \mu = \frac{E}{2(1+\nu)} . \qquad (13)$$

To describe the process of deformation beyond the elasticity of a material whose physical properties depend on temperature, we use the theory of plastic flow [1].

It is supposed that the material is homogeneous and isotropic in the initial state, plastic non-compressed and change of material's volume is linear-elastic:

$$d\varepsilon^{p}_{ij} = 0, \quad d\varepsilon_{ij} = d\varepsilon^{e}_{ij} . \qquad (14)$$

The increment of complete deformation $d\varepsilon_{ij}$ is equal to amount of elastic deformation $d\varepsilon^{e}_{ij}$, temperature deformation $d\varepsilon^{T}_{ij}$ and deformation of plasticity $d\varepsilon^{p}_{ij}$:

$$d\varepsilon_{ij} = d\varepsilon_{ij}^{e} + d\varepsilon_{ij}^{p} + d\varepsilon_{ij}^{T} . \tag{15}$$

Elastic deformations are related to the stress of the Hooke law (10). The area of elastic deformation is limited in the space of stresses by the yield surface:

$$f_p(\sigma^{ij}, \chi, T) = 0 . \tag{16}$$

In accordance with the hypothesis of isotropic hardening under the conditions of Mises' fluidity, the equations of the yield surface are as follows:

$$f_p = \frac{1}{2} s_{ij} s^{ij} - \tau_s^2(\chi, T) = 0 , \tag{17}$$

where $\tau_s(\chi, T)$ - yield limit under pure shear, χ - Odquist's strengthening parameter:

$$\chi = \int\limits_{\varepsilon_{ij}^{p}} \sqrt{\frac{2}{3} d\varepsilon_{ij}^{p} d\varepsilon_{p}^{ij}} . \tag{18}$$

The components of the stress deviator included in expression (17) are determined by the formula:

$$s^{ij} = \sigma^{ij} - \frac{1}{3} \delta_{mn} \sigma^{mn} g^{ij} . \tag{19}$$

Stress deviator is associated with an increase in plastic deformation in accordance with the associated law of plastic yield:

$$d\varepsilon_{ij}^{p} = \lambda_p \frac{\partial f_p}{\partial s^{ij}} = \lambda_p s_{ij} . \tag{20}$$

In case of creep deformations presence the equations of state are adopted in accordance with the theory of strengthening [8]. It is assumed that the complete increments of deformation are defined as the sum of four components:

$$d\varepsilon_{ij} = d\varepsilon_{ij}^{e} + d\varepsilon_{ij}^{T} + d\varepsilon_{ij}^{p} + d\varepsilon_{ij}^{c} . \tag{21}$$

The creep surface equation looks like:

$$f_c = \frac{3}{2} s_{ij} s^{ij} - \tau_c^2(\psi, T, \varepsilon_i) = 0 . \tag{22}$$

The creep limit is determined by the formula:

$$\tau_c = \left[\frac{\varepsilon_i^c}{\alpha} (\psi)^{\beta} \right]^{\frac{1}{\gamma}} , \tag{23}$$

where α, β, γ are temperature dependent constants which characterized a creep properties of material; ψ - strengthening parameter:

$$\psi = \int_{\varepsilon_{ij}^{c}} \sqrt{\frac{2}{3} d\varepsilon_{ij}^{c} d\varepsilon_{c}^{ij}} . \tag{24}$$

The increase of creep deformations is found by the components of the stress deviator:

$$d\varepsilon_{ij}^{c} = \lambda_c \frac{\partial f_c}{\partial s^{ij}} = \lambda_c s_{ij} . \tag{25}$$

Determination of deformations in geometrically nonlinear problems. We will still use [4, 5, 6] the basic Cartesian coordinate system $Z^{i'}$ when considering spatial objects in geometrically nonlinear formulation and the local coordinate system x^i, provided that it is "frozen" into the medium and deformed with it. The positions of each particle of body at any time are determined by the radius vector:

$$\bar{r} = \bar{r}(Z^i, t) . \qquad (26)$$

We suppose that the reference initial configuration is formed by vectors $\bar{r}_0$ at time t_0, topical – vector $\bar{r}_t = R$ at time t. We also introduce the reference variable configuration that corresponds to the time $\tilde{t}$ which is close enough to t:

$$t = \tilde{t} + \Delta t . \qquad (27)$$

We denote the metric tensors of these states €, €, € respectively (Fig. 2).

