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WAVELET TRANSFORM USING FOR ANALYSIS OF VIBROIMPACT SYSTEM CHAOTIC BEHAVIOR

V.A. Bazhenov

O.S. Pogorelova

T.G. Postnikova[1]

O.O. Lukianchenko

Kyiv National University of Construction and Architecture
31, Povitroflotskiy avenu, Kyiv, Ukraine

Chaotic behaviour of dynamical systems, their routes to chaos, and the intermittency are interesting and investigated subjects in nonlinear dynamics. The studying of these phenomena in non-smooth dynamical systems is of the special scientists' interest. In this paper we apply relatively young mathematical tool – continuous wavelet transform CWT – for investigating the chaotic behavior and intermittency in particular in strongly nonlinear non-smooth discontinuous 2-DOF vibroimpact system. We show that CWT applying allows to detect and determine the chaotic motion and the intermittency with great confidence and reliability, gives the possibility to demonstrate route to chaos via intermittency, to distinguish and analyze the laminar and turbulent phases.

Keywords: vibroimpact system, chaotic behaviour, intermittency, continuous wavelet transform, surface of wavelet coefficients.

1. Introduction

The wavelet transform (WT) is a relatively new mathematical tool for analysis or synthesizing a wide variety of generic signals at different frequencies and with different resolution. WT arose in 80-th years of XX century. Now it is state-of-art technique for nonstationary signals analysis. There are quite a few articles and books and textbooks written on them [1-6]. There is developed Software: Wavelet Toolbox in Matlab, Mathcad and so on [7, 8].

Mathematical transformations are applied to signals to obtain a further information from that signal that is not readily available in the raw signal. There is number of transformations that can be applied, among which the Fourier transforms (FT) are probably by far the most popular.

The FT gives the frequency information of the signal, which means that it tells us how much of each frequency exists in the signal, but it does not tell us when in time these frequency components exist. This information is not required when the signal is so-called stationary. When the signal is not stationary it is suitably to use the WT, more exactly when the time localization of the spectral components are needed, a transform giving the time-frequency representation of the signal is needed. The Wavelet transform is a transform of this type. It provides the time-frequency representation. (There are other transforms which give this information too, such as short time Fourier transform, Wigner distributions, etc.). Wavelet transform is capable of providing the time and

[1] Corresponding author.
E-mail address: posttan@ukr.net

frequency information simultaneously, hence giving a time-frequency representation of the signal. The WT was developed as an alternative to the short time Fourier Transform (STFT).

Like the FT the continuous wavelet transform (CWT) uses inner products to measure the similarity between a signal and an analyzing function. In the FT the analyzing functions are the complex exponents $e^{j\omega t}$. The resulting transform is a function of a single variable ω. In the STFT the analyzing functions are windowed complex exponentials $w(t)e^{j\omega t}$, and the result is the function of two variables. The STFT coefficients $F(\omega,\tau)$ represent the match between the signal and a sinusoid with angular frequency ω in an interval of a specified length centered at τ.

In CWT the analyzing function is a wavelet ψ. The CWT compares the signal to shifted and compressed or stretched versions of a wavelet. Stretching or compressing a function is collectively referred to as dilatation or scaling and corresponds to the physical notion of scale. By comparing the signal to the wavelet at various scales and positions we obtain a function of two variables. There are many different admissible wavelets that can be used in the CWT. While it may seen confusing that there are so many choices for the analyzing wavelet it is actually a strength of wavelet analysis. Depending on what signal features we are trying to detect, we are free to select a wavelet that facilitates our detection of that feature.

We apply the continuous wavelet transform CWT in order to study the chaotic behavior in general and route to chaos via intermittency in particular for vibroimpact system.

Chaotic behavior occurs in many phenomena: mechanical, engineering, experimental physical, medical, biology, and so on. The studying of such phenomena was begun in 80-th years of XX century too when the behaviour of different dynamical systems began to be described by nonlinear differential equations. Now the theory of chaotic vibrations is well developed and is continuing to develop further. There are many textbooks, monographs and papers about it [9, 10]. There are many special journals devoted to different questions on nonlinear dynamics in general and on chaos in particular (for example, an Interdisciplinary Journal of Nonlinear Science “Chaos, Solitons & Fractals”, an International Journal of Nonlinear Dynamics and Chaos in Engineering Systems “Nonlinear Dynamics”). The numerous conferences are holding in many countries of the world (for example, 7th International conference on Nonlinear Science and Complexity (NSC2018) in México, Fourth International Conference on Recent Advances in Nonlinear Mechanics RANM 2019 in Poland).

