УДК 539.3

STOCHASTIC STABILITY OF PARAMETRIC OSCILLATIONS OF ELASTIC SHELLS

O.O. Lukianchenko,
Doctor of Engineering Sciences

D.V. Poshyvach

I.D. Kara,
Candidate of Technical Sciences

Kyiv National University of Construction and Architecture

DOI: 10.32347/2410-2547.2024.113.63-74

Abstract. A review of studies of parametric oscillations of shells and their stability showed that most of them concern parametric oscillations of shells excited by periodic loads. The problem of studying stochastic parametric oscillations of shell structures remains relevant due to the complexity of forming calculation models of parametric oscillations and solving the problem of their stochastic stability. Results of numerical researches of stability of parametric oscillations of the cylindrical and shallow shells under different stochastic influences were presented at this article. Parametric oscillations models of the shells were formed on the basis of the asymptotic or functional approaches and Monte-Carlo method using the calculation procedures of finite element analysis software. Stochastic stability of elastic shells was formulated as stability in probability, on average and with respect to the moment functions of different order. The critical values of stochastic load intensity and the regions of stochastic stability of shells were obtained by the Runge-Kutta method of the fourth order and the continuation by parameter method.
Keywords: shell, random parametric load, stochastic stability, probability, moment functions.

Introduction. There is a powerful mathematical apparatus which appeared up on the basis of the theory of Brownian motion of Markov processes and processes of diffusional type nowadays [1-13]. It allows deciding intricate dynamic problems taking into account fluctuation processes. A classic result in this area is the article of O.O. Andronov, L.S. Pontryagin and O.A. Vitt [1], in which firstly the methods of the theory of Markov processes were applied to research of problems of statistical dynamics of the nonlinear systems. Later, the strict mathematical theory of stochastic differential equations of Ito was presented in an article of I.I. Gikhman and A.V. Skorokhodov [5]. An important step in application of this theory to research of dynamic problems of the elastic systems was become researches of R.L. Stratonovich [11]. These researches were based on combination of Krilov-Bogolyubov method of averaging with the method of theory of Markov processes [2]. The strict ground of this approach was done by R.Z. Khasminski [12]. A significant contribution to the development of the theory of stochastic systems and the introduction of probabilistic methods for the calculation of structures was made by V.V. Bolotin and his followers. V.V. Bolotin performed significant work on the application of probabilistic methods to the calculation of structures [3, 4]. Also important are studies of stochastic parametric oscillations of various systems by Dimentberg [6], V.I. Klyatskin [8] and others. Mathematical aspects of the theory of stochastic stability of oscillations of elastic systems are given in books [2-6, 11, 23].

From the beginning of 80-th of the last century the scientists of Structural and Theoretical mechanics department of the Kyiv National University of Construction and Architecture are engaged in development of the numeral research of stochastic stability of shell parametric oscillations [14-26]. The stability areas of parametric oscillations of the elastic shells under actions of different random loads, which were received by authors, were presented in a monograph [23]. The results of research of stochastic stability of elastic shells, which is formulated as stability in probability, on average and with respect to the moment functions of different order are presented at this article.

1. Mathematical definition of stochastic stability in probability, on average and with respect to the moment functions of different order from phase variables. Let us consider the mathematical aspect of the problem of stochastic stability of oscillations of an elastic system. Let the system be stochastic in the sense that at each instant of time t its state is described in some phase space U by a random vector $\vec{x}(t)$ whose components are dynamic variables of the system and its velocities. Let's set the dimension of the space U equal to $2n$. The stochastic nature of the system's behavior can be a consequence of the random setting of the initial conditions, as well as the action of random factors in the process of the system's movement. In the second case, the evolution of the system is described by a differential equation for the realization of a random process

$$\frac{d\vec{x}}{dt} = \vec{f}\left(\vec{x},t\right), \tag{1}$$

where $\vec{f}\left(\vec{x},t\right) = \left(f_1\left(\vec{x},t\right), f_2\left(\vec{x},t\right),..., f_{2n}\left(\vec{x},t\right)\right)$ − a vector function that determines the behavior of a dynamic system.

Let $\vec{x}(t) \equiv 0$ be the solution of this equation. We consider the initial condition $x(t_0) = x_0$ to be deterministic. By analogy with the classical definition of stability according to Lyapunov [10], we introduce the following definition of stability according to probability.