The increase of time Δt chosen in a such way that during the transition from the reference variable configuration to the actual metric tensor components were corresponded to the ratio:

$$\Delta € = € - €, \quad \Delta G_{ij} \langle\langle G_{ij} . \qquad (28)$$

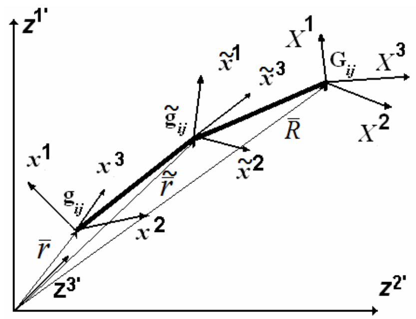

Fig. 2. Three configurations of coordinate system

The covariance components of metric tensors of configurations being entered into consideration are calculated similarly (4) through the transformation components tensor of the respective configurations.

To identify the components $\Delta €$ we will write an expression for the radius vector of a point in the current configuration $\bar{R}$, as the sum of the vector $\bar{r}_{\tilde{t}} = \bar{\tilde{r}}$ in the variable reference configuration and displacement vector $\bar{u}$ (Fig. 3):

$$\bar{R} = \bar{\tilde{r}} + \bar{u} , \qquad (29)$$

or, using of index notation:

$$Z^{m'} = \tilde{Z}^{m'} + u^{m'} . \qquad (30)$$

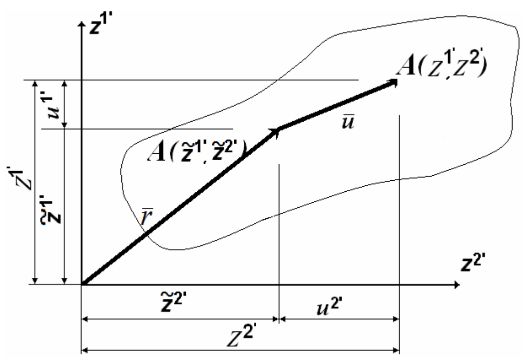

Fig. 3. Changing the position of a point according to the entered reference variable configuration

The components of the transformation tensor that determine the relationship between the local and basic coordinate systems in the current configuration are determined by the formula:

$$Z^{m'}_{,i} = \tilde{Z}^{m'}_{,i} + u^{m'}_{,i} . \tag{31}$$

The covariant components of the metric tensor of the actual configuration are represented using of formula (4):

$$G_{ij} = Z^{m'}_{,i} Z^{m'}_{,j} . \tag{32}$$

Turning (32) and taking into account (31), we obtain:

$$G_{ij} = \tilde{Z}^{m'}_{,i} \tilde{Z}^{m'}_{,j} + \tilde{Z}^{m'}_{,i} u^{m'}_{,j} + u^{m'}_{,i} \tilde{Z}^{m'}_{,j} + u^{m'}_{,i} u^{m'}_{,j} = \tilde{g}_{ij} + \Delta G_{ij} , \tag{33}$$

where

$$\Delta G_{ij} = \tilde{Z}^{m'}_{,i} u^{m'}_{,j} u^{m'}_{,i} + u^{m'}_{,i} u^{m'}_{,j} . \tag{34}$$

Counter-variant components ΔG_{ij} are determined by the condition:

$$G^{ij} G_{jl} = \delta^{i}_{l} \tag{35}$$

or

$$(\tilde{g}^{ij} + \Delta G^{ij})(g_{jl} + \Delta G_{jl}) - \delta^{i}_{l} = 0 . \tag{36}$$

Neglecting small increments of $\Delta G^{ij} \Delta G_{jl}$ value, we get:

$$\Delta G^{ij} g_{jl} + \tilde{g}^{ij} \Delta G_{jl} = 0 , \tag{37}$$

where

$$\Delta G^{ik} = -\tilde{g}^{ij} \Delta G_{jl} \tilde{g}^{lk} . \tag{38}$$

We write the expressions for the strain tensor in the current configuration using the Finger measure $\tilde{F}$ [2, 3]:

$$\tilde{\varepsilon} = \frac{1}{2}(F - \tilde{G}) . \tag{39}$$

Counter-variant components of the Finger measure F^{ij} is equal to the corresponding components of the metric tensor g^{ij} of reference initial configuration.