Vibroimpact system is strongly nonlinear dynamical system, its motion is describing by nonlinear differential equations. So its behaviour is typically nonlinear one: the stable motion regions are changing by unstable ones, periodic motion is replacing by quasiperiodic one, which then turns into chaotic [10-12]. The analysis gets complicated by the non-smoothness of vibroimpact system because its motion equations are discontinuous due the impacts. It is known three main routes to chaos in nonlinear systems – the Feigenbaum route via period doubling, quasiperiodic route to chaos [13, 14], and route to chaos via

intermittency [15]. This later route has big complexity for analysis. At first it occurs much less then route via period doubling (which occurs the most often and is studied in the best way). At second "the catching" of intermittency in system motion is not such simple task. The continuous wavelet transform CWT is useful exactly for this task solving.

The chaotic motion and the intermittency in different mechanical and physical systems were studied in [16-22] with WT applying.

The aim of this paper is: to apply the wavelet transform WT for studying of vibroimpact system motion and to show its use for intermittency "catching" and chaoticity anlysis.

2. The initial equations

For this goal achievement we consider the model of 2-DOF two-body vibroimpact system which we have studied it our previous works [13, 14, 23] and have obtained the-frequency response [23] in wide range of control parameter by parameter continuation method (Fig. 1). Therefore here we'll give only short model description.

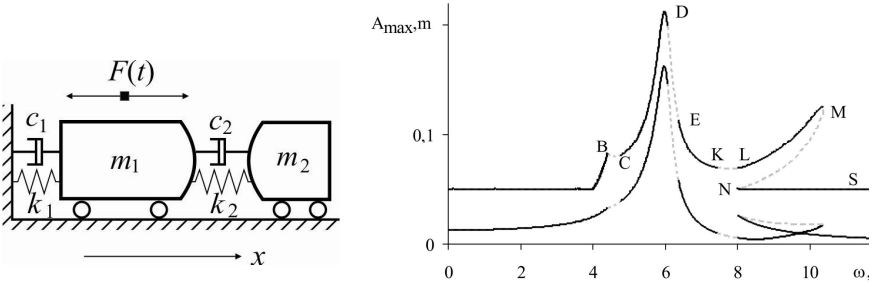

Fig. 1. Vibroimpact system model and amplitude-frequency response

This model is formed by the main body m1 and attached one m_2, which can play the role of percussive or non-percussive dynamic damper. Bodies are connected by linear elastic springs with stiffness k_1 and k_2 and dampers with damping coefficients c_1 and c_2. (The damping force is taken as proportional to first degree of velocity with coefficients c_1 and c_2.)

The differential equations of its movement are:

$$\ddot{x}_1 = -2\xi_1\omega_1\dot{x}_1 - \omega_1^2 x_1 - 2\xi_2\omega_2\chi(\dot{x}_1 - \dot{x}_2) - \\ -\omega_2^2\chi(x_1 - x_2 + D) + \frac{1}{m_1}[F(t) - F_{con}(x_1 - x_2)] \qquad (1)$$

$$\ddot{x}_2 = -2\xi_2\omega_3(\dot{x}_2 - \dot{x}_2) - \omega_1^2(x_2 - x_1 - D) + \frac{1}{m_1}F_{con}(x_1 - x_2)$$

where $\omega_1 = \sqrt{\frac{k_1}{m_1}}$, $\omega_2 = \sqrt{\frac{k_2}{m_2}}$; $\xi_1 = \frac{c_1}{2m_1\omega_1}$, $\xi_2 = \frac{c_2}{2m_2\omega_2}$; $\chi = \frac{m_2}{m_1}$.

External loading is periodic one: $F(t) = P\cos(\omega t + \varphi_0)$, $T = 2\pi/\omega$ is its period.

