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THE EFFICIENCY OF USING A SEMI-ANALYTICAL FINITE ELEMENTS METHOD IN GEOMETRICALLY NONLINEAR PROBLEMS OF ELASTIC-PLASTIC DEFORMATION

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The reliability and efficiency of the semi-analytic finite element method in the problems of geometrically nonlinear elastic-plastic deformation of axisymmetric structures under the influence of dynamic loads are considered. The possibilities of the technique are demonstrated on the examples of numerical simulation of the building structures' stress-strain state with large linear deformations and technological operation analysis of impulse metal processing.

Keywords: dynamics, geometric nonlinearity, plastic deformations, a solid of revolution, axisymmetric structures, semi-analytic finite element method (SAFEM).

Introduction. Among the variety of objects considered by analytical and numerical methods of particular interest are solid of revolution of complex shape and cross-sectional structure formed by the movement of some solid surface along a closed or disclosed broken line without breaks. The selected geometric class is used as a natural structure, nodes and components in the construction and various fields of mechanical engineering. Examples of facilities include water towers, chimneys, tanks for various purposes, nuclear reactor casings, various components and parts of power and transport engineering. The sufficiently high prevalence of these forms on the one hand, and the possibility of greatly simplifying the computational relationships based on their geometric features on the other, is attracting the increasing attention of researchers.

It is known that today the finite element method (FEM) is the most powerful tool for analyzing the problems of structural mechanics and deformation of a solid bodies. Over the last few years, the dimension of FEM models has increased dramatically, driven by increased requirements for the accuracy and reliability of results, prompting the use of more and more detailed calculation schemes. In addition, the difficulties of studying the behavior of structures in the presence of dynamic loads are increased in comparison with static analysis many times. To overcome these problems in many cases they introduce additional hypotheses, which, as a rule, narrow the class of objects and processes under study, but can significantly improve efficiency and significantly reduce the duration of the calculation. Semi-analytic finite element method (SAFEM) is such an approach that is widely used for solving problems which objects are prismatic and solids of revolutionary of complex shape and cross-sectional structure. High efficiency of SAFEM for a certain range of objects has been demonstrated in the field of static and dynamic [2] analysis, continuous fracture mechanics under creep conditions [1], processes of nonlinear deformation of reinforced concrete structures [3]. The analysis of the results on this issue obtained by domestic and foreign scientists shows that most analytical and numerical methods of scientific research are usually oriented to geometrically nonlinear problems at static load [5, 12, 13]. Aspects of solving problems of dynamic deformation of axisymmetric solids with significantly expressed nonlinear mechanical characteristics, taking into account geometric nonlinearity and conditions of contact interaction are considered in the works of the authors [4, 10, 11]. The purpose of this work is to analyze the calculation results reliability of the specified classes of problems on the basis of the constructed approach and study of complex technological processes.

1. Analysis of the reliability of calculation results based on test examples
An important task of designing the shell of the electromagnetic drive is the correct determination of the thickness of the cylindrical screen wall, the value of which depends on its strength. At the same time, the thickness of the wall determines the magnitude of the magnetic gap, in proportion to the square of which the power of the magnetic drive falls. In this regard the importance of the task of reducing the thickness of the shell wall becomes clear. As shown by experimental studies, the use of a two-layer shell allows to increase the strength of the structure compared to all-metal one. Thus, in the first case, the magnitude of the breaking load is 10MPa, and in the second - 8MPa. In Fig.1 shown the calculation scheme and the results of numerical calculation in the form of displacements, depending on the intensity of the internal pressure in the linear calculation and taking into account the material fluidity.

The problem of deposition of a cylindrical workpiece in the cold state between two rigid plates is considered. The workpiece is a cylinder height 1.9m, diameter 2.38m. The material is elastic-plastically reinforced.