The solution $\vec{x}(t) \equiv 0$ of equation (1) is called stable in probability if for any one $\varepsilon > 0, \rho > 0$ can find such $\delta\left(\varepsilon,\rho\right) > 0$ that from $\left\|\vec{x}_0\right\| < \delta$

$$P\left\{\sup_{t_0<t<\infty}\left\langle\left\|\vec{x}\right\|\right\rangle < \varepsilon,\right\} > 1-\rho, \tag{2}$$

Where the symbol $\|\,.\,\|$ − is some norm in space U.

The meaning of equation (2) is that with a stable solution $\vec{x}(t) \equiv 0$, the initial perturbation can be chosen in such a way that the probability of a predetermined small deviation of the system from the initial coordinates at $t > t_0$ will be less than any predetermined value ρ.

A solution $\vec{x}(t) \equiv 0$ is called asymptotically stable in probability if it is stable in probability and for any $\varepsilon > 0$

$$\lim_{t\to\infty} \mathrm{P}\left\{\left\|\vec{x}\right\| < \varepsilon\right\} = 1. \tag{3}$$

The solution $\vec{x}(t) \equiv 0$ of equation (1) is called stable according to the mathematical expectation of the norm $\left\|\vec{x}\right\|$ in the space U if for any one $\varepsilon > 0$ can find such $\delta\left(\varepsilon\right) > 0$ that from $\left\|\vec{x}_0\right\| < \delta$

$$\sup_{t_0<t<\infty}\left\langle\left\|\vec{x}\right\|\right\rangle < \varepsilon, \tag{4}$$

where the symbol $\left\langle\,.\,\right\rangle$ is an averaging operation over an ensemble of implementations.

If the solution $\vec{x}(t) \equiv 0$ is stable according to the mathematical expectation of the norm $\left\|\vec{x}\right\|$ in the space U and a condition is executed

$$\lim_{t\to\infty}\left\langle\left\|\vec{x}\right\|\right\rangle = 0, \tag{5}$$

then the solution is called asymptotically stable according to the mathematical expectation of the norm.

If the norm $\left\|\vec{x}\right\|$ is in the form

$$\left\|\vec{x}\right\| = \left(\sum_{k=1}^{2n}\left|x_k\right|^{\rho}\right)^{1/\rho}, \tag{6}$$

then they talk about ρ - stability of the system. If $\rho = 2$, then stability in rms is considered.

The concept of stability with respect to moment functions of different order from phase variables has received wide practical application. The moment functions of a vector random process $\vec{x}(t)$ are considered, which are equal to the mathematical expectations of the components of the process and their product at the same moment in time. There are moment functions of the first order − mathematical expectation of components, moment functions of the second order − mathematical expectation of squares and even products of components, etc.

A vector of moment functions of the r-th order is entered

$$m_r(t) = \{m_{11\ldots1}(t), m_{11\ldots2}(t), \ldots\}, \tag{7}$$

where $m_{jkl\ldots}(t) = \langle x_j(t) x_k(t) x_l(t) \ldots \rangle$, here the number of indices is equal to the order of the moment function. The corresponding vector space is denoted by M_r, and the norm in this space is denoted by $\|m_r\|$.

The solution $\vec{x}(t) \equiv 0$ of equation (1) is called stable with respect to moment functions of the r-th order, if for each one $\varepsilon > 0$ can be found such $\delta(\varepsilon) > 0$ that from $\|m_r(t_0)\| < \delta$

$$\sup_{t_0 < t < \infty} \langle \|m_r(t)\| \rangle < \varepsilon. \tag{8}$$

If the solution $\vec{x}(t) \equiv 0$ is stable with respect to moment functions of the r-th order and the condition is fulfilled

$$\lim_{t \to \infty} \langle \|m_r(t)\| \rangle = 0, \tag{9}$$

then the solution is called asymptotically stable with respect to moment functions of the r-th order.

In some problems, moment functions of different orders are connected in such a way that it is impossible to consider them separately in time. In this case, it is advisable to modify the definition of system stability as follows. A vector is entered

$$m_1^r(t) = \{m_1(t), m_2(t), \ldots, m_r(t)\}, \tag{10}$$

the components of which are the moment functions of the process from the first to the r-th order inclusively. The dimensionality of this vector is reduced taking into account the symmetry of the moment functions.