Impact is simulated by contact interaction force F_{con} according to contact quasistatic Hertz's law:

$$F_{con}(z) = K[H(z)z(t)]^{3/2},$$
$$K = \frac{4}{3}\frac{q}{(\delta_1+\delta_2)\sqrt{A+B}},\quad \delta_1 = \frac{1-\mu_1^2}{E_1\pi},\quad \delta_2 = \frac{1-\mu_2^2}{E_2\pi}, \tag{2}$$

where $z(t)$ is the relative closing in of bodies, $z(t) = x_2 - x_1$, A, B, and q are constants characterizing the local geometry of the contact zone; μ_i and E_i are respectively Poisson's ratios and Young's modulus for both bodies, $H(z)$ is the discontinuous step Heviside function. The numerical parameters of this system are following:

m_1 =1000 kg, ω_1=6.283 rad·s^{-1}, ξ_1= 0.036, E_1= 2.1·1011 N·m^2, μ_1=0.3,
m_2 = 100 kg, ω_2=5.646 rad·s^{-1}, ξ_2= 0.036, E_2= 2.1·1011 N·m^2, μ_2=0.3,
P=500 N, A=B=0.5 m^{-1}, q=0.318.

3. Chaoticity analysis

Here we consider the region DE where the main (1,1)-periodic regime (regime with period 1T and 1 impact per cycle) is unstable one. Let us have a look what regimes are realising at this region. The largest Lyapunov exponent dependence on control parameter ω is depicted at Fig.2 [24].

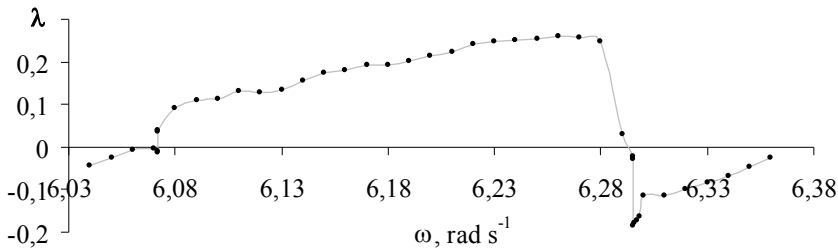

Fig.2. The largest Lyapunov exponent dependence on control parameter ω

The regions ω < 6.07 rad·s^{-1} and ω > 6.29 rad·s^{-1} correspond to periodic motions because the largest Lyapunov exponents are negative. The region 6.07 rad·s^{-1}<ω<6.29 rad·s^{-1} corresponds to chaotic regime because the largest Lyapunov exponents are positive. How is the transition to chaos carried out? Here we'll not discuss this problem. We'll tell only that (1,1)-regime under ω=6.07 rad·s^{-1} and ω=6.29 rad·s^{-1} becomes the (2,2)-regime (regime with period 2T and 2 impact per cycle), under ω=6.3 rad·s^{-1} it becomes the (2,3)-regime. We don't observe the further period doubling, Feigenbaum's route to chaos doesn't realize under these frequencies. But then we observe intermittency under some frequencies inside the chaotic motion. This phenomenon will be described in sec.4.

Now we'll look more in details at chaotic motion.

Let us look how the continuous wavelet transform (CWT) helps to detect this oscillatory regime.

At first for the comparison we'll show how (1,1) and (2,2) regimes look at wavelet surface projection.

At Fig. 3 and Fig. 4 the time series and wavelet surface projections for these regimes are depicted. We fulfilled CWT with Morlet wavelet using.

It is well seen one high frequency at Fig. 3 which is constant in time, it doesn't change in time. At Fig. 4 there are two high frequencies which are constant in time, they don't change in time.