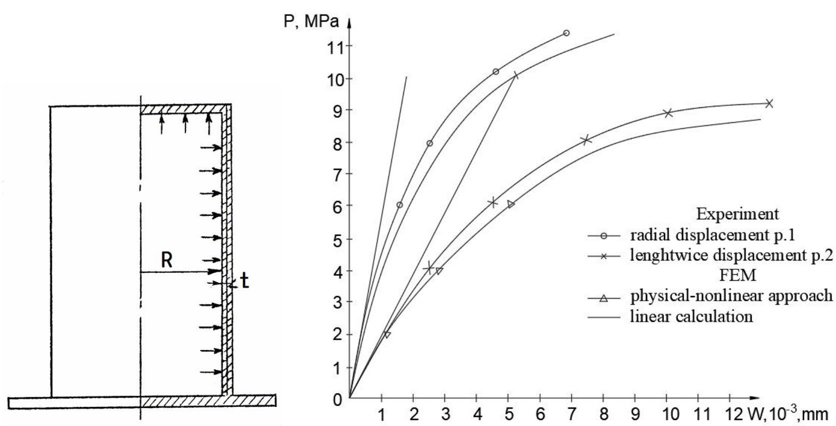

Fig. 1. The calculation scheme and the results of the numerical calculation of the electromagnetic drive shell

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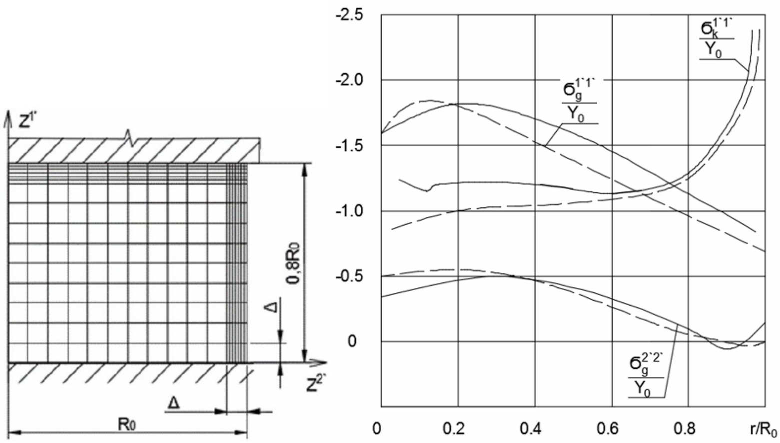

Fig. 2. The design scheme of the cylindrical workpiece and the graph of the distribution of axial and tangential stresses

A full adhesion is provided at the interface between the tool and the workpiece. The process of deposition of the cylindrical workpiece was performed to a degree of precipitation of 10%. The calculation scheme of the object and the results of the calculation in the form of graphs of the distribution of axial and tangential stresses in the equatorial plane are shown in Fig. 2. The tangent stresses coincide quite well, the discrepancy is observed only at points lying on the axis of symmetry. The plot of normal stresses on the surface of contact with the tool also conforms well with the results given in [9].

2. The investigation of the deformation process of monolithic reinforced concrete chimney under the influence of wind load

An example of calculating a reinforced concrete chimney (Fig. 1) with height $H = 120\text{m}$, loaded with its own weight and wind pressure was considered, which was considered as normal static pressure applied to the outer surface of the object.

The design scheme is a conical shell of variable thickness (Fig. 3(a)), which takes into account a vertical roll of magnitude 0.01 from the height of the structure. The normative value of the average wind load was calculated by the known formula $w_m = w_0 k c$, where k is the altitude coefficient, c is the aerodynamic coefficient for the cylindrical surface and 0.23 kPa was adopted. The distribution of wind pressure is shown in Fig. 3(b).