We present the counter-variant components of the deformation tensor in the current configuration as follows:

$$\varepsilon^{ij} = \frac{1}{2}(F^{ij} - G^{ij}) = \frac{1}{2}(g^{ij} - G^{ij}) . \tag{40}$$

Using a variable reference configuration, we represent (40) as amount of:

$$\varepsilon^{ij} = \frac{1}{2}(g^{ij} - \tilde{g}^{ij} + \tilde{g}^{ij} - G^{ij}) = \tilde{\varepsilon}^{ij} + \Delta\varepsilon^{ij} . \tag{41}$$

The components of the strain tensor $\tilde{\varepsilon}$ in the variable reference configuration relative to the initial reference one are indicated there as $\tilde{\varepsilon}^{ij}$:

$$\tilde{\varepsilon}^{ij} = \frac{1}{2}(g^{ij} - \tilde{g}^{ij}) , \tag{42}$$

and components of the strain tensor in the transition from the variable reference to the actual configuration are indicated through $\Delta\varepsilon^{ij}$:

$$\Delta\varepsilon^{ij} = \frac{1}{2}(\tilde{g}^{ij} - G^{ij})\,. \tag{43}$$

Counter-variant components of the deformation increment during transition from the reference variable to the actual configuration, taking into account (38), represented by the relations:

$$\Delta\varepsilon^{ij} = \frac{1}{2}(\tilde{g}^{ij} - G^{ij}) = \frac{1}{2}(\tilde{g}^{ij} - \tilde{g}^{ij} - \Delta G^{ij}) = -\frac{1}{2}\tilde{g}^{im}\Delta G_{mn}\tilde{g}^{jn}\,, \tag{44}$$

and the covariance components are:

$$\Delta\varepsilon_{kl} = \Delta\varepsilon^{ij}G_{ik}G_{jl} \approx \Delta\varepsilon^{ij}\tilde{g}_{jl} = \frac{1}{2}\Delta G_{kl}\,. \tag{45}$$

Using expression (34), we write the covariance components of the strain tensor in the current configuration through displacements:

$$\Delta\varepsilon_{ij} = \frac{1}{2}(\tilde{Z}^{m'}_{,i}u^{m'}_{,j} + u^{m'}_{,i}\tilde{Z}^{m'}_{,j} + u^{m'}_{,i}u^{m'}_{,j})\,. \tag{46}$$

On the other hand, the increment of the strain tensor $\Delta\text{€}$ can be expressed as the product of the strain rate tensor at Δt .

$$\Delta\text{€} = \text{€}^{ol}\cdot\Delta t\,. \tag{47}$$

The Aldroid derivative of the tensor € we represent with the relation [7]:

$$\text{€}^{ol} = \dot{\text{€}} - \nabla\overline{\vartheta}^{T}\text{€}\,. \tag{48}$$

Taking into account (39) and equivalence to zero of the operator $\nabla\overset{\text{€}}{G} = 0$, we get:

$$\text{€}^{ol} = \frac{1}{2}[(\dot{\overset{\text{€}}{F}} - \dot{\overset{\text{€}}{G}}) - \nabla\overline{\vartheta}^{T}(\overset{\text{€}}{F} - \overset{\text{€}}{G}) - (F - \overset{\text{€}}{G})\nabla\overline{\vartheta}] = \frac{1}{2}\Big[\nabla\overline{\vartheta}^{T}\overset{\text{€}}{F} + \overset{\text{€}}{F}\nabla\overline{\vartheta} -$$

$$-\dot{\overset{\text{€}}{G}} - \nabla\overline{\vartheta}^{T}\overset{\text{€}}{F} + \nabla\overline{\vartheta}^{T}\overset{\text{€}}{G} - \overset{\text{€}}{F}\nabla\overline{\vartheta} + \overset{\text{€}}{G}\nabla\overline{\vartheta}\Big] = -\frac{1}{2}(\dot{\overset{\text{€}}{G}} + \nabla\overset{\text{€}}{G}) = \frac{1}{2}\frac{\partial\overset{\text{€}}{G}}{\partial t}. \tag{49}$$

Then at $\Delta t \to 0$:

$$\Delta\text{€} = -\frac{1}{2}\frac{\partial\overset{\text{€}}{G}}{\partial t}\Delta t = -\frac{1}{2}\Delta\overset{\text{€}}{G}\,, \tag{50}$$

which is equivalent to component form (45).