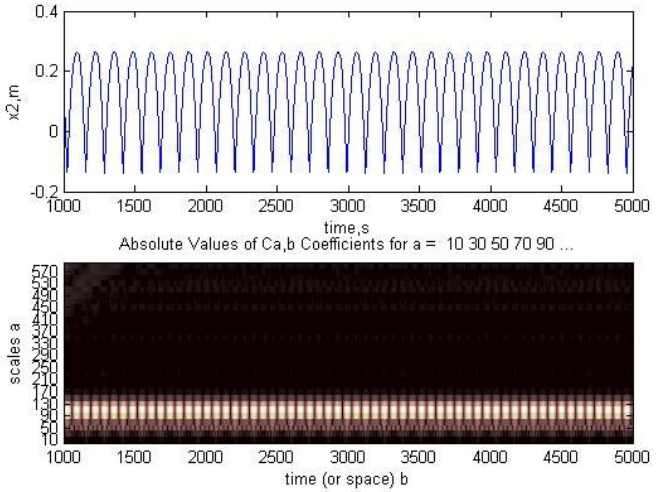

Fig. 3. Time series and wavelet surface projection for (1,1)-regime under ω= 6.06 rad·s^{-1} (Color online)

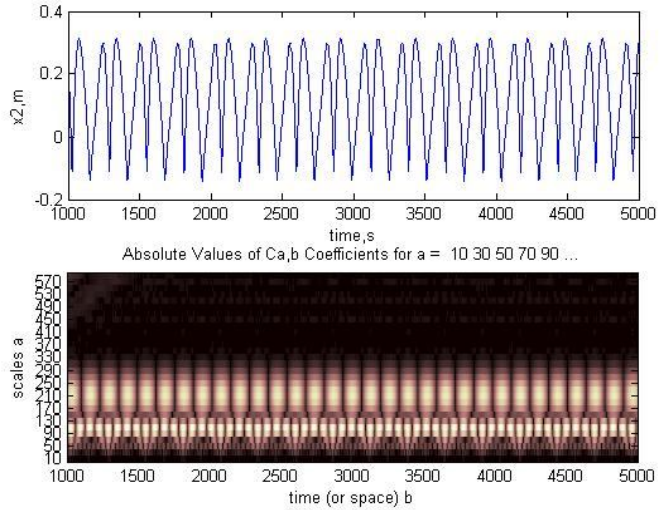

Fig. 4. Time series and wavelet surface projection for (2,2)-regime under ω=6.07 rad·s^{-1} (Color online)

At Fig. 5 the surface of wavelet coefficients for (1,1)-regime is shown for comparison. We see well that one high frequency is constant in time, it doesn't change in time.

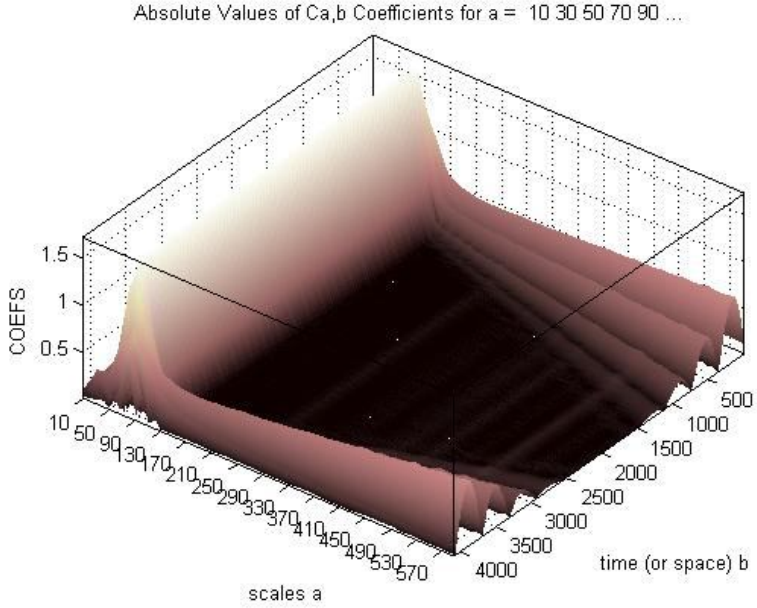

Fig. 5. Surface of wavelet coefficients (3D plot) for (1,1)-regime under ω=6.06 rad·s^{-1} (Color online)

Here and further all plots are fulfilled for attached body. Its mass is much less the main body mass. So its oscillatory amplitudes are more big and their changes are seen better, so the plots are more obvious ones.

For chaotic motion we show time series and wavelet surface projection at Fig. 6.