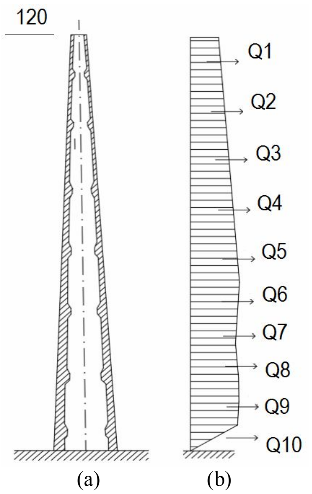

Fig. 3. General view of the pipe (a) and the diagram of the distribution of wind pressure (b)

The results of the calculation of the

stress-strain state of the structure are shown in the form of graphs in Fig. 4(a) and 4(b).

The obtained data show an increase of the expected results by 20% for the meridional stresses, and for the radial displacements - almost twice taking into account the geometric nonlinearity in comparison with the linear calculation.

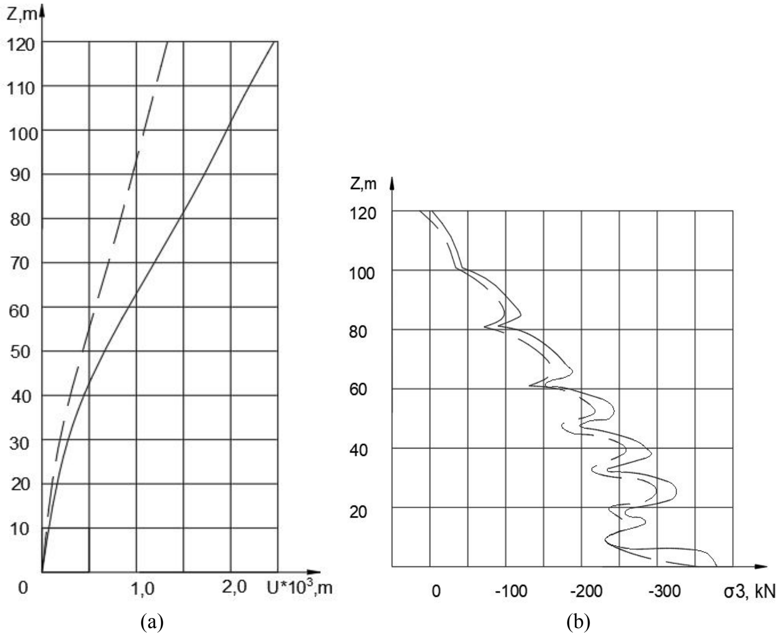

Fig. 4. Graphs of radial displacements U (a) and meridional stresses σ_3 (b). The dashed line corresponds to the consideration of the problem in a linear formulation, solid - in a geometrically nonlinear

3. Applied tasks of numerical modeling of technological operations of impulse metal processing

The possibilities of the developed methodology are demonstrated by the examples of numerical study of technological operation of impulse metal processing. The problems are solved in an axisymmetric formulation taking into account the physical and geometric nonlinearity, contact interaction and the boundary state of the material. In modeling of separate operations, the phenomenological theory of Kolmogorov destruction is used to estimate the boundary condition, which is a model of accumulation of metal damage during its plastic deformation [8]. The condition of destruction is determined achieve by the level of damage function $H = \int_0^{\varepsilon_i^p} \frac{\mathrm{d}\varepsilon_i^p}{\widehat{\varepsilon}_i^p(\gamma)} = 1$. Here $d\varepsilon_i^p$ - the intensity of plastic deformation growth, ε_i^p - the intensity of accumulated plastic deformation, $\widehat{\varepsilon}_i^p(\gamma)$ - the marginal intensity of plastic deformation at the time of fracture, which is determined by the experimental deformation diagrams for each specific material.

It's considered the problem of forming a flange at the end of a tubular workpiece at its opening by an internal pressure impulse. The design scheme of the flange forming process is a cross-section of the pipe in the plane of formation, the external pressure *P*(*t*), as a function of time *t* is also presented in Fig. 5. The physical and mechanical properties of the materials are as follows: modulus of elasticity $E = 6.8\times10^4$ MPa, Poisson's ratio $\nu = 0.3$, material density $\rho = 2.64\times10^3$ kg/m3, limit of yield $\sigma_s = 182$ MPa, confidence interval (CI) for the power law of state $\sigma = k\times\varepsilon^n$, taking into account plastic flow with isotropic hardening, is: $k = 374\div433$ MPa, $n = 0.118\div0.177$, marginal intensity of the deformation at the time of destruction $\hat{\varepsilon}_i^p = 0.69$.