Conclusion. The initial relations for physically and geometrically nonlinear problems of deformation process for space prismatic bodies being formulated above. It will allow to create new types of finite elements and to obtain corresponding ratios for calculating the coefficients of stiffness matrices and nodal reactions for a new class of problems.

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Стаття надійшла до редакції 03.02.2020 р.

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MAIN RELATIONSHIPS FOR PHYSICALLY AND GEOMETRICALLY NONLINEAR PROBLEMS OF DEFORMATION OF PRIMATIC BODIES

A number of responsible structures elements, which are prismatic bodies, are undergoing a significant shaping in the process of manufacturing and operation, which often take place at high temperatures, which leads to changes in the physical and mechanical characteristics of the material and the development of various types of deformations. The solution authenticity of such problems of the deformable body mechanics depends essentially on the adequacy of the physical relations used to the considered processes of the material deformation, in particular taking into account the presence of large deformations.

The initial relations of thermo elastic-plastic deformation of prismatic bodies are given in the paper. A Cartesian coordinate system used as a basis for the study of prismatic bodies. The relation for determining the deformation components through displacement values in the local coordinate system are formulated. The components of the complete thermo elastic-plastic and creep deformation tensor are taken as amount of appropriate deformation components. The plastic deformation described with associated law of plastic yield, a creep deformation – in accordance with the theory of strengthening The basic concepts, indifference of deformation tensors, with the condition of energy conjunction in description of the shaping process are laid out on the basis of classical work.

Keywords: prismatic bodies, physical and geometric nonlinearity, thermo elasticplastic deformation, shaping process, Finger measure, Aldroid derivative.

Максимюк Ю.В., Пискунов С.О., Шкрыль А.А., Максимюк О.В.

ОСНОВНЫЕ СООТНОШЕНИЯ ДЛЯ ФИЗИЧЕСКИХ И ГЕОМЕТРИЧЕСКИХ НЕЛИНЕЙНЫХ ЗАДАЧ ДЕФОРМИРОВАНИЯ ПРИЗМАТИЧЕСКИХ ТЕЛ

В работе приведены исходные соотношения термовязкоупругопластического деформирования призматических тел. На основе классических работ изложены основные понятия, индифферентность тензоров деформаций при условии энергетической сопряженности для описания процесса формоизменения.

Ключевые слова: призматические тела, физическая и геометрическая нелинейность, термовязкоупругопластическое деформирование, формоизменение, мера Фингера, производная Олдроида.

УДК 539.375

Максим'юк Ю.В., Пискунов С.О., Шкриль О.О., Максим'юк О.В. **Основні співвідношення для фізично і геометрично нелінійних задач деформування призматичних тіл** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – Київ: КНУБА, 2020. – Вип. 104. – С. 255-264.

В роботі наведені вихідні співвідношення термов'язкопружнопластичного деформування призматичних тіл. На основі класичних робіт викладені основні поняття, індиферентність тензорів деформацій при умові енергетичної сполученості для опису процесу формозмінення.
Табл. 0. Іл. 3. Бібліогр. 8 назв.

UDC 539.375

Maksimyuk Yu.V., Pyskunov S.O., Shkril' A.A., Maksimyuk O.V., **Basic relations for physically and geometrically nonlinear problems of deformation of prismatic bodies** // Strength of Materials and Theory of Structures: Scientific-&-Technical collected articles – Kyiv: KNUBA, 2020. – Issue 104. – P. 255-264. – Engl.

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УДК 539.375

Максимюк Ю.В., Пискунов С.О., Шкрыль А.А., Максимюк О.В. Основные соотношения для физических и геометрических нелинейных задач деформирования призматических тел // Сопротивление материалов и теория сооружений: науч.-тех. сборн. – К.: КНУСА, 2020. – Вып. 104. – С. 255-264. – Англ.

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Табл. 0. Ил. 3. Библиогр. 8 назв.

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