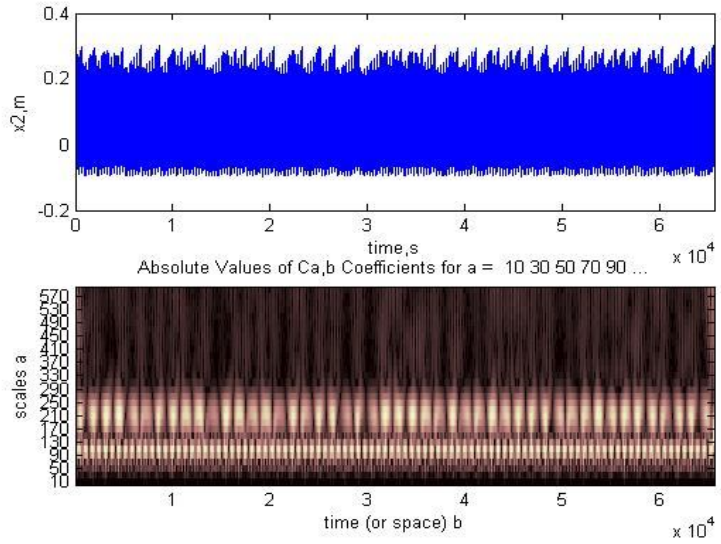

Fig. 6. Time series and wavelet surface projection for chaotic regime under ω= 6.20 rad·s^{-1} (Color online)

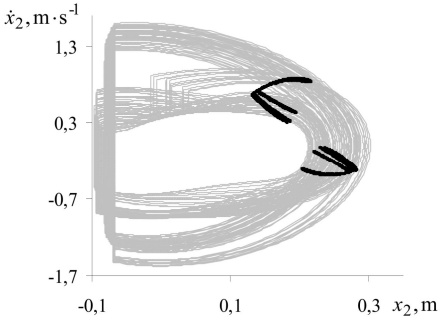

Fig. 7. Phase trajectories and Poincare map for chaotic motion under ω=6.2 rad·s^{-1}

For confirming the chaoticity of this motion we show its phase trajectories and Poincare map at Fig. 7

At Fig. 8 the surface of wavelet coefficients is depicted. It is seen well not a regular set of frequencies which are not constant in time, they change in time. We see also many not regular low frequencies which are not constant in time too.

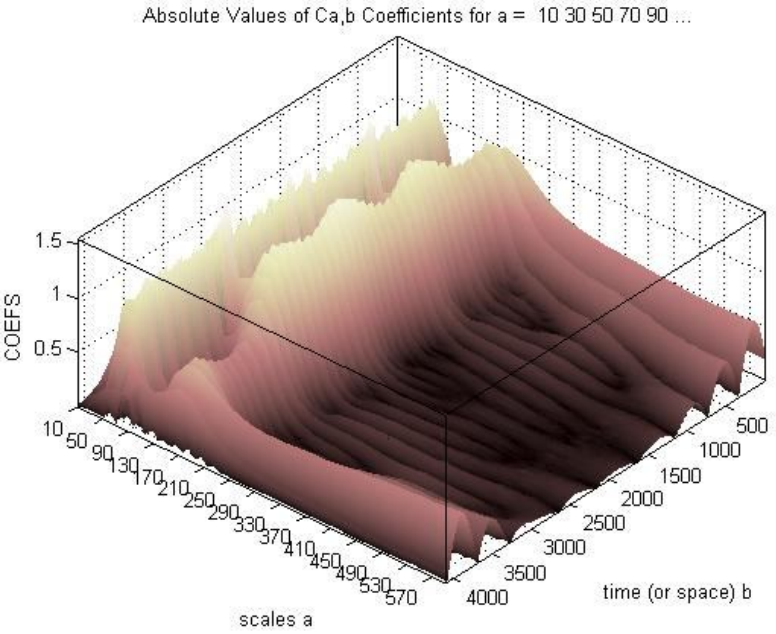

Fig. 8. Surface of wavelet coefficients (3D plot) for chaotic motion under ω=6.2 rad·s^{-1} (Color online)

We see that CWT gives well and reliable information about chaotic motion.