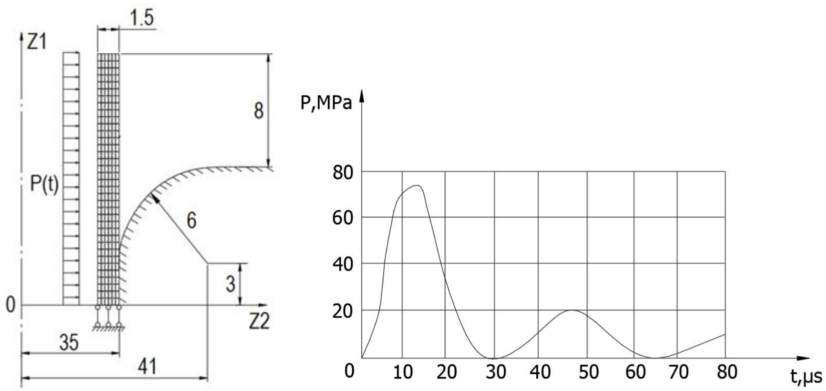

Fig. 5. Calculation scheme of flange formation at the end of the tubular workpiece

The results of the calculation (Fig. 6) showed that the model of ideally plastic material does not give sufficiently reliable results, so for the considered processes and materials, it is necessary to use more complex state laws that take into account the strengthening. It is seen that the experimental curves [6] lie in the region bounded by the two calculated curves obtained for given limit values of the parameters of the strengthening *k*. At *k* = 433MPa and *n* = 0.177 there is a qualitative and quantitative convergence of calculation results and experimental data.

Two technological circuits of separation operations are considered: cutting and end-of-pipe method at compression field impulse by force. Cutting technology (Fig. 7(a)) involves the formation of annular waste in the area of free deformation between the two cutting edges. The process of milling (Fig. 7(b)) is technologically different from cutting with the absence of the second cutting edge and the presence of a longer free end part. The model of isotropic hardening is adopted according to the power law $\sigma = k\times\varepsilon^n$ at the values of the hardening parameters *k* = 433MPa and *n* = 0.177.

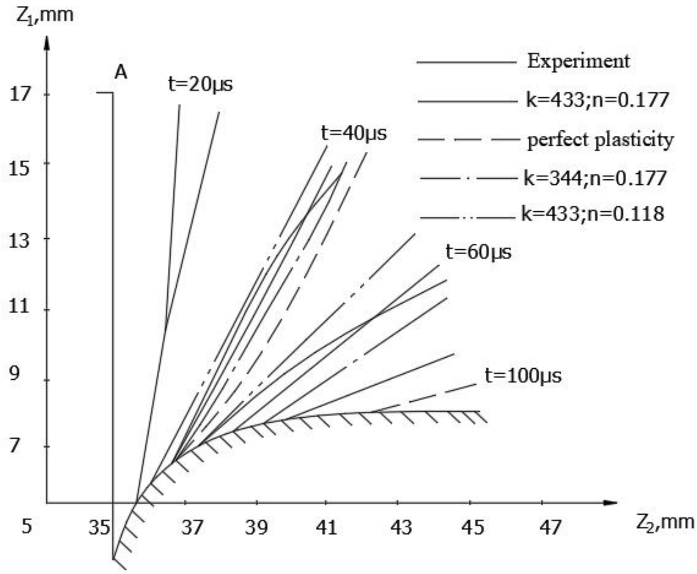

Fig. 6. Time external configuration of the workpiece surface

In Fig. 8 the graphs of changes in the intensity of the external pressure and the configuration of the workpiece in the deformation process are presented. As we can see, for both schemes of separation operations, the process of separation of the workpiece is completed by the time of reaching the maximum load applied. At the same time in the problem of end face, the movement of the free end part at a sufficient distance from the cutting edge (about half the length of end face) occurs almost parallel to the initial position of the face.