We denote the corresponding vector space by M_1^r, and the norm in this space by $\|m_1^r\|$. For example, the Euclidean norm is written in the form

$$\|m_1^r\| = \left(\sum_{j=1}^{n} m_j^2 + \sum_{j=1}^{k} \sum_{k=1}^{n} m_{jk}^2 + \ldots \sum_{j=1}^{k} \sum_{k=1}^{l} \ldots m_{jkl}^2 \right)^{1/2}. \tag{11}$$

A solution $\vec{x}(t) \equiv 0$ is said to be stable with respect to moment functions up to the r-th order if for each $\varepsilon > 0$ it is possible to find such $\delta(\varepsilon) > 0$ that from $\|m_1^r(t_0)\| < \varepsilon$

$$\sup_{t_0 < t < \infty} \langle \|m_1^r(t)\| \rangle < \varepsilon. \tag{12}$$

If the solution $\vec{x}(t) \equiv 0$ is stable with respect to moment functions up to and including the r-th order and the condition is fulfilled

$$\lim_{t \to \infty} \langle \|m_1^r(t)\| \rangle = 0, \tag{13}$$

then the solution is called asymptotically stable with respect to moment functions up to and including the r-th order.

Parametric resonance occurs with such probabilistic characteristics of the random process, in which the trivial solution of system (1) becomes unstable in the sense of the accepted definition of stochastic stability in probability, on average, and relative to moment functions.

2. Stochastic stability in probability of a circular cylindrical shell under longitudinal loading. A circular cylindrical shell had the following characteristics: radius $R = 0.16$ m, length $l = 0.43$ m, thickness $h = 0.005$ m, surface density $\rho h = 13.5$ kg/m^2, modulus of elasticity of the material $E = 0.7 \cdot 10^{11}$ Pa, Poisson's ratio $\nu = 0.3$. Fastening allowed free displacement of the edges in the longitudinal direction, but limited the displacement in the circular direction. Along the ends of the shell, a uniformly distributed longitudinal load was applied, which was given by a random function $P(t)$.

The calculation model of stochastic parametric oscillations of the cylindrical shell can be performed analytically according to the asymptotic method [3] and numerically [23]. The system of equations that describes the dynamic stability of shell parametric oscillations was reduced to a system of uncoupled equations of the form

$$\ddot{y}_i(t) + 2\xi\omega_{0i}\dot{y}_i(t) + \omega_{0i}^2\left[1 + P_i(t)\right] y_i(t) = 0,\ \ i = 1,2,...,m\,, \qquad (14)$$

where $y_i(t), \dot{y}_i(t)$ – the generalized coordinates and generalized velocities (m – the number of retained base vectors); $P_i(t) = 2\mu_i\zeta(t)$, $\mu_i = \dfrac{g_i p_0}{2\omega_{0i}^2}$ – respectively, the longitudinal load and its intensity, $\zeta(t)$ – a random function with variance $D_\zeta = 1/2$, ω_{0i} – the shell natural frequency, g_i – geometric stiffness coefficient, $\varepsilon = 0{,}01$ – the damping parameter, which was assumed to be the same for all forms of shell natural oscillations.

Equation (14) is a stochastic analogue of the Mathieu-Hill equation. The question of the occurrence of parametric resonance is reduced to the problem of the stability of trivial solutions of equation (14). Using the finite element method, which is implemented in the NASTRAN software [27], a finite element model of a circular cylindrical shell was constructed. Natural frequencies were determined numerically by the Lanczos method using the computational procedure of the NASTRAN software: $\omega_{01} = \omega_{02} = 4734$ rad/s, $\omega_{03} = \omega_{04} = 5263$ rad/s, $\omega_{05} = \omega_{06} = 7275$ rad/s.
Natural frequencies were compared to analytical ones [3]:

$$\omega_{1,3}^{(1)} = 4764{,}21 \text{ rad/s},\ \omega_{1,3}^{(2)} = 64789{,}28 \text{ rad/s},\ \omega_{1,3}^{(3)} = 111703{,}9 \text{ rad/s},$$

$$\omega_{1,4}^{(1)} = 5341{,}46 \text{ rad/s},\ \omega_{1,4}^{(2)} = 82885{,}32 \text{ rad/s},\ \omega_{1,4}^{(3)} = 142599{,}6 \text{ rad/s},$$

$$\omega_{1,2}^{(1)} = 7278{,}66 \text{ rad/s},\ \omega_{1,2}^{(2)} = 48390{,}12 \text{ rad/s},\ \omega_{1,3}^{(3)} = 82364{,}68 \text{ rad/s}.$$