4. Intermittency "catching"

We observed intermittency inside the chaotic motion. Let us have a look at this phenomenon more in details.

Intermittency was discovered by French scientists Yves Pomeau and Paul Manneville [15] in 1980 year. They had written:"…the fluctuations remain apparently periodic during long time intervals (which we'll call "laminar phases" but this regular behavior seems to be randomly and abruptly disrupted by a "burst" on the time record. This "burst" has a finite duration, it stops and a new laminar phase starts and so on". In other words one observe long periods of periodic motion with bursts of chaos under one value of control parameter that is

external load frequency that is the zones of turbulent and laminar motion alternate in such regime under one frequency value. As one varies a control parameter the chaotic bursts become more frequent and longer.

We'll show the intermittency which we observe under $6.1\,\text{rad}\cdot\text{s}^{-1} < \omega < 6.14\,\text{rad}\cdot\text{s}^{-1}$. At Fig. 9 the time series and wavelet surface projection for this regime are depicted under ω=6.13 rad·s^{-1}.

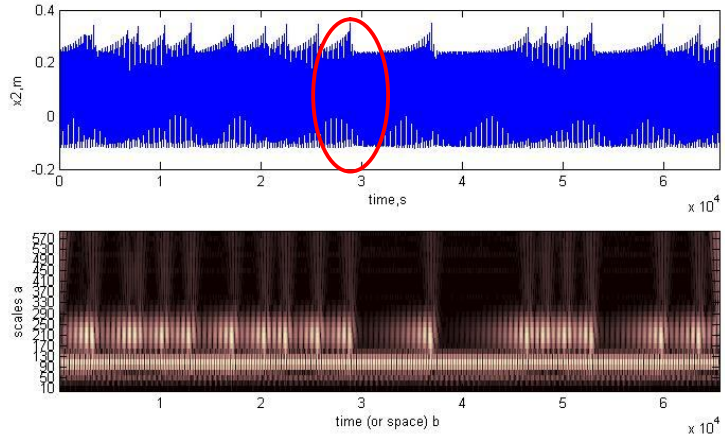

Fig. 9. Time series and wavelet surface projection for intermittency regime under ω=6.13 rad·s^{-1} (Color online)

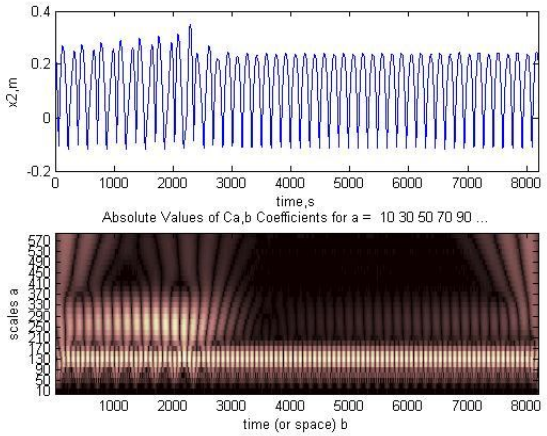

Fig. 10. Time series and wavelet surface projection for intermittency regime under ω=6.13 rad·s^{-1} (region inside red oval at Fig. 9)

It is well seen the regions where chaotic motion and it's high and low frequencies are interrupted and only one high frequency remains.

At Fig. 10 we show the small region that is picked out by red oval. At this Fig. we see very obviously the sharp change of chaotic motion into periodic one.

The surface of wavelet coefficients is shown at Fig. 11. We see very obviously how chaotic motion with many different frequencies is changing by the periodic motion with only one high frequency.