The same effect is observed in the initial stage of the flange forming process, which does not fully correspond to the experimental data. This discrepancy is due to the presence in the actual processes of boundary effects of force field pressure, which are not taken into account in the calculations.

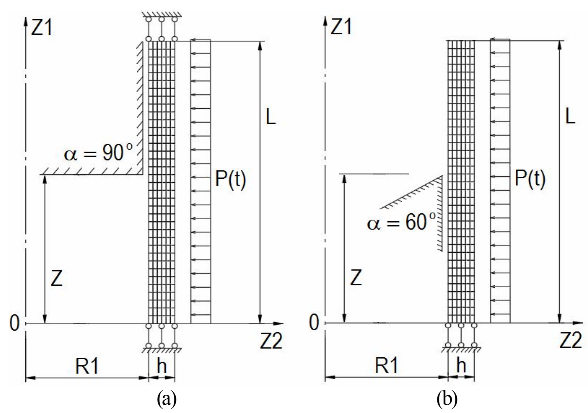

Fig. 7. Models of cutting (a) and end face (b) of cavity parts

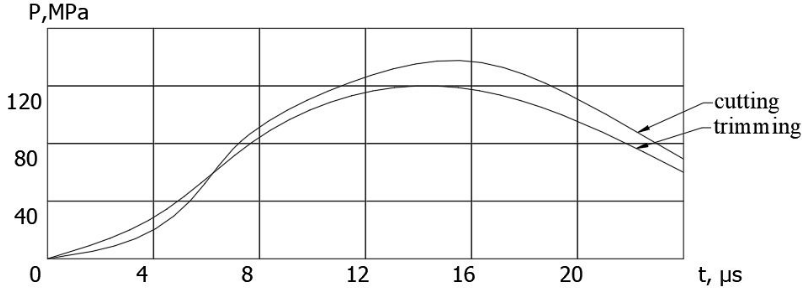

Fig. 8. Schedule of change of intensity of external pressure

It is significant that the distribution of plastic deformations in the thickness of the workpiece is significantly uneven. As a result, the development of fracture zones is quite complex. For example, in the process of cutting (Fig. 9) the initial zone of destruction reaches half the thickness of the workpiece for the time 19 - 20.7 μs. Then, at the time of 20.9 μs a small zone of destruction appears on the opposite edge of the cutting surface of the workpiece, after which the development of the initial zone continues, and by the time of 22.1 μs the process of cutting ends. At the same time, near the cutting edge, the greatest contribution to the intensity of the accumulated plastic deformations is made by shear deformations, the value of which is twice as much as the relative elongations, while near the opposite surface of the shear deformation by 2 - 3 orders of magnitude below the relative elongations.

Thus, from the side of the cutting edge there is a shear failure, and on the opposite surface - due to stretching.