The values of the elements of the geometric stiffness matrix were determined numerically using the NASTRAN software [27] $g_1 = 3{,}953931\,\text{kg}^{-1}$ and compared with the analytical values [3]:

$$g_{11}=g_{22}=3{,}513,\ g_{33}=g_{44}=3{,}689,\ g_{55} = g_{66} = 3{,}136\,.$$

The random function of the parametric loading $P(t)$ was given with the spectral density

$$G_P(\omega) = \frac{2\sigma_P^2}{\pi}\cdot\frac{\alpha(\omega^2+\theta^2)}{(\omega^2-\theta^2)^2+(2\alpha\omega)^2}\,, \qquad (15)$$

where α – correlation parameter, θ – characteristic load frequency, $\sigma_P = \mu\sqrt{2} = \dfrac{gp_0}{\omega_0^2\sqrt{2}}$ – standard deviation of a random process.

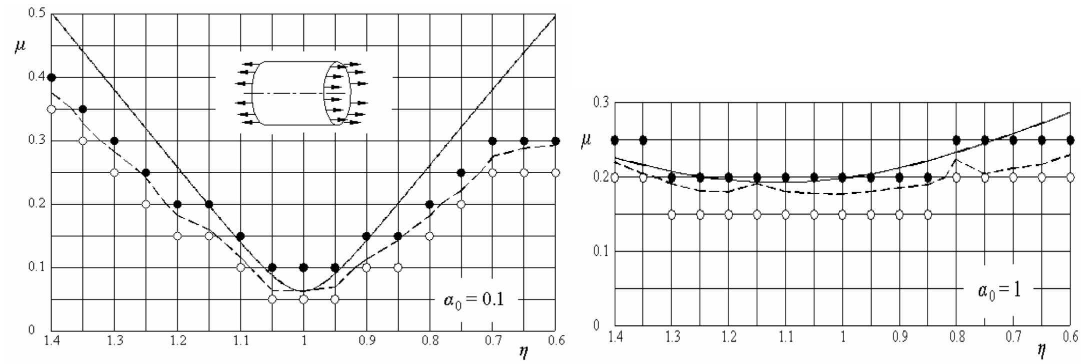
In the case of a broadband random process according to the Stratonovich-Khasminsky approximate theory [11, 12], the condition of asymptotic stability in terms of the probability of parametric oscillations of the envelope can be written by the expression

$$G_P(2\omega_0) = \frac{8\varepsilon}{\pi\omega_0}\,, \qquad (16)$$

By substituting the function (15) into the relation (16), was obtained the boundary equation of the region of stability of the shell parametric oscillations. In the coordinates of the relative frequency $\eta = \theta/(2\omega_0)$ and reduced intensity μ, when $\alpha_0 = \alpha/\omega_0$ the following equation takes the form

$$\mu = 2\sqrt{2\varepsilon\frac{(1-\eta^2)^2 + \alpha_0^2}{\alpha_0(1+\eta^2)}}\,. \qquad (17)$$

It is possible to determine the limits of the loss of stability numerically by direct simulation of the oscillatory process. For this, the realization of a stationary random function was modeled by the spectral density (15) and equation (14) was integrated using the Runge-Kutta method for some small initial disturbances. In the range of values of the external influence parameters, which correspond to the stability region of parametric fluctuations, the initial disturbances during the integration of equation (14) were attenuated. Beyond this region, the initial perturbations increase. Results of analytical and numerical studies of stability according to the probability of shell parametric oscillations the under longitudinal loading with the corresponding values of α_0=0,1 and α_0=1 were presented in Fig. 1. The study showed that the lowest values of the critical load intensity correspond to equation (14) with the natural frequency $\omega_0 = \omega_{01} = \omega_{1,3}^{(1)}$. The solid line shows the boundaries of the stability region of this form of oscillations under parametric loading with different values α_0 constructed according to the analytical formula (17). As a result of the numerical study, points corresponding to shell stable oscillations (empty markers) and points corresponding to un stable oscillations (colored markers) were obtained. The dashed line shows the experimental boundary of the stability region of shell parametric oscillations.