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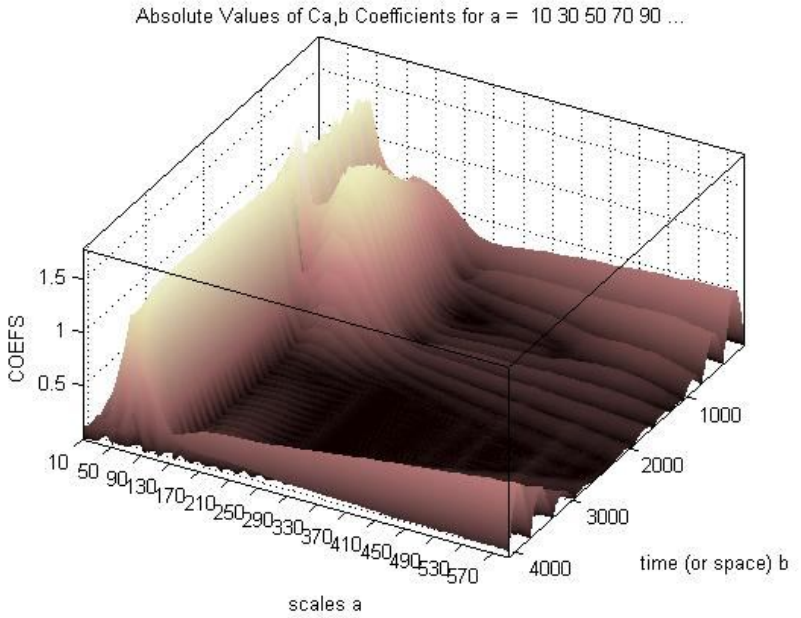

Fig. 11. Surface of wavelet coefficients (3D plot) for intermittency under ω=6.13 rad·s^{-1} (Color online)

At Fig. 12 the phase trajectories and Poincare maps are shown for regions of chaotic (turbulent phase) and periodic (laminar phase) motions under intermittency. These plots underline the regimes changing and confirm and give the confidence in motion periodicity at this region.

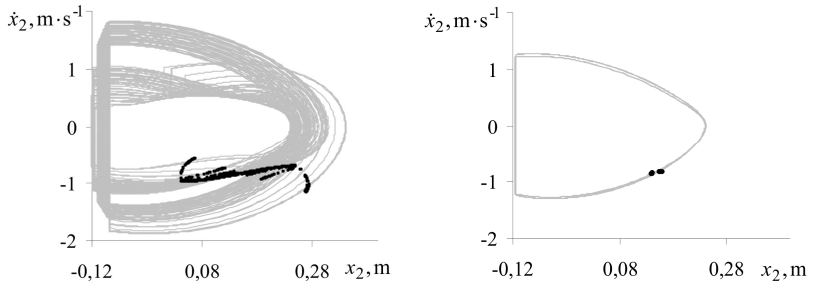

Fig. 12. Phase trajectories and Poincare maps for the regions of chaotic and periodic motions under intermittency (ω=6.13 rad·s^{-1})

Thus we see that surfaces of wavelet coefficients and their projections obtained by continuous wavelet transform CWT give the possibility to find and "catch" the intermittency with great confidence and reliability.

In [21] the authors study intermittency in Lorenz model also by CWT using. We succeeded in finding the intermittency in nonsmooth strongly nonlinear vibroimpact system. The CWT was very useful for this studying.

4. Conclusions

The continuous wavelet transform CWT allows to detect and determine the chaotic motion and the intermittency with great confidence and reliability. Wavelet transform applying gives the possibility to demonstrate the route to chaos via intermittency and to distinguish and analyze the laminar and turbulent phases. The plots of wavelet coefficients surfaces and their projections give very obvious presentation of these regimes, especially the color plots online. The wavelet theory and existing Software are very useful for these phenomena studying.

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Баженов В.А., Погорелова О.С., Постнікова Т.Г., Лукьянченко О.О.
АНАЛІЗ ХАОТИЧНОЇ ПОВЕДІНКИ ВІБРОУДАРНОЇ СИСТЕМИ З ВИКОРИСТАННЯМ ВЕЙВЛЕТ ПЕРЕТВОРЕННЯ

Хаотична поведінка динамічних систем, сценарії їхніх переходів до хаосу, явище переміжності – це є сфера нелінійної динаміки, що широко досліджується вченими різних країн. Особливу цікавість викликає вивчення цих явищ в негладких динамічних системах, якими і є віброударні системи. В цій статті ми використовуємо відносно молодий математичний апарат – безперервне вейвлет перетворення CWT – для дослідження хаотичної поведінки та зокрема переміжності в сильно нелінійної негладкій розривний віброударній системі з двома ступнями вільності. Ми показуємо, що застосування CWT дозволяє упевнено та надійно визначити хаотичну поведінку та переміжність, дає можливість демонструвати сценарій переходу до хаосу через переміжність та розрізнювати і аналізувати ламінарну і турбулентну фази.