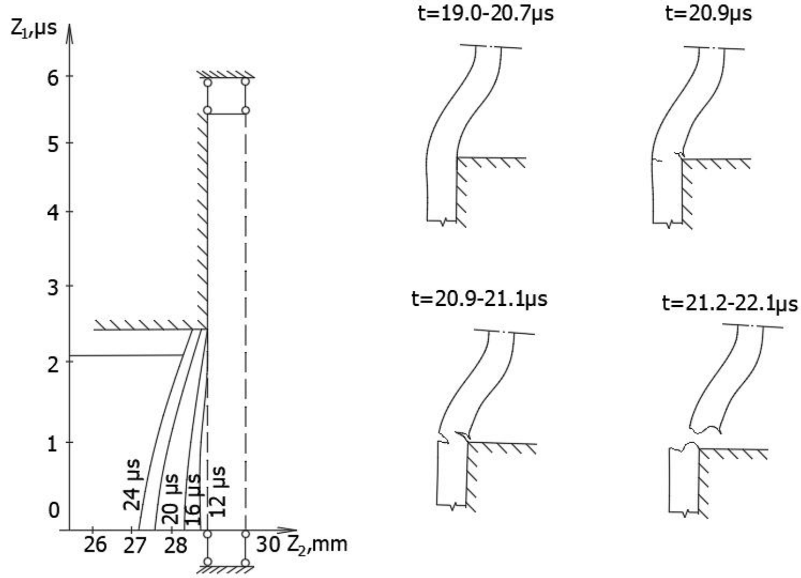

Fig. 9. Crimping

In the problem of end face crimping (Fig. 10), initially at t = 17.4 - 17.8 μs there is also a slight undercutting of the material by the cutting edge due to shear deformations.

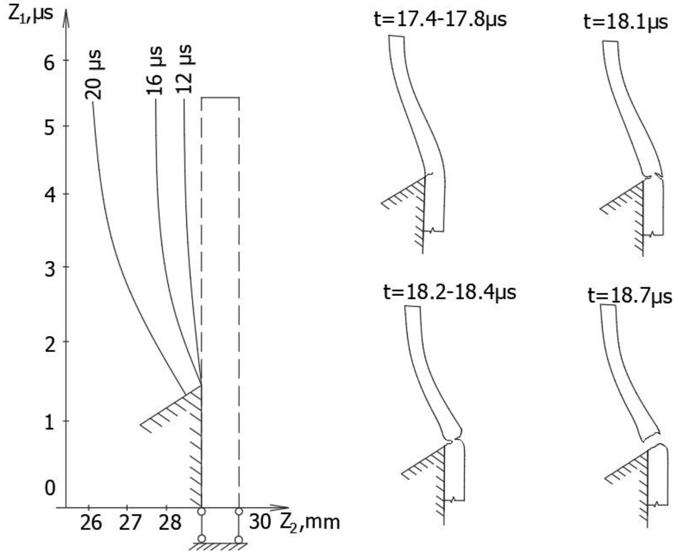

Fig. 10. Compression molding

Then the main zone of destruction is developed on the opposite side of the workpiece due to stretching. By the time t = 18.1 - 18.7 μs, trimming process is complete.

Conclusion. Within the framework of the proposed methodology a demonstration of the possibilities of the approach is made on the basis of the semi-analytic variant of the finite element method. Numerical studies have shown a significant influence of geometrical nonlinearity on determining the displacements of the top of a monolithic reinforced concrete pipe. Analyzing the results of studies of technological operations of impulse metal processing, it can be argued that in the separation operations the destruction of the workpiece when cutting is mainly due to shear deformation, scilicet there is a fracture of the cut, while the workpiece partitioning at the expense of tensile, that is tear fracture. On this basis, the largest elongation criterion of Mariotte [7], which is commonly used in calculations, give accurate results only for numerical modeling of face processes. To solve cutting problems, the Kolmogorov criterion is required, but, as the nature of plastic deformation shows, the dependence of the parameters on the type of stress state must be taken into account.

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Solodei I.I., Vabishchevich M.O., Stryhun R.L.

SEMIANALYTICAL FINITE ELEMENTS METHOD EFFICIENCY IN THE GEOMETRICALLY NONLINEAR ELASTIC-PLASTIC PROBLEMS

Particular interest, among the variety of objects considered using analytical and numerical methods, are bodies of revolution with complex shape and cross-sectional structure. The selected geometric class is used as natural structure components in the construction and various fields of mechanical engineering. The sufficiently high prevalence of these forms in the construction and machine-building sectors on the one hand, and the possibility of a significant simplification math equestions by taking into account their geometric features on the other, is attracting increasing attention of researchers.

It is known that today the finite element method (FEM) is the most powerful tool for analyzing the problems of building mechanics and deformable solid mechanics. Over the past few years, the dimension of FEM models has grown dramatically, driven by increased demands for accuracy and reliability of results. In addition, the difficulties of studying the behavior of structures in the presence of dynamic loads are many times increased in comparison with static analysis. To overcome these problems, in many cases, it introduce additional hypotheses, which, as a rule, narrow the class of objects and processes under study, but can significantly improve efficiency and significantly reduce the duration of the calculation. Semi-analytical finite element method (SAFEM) is one of such approaches that is widely used for solving problems whose objects are prismatic and rotational bodies. High efficiency of SAFEM for a certain range of objects was demonstrated in the field of static analysis, continuous mechanics of fracture under creep conditions, processes of nonlinear deformation of reinforced concrete structures. The analysis of the results obtained by domestic and foreign scientists on this issue shows that most analytical and numerical methods of scientific research are usually oriented to geometrically nonlinear problems at static load. The veracity and effectiveness of the semi-analytical finite element method in the problems of geometrically nonlinear elastoplastic deformation of axisymmetric structures under dynamic loads is considered. The capabilities of methodology are demonstrated by examples of numerical modeling of the stress-strain state of building structures with large linear strains and analysis of technological processes of pulsed metal processing.

Keywords: dynamics, geometric nonlinearity, plastic deformations, bodies of revolution, axisymmetric constructions, semi-analytical finite element method.

Солодей И.И., Вабищевич М.О., Стригун Р.Л.

ЭФФЕКТИВНОСТЬ ИСПОЛЬЗОВАНИЯ ПОЛУАНАЛИТИЧЕСКОГО МЕТОДА КОНЕЧНЫХ ЭЛЕМЕНТОВ В ГЕОМЕТРИЧЕСКИ НЕЛИНЕЙНЫХ ЗАДАЧАХ УПРУГОПЛАСТИЧЕСКОГО ДЕФОРМИРОВАНИЯ

Рассмотрена достоверность и эффективность полуаналитического метода конечных элементов в задачах геометрически нелинейного упругопластического деформирования осесимметричных конструкций под действием динамических нагрузок. Возможности методики демонстрируются на примерах численного моделирования напряженно-деформированного состояния строительных конструкций при больших линейных деформациях и анализа технологических процессов импульсной обработки металлов.

Ключевые слова: динамика, геометрическая нелинейность, пластические деформации, тела вращения, осесимметричные конструкции, полуаналитического метод конечных элементов.

УДК 539.3

Солодей І.І., Вабіщевич М.О., Стригун Р.Л. **Ефективність використання напіваналітичного методу скінченних елементів в геометрично нелінійних задачах пружно-пластичного деформування** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2019. – Вип. 103.– С. 71-81.

Розглянута достовірність та ефективність застосування напіваналітичного методу скінченних елементів в задачах геометрично нелінійнійного пружнопластичного деформування вісесиметричних конструкцій під дією динамічних навантажень. Можливості методики демонструються на прикладах чисельного моделювання напружено-деформованого стану будівельних конструкцій при великих лінійних деформаціях та аналізу технологічних процесів імпульсної обробки металів.

Іл. 9. Бібліогр. 13 назв.

UDC 539.3
Solodei I.I., Vabishchevich M.O., Stryhun R.L. **Semianalytical finite elements method efficiency in the geometrically nonlinear elastic-plastic problems** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles – Kyiv: KNUBA, 2019. – Issue 103. – P. 71-81.

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Fig. 9. Ref. 13.

УДК 539.3
Солодей И.И., Вабищевич М.О., Стригун Р.Л. **Эффективность использования полуаналитического метода конечных элементов в геометрически нелинейных задачах упругопластического деформирования** // Сопротивление материалов и теория сооружений: науч.-тех. сборн. – К.: КНУСА, 2019. - Вып. 103. - С. 71-81.

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