Ключові слова: віброударна система, хаотична поведінка, переміжність, безперервне вейвлет перетворення, поверхня вейвлет коефіцієнтів.

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Chaotic behaviour of dynamical systems, their routes to chaos, and the intermittency are interesting and investigated subjects in nonlinear dynamics. The studying of these phenomena in non-smooth dynamical systems is of the special scientists' interest. In this paper we apply relatively young mathematical tool – continuous wavelet transform CWT – for investigating the chaotic behavior and intermittency in particular in strongly nonlinear non-smooth discontinuous 2-DOF vibroimpact system. We show that CWT applying allows to detect and determine the chaotic motion and the intermittency with great confidence and reliability, gives the possibility to demonstrate route to chaos via intermittency, to distinguish and analyze the laminar and turbulent phases.
Fig. 12. Ref. 24

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Баженов В.А., Погорелова О.С., Постнікова Т.Г., Лукьянченко О.О. **Аналіз хаотичної поведінки віброударної системи з використанням вейвлет перетворення** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2018. – Вип. 101. – С. 14-25.

Хаотична поведінка динамічних систем, сценарії їхніх переходів до хаосу, явище переміжності – це є сфера нелінійної динаміки, що широко досліджується вченими різних країн. Особливу цікавість викликає вивчення цих явищ в негладких динамічних системах, якими і є віброударні системи. В цій статті ми використовуємо відносно молодий математичний апарат – безперервне вейвлет перетворення CWT – для дослідження хаотичної поведінки та зокрема переміжності в сильно нелінійної негладкій розривний віброударній системі з двома ступнями вільності. Ми показуємо, що застосування CWT дозволяє упевнено та надійно визначити хаотичну поведінку та переміжність, дає можливість демонструвати сценарій переходу до хаосу через переміжність та розрізнювати і аналізувати ламінарну і турбулентну фази.
Іл. 12. Бібліог. 24 назв.

Автор (вчена ступень, вчене звання, посада)*: доктор технічних наук, професор, академік Національної академії педагогічних наук України, директор НДІ будівельної механіки БАЖЕНОВ Віктор Андрійович*
Адреса робоча: *03680 Україна, м. Київ, Повітрофлотський проспект 31, Київський національний університет будівництва і архітектури, БАЖЕНОВУ Віктору Андрійовичу*
Робочий тел.: *+38(044) 245-48-29;*
Мобільний тел.: *+38(067) 111-22-33;*
E-mail: vikabazh@ukr.net
ORCID ID: https://orcid.org/0000-0002-5802-9848

Автор (вчена ступень, вчене звання, посада)*: кандидат фізико-математичних наук, старший науковий співробітник, провідний науковий співробітник НДІ будівельної механіки ПОГОРЕЛОВА Ольга Семенівна*
Адреса робоча: *03680 Україна, м. Київ, Повітрофлотський проспект 31, Київський національний університет будівництва і архітектури, ПОГОРЕЛОВІЙ Ользі Семенівні.*
Робочий тел.: *+38(044) 245-48-29*
Мобільний тел.: *+38(067) 606-03-00*
E-mail: pogos13@ukr.net
ORCID ID: https://orcid.org/0000-0002-5522-3995

Автор (вчена ступень, вчене звання, посада)*: кандидат технічних наук, старший науковий співробітник, старший науковий співробітник НДІ будівельної механіки ПОСТНІКОВА Тетяна Георгіївна*
Адреса робоча: *03680 Україна, м. Київ, Повітрофлотський проспект 31, Київський національний університет будівництва і архітектури, ПОСТНІКОВІЙ Тетяні Георгіївні.*
Робочий тел.: *+38(044) 245-48-29*
Мобільний тел.: *+38(050) 353-47-19*
E-mail: posttan@ukr.net
ORCID ID: https://orcid.org/0000-0002-6677-4127