(a) (b)

Fig. 1

A better agreement between the analytical and numerical results can be seen in Fig. 2 than in Fig. 1. This can be explained by the better fulfillment of the condition $\alpha_0 >> \varepsilon$ necessary for the application of formula (17). There is also a better convergence of the results of analytical and numerical studies nearby $\eta = 1$. This is explained by the fact that formula (17) has an approximate character, that is, it is valid in the immediate vicinity of the main parametric resonance, when the characteristic frequency of the load θ coincides with the frequency of the main parametric resonance of the shell $2\omega_0$.

3. Stochastic stability of the shallow shell under surface delta-correlated disturbance. The question of stochastic stability of parametric oscillations of a shallow shell was formulated with respect to moment functions of phase coordinates of the second order. A square shallow shell with the following geometric and mechanical characteristics was considered: sides $a = b = 0,48$ m, thickness h=0,004 m, modulus of elasticity E =7,2·10^{10} Pa, specific gravity ρ = 2700 kg/m^3, Poisson's ratio $\mu = 0,3$. The maximum deflection in the middle of the shell was $f \le \frac{1}{5}a$=0.096 m. The main curvatures of the middle surface of the shell along the forming axes were assumed to be the same $k_x = k_y = const$.

The reduced model of shell parametric oscillations has the form of a system of linearized ordinary differential equations

$$M^{*}\ddot{y}_i(t) + C^{*}\dot{y}_i(t) + K^{*}y_i(t) + q(t)K_G^{*}y_i(t) = 0,\ i = 1,2,...,m, \qquad (18)$$

where M^*, C^*, K^* and K_G^* – reduced matrices of masses, damping, stiffness and geometric stiffness of the dimension $m\times m$ respectively (m – the number of basic vectors retained); $y_i(t), \dot{y}_i(t)$ – the generalized coordinates and generalized velocities of the nodes of the shell finite element model.

In equation (18) $q(t) = q_0 + \tilde{q}(t)$ is the surface distributed stochastic load, where q_0 – the constant component of the load, $\tilde{q}(t)$ – the delta-correlated random component of the load with the correlation function

$$K(\tau) = \sigma_0^2 e^{-\alpha\tau}\left(\cos\theta_\alpha\tau + \frac{\alpha}{\theta_\alpha}\sin\theta_\alpha\tau\right), \tag{19}$$

and a finite radius of correlation

$$\tau_0 = \frac{1}{\sigma_0^2}\int K(\tau)d\tau = \frac{1}{\sigma_0^2}\int \sigma_0^2 e^{-\alpha\tau}\left(\cos\theta_\alpha\tau + \frac{\alpha}{\theta_\alpha}\sin\theta_\alpha\tau\right)d\tau = \frac{\alpha}{\alpha^2+\theta_\alpha^2} + \frac{\alpha}{\theta_\alpha}\left(\frac{\alpha}{\alpha^2+\theta_\alpha^2}\right) = \frac{2\alpha}{\alpha^2+\theta_\alpha^2}, \tag{20}$$

where σ_0^2 – the intensity of stochastic influence; α – the correlation parameter, θ_α – the frequency of the hidden periodicity.

If we enter a $2m$-dimensional vector of phase variables $\vec{\zeta}(t) = (\zeta_1(t), \zeta_2(t), ..., \zeta_{2m}(t))^T = (y_1(t), y_2(t), ..., y_m(t), \dot{y}_1(t), \dot{y}_2(t), ..., \dot{y}_m(t))^T$, then system (18) can be rewritten in normal form

$$\frac{d}{dt}\vec{\zeta}(t) = A\vec{\zeta}(t) + q(t)B\vec{\zeta}(t). \tag{21}$$

Matrices A and B are calculated by formulas

$$A = \begin{bmatrix} 0 & E \\ -(M^*)^{-1}K^* & -(M^*)^{-1}C^* \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ -(M^*)^{-1}K_G^* & 0 \end{bmatrix},$$

where E – unit matrix with dimension $m\times m$.

For system (21), we consider the Cauchy problem with initial conditions $\vec{\zeta}(0) = \vec{\zeta}_0$, where the vector $\vec{\zeta}_0 = (\zeta_{01}, \zeta_{02}, ..., \zeta_{02m})^T$ is considered deterministic.

The question about the loss of dynamic stability of the shell is equivalent to the question about the stability of trivial solutions of equations (21). When averaging over the ensemble of realizations of system (21), the equations take the form

$$\frac{d}{dt}\langle\vec{\zeta}(t)\rangle = A\langle\vec{\zeta}(t)\rangle + B\langle q(t)\vec{\zeta}(t)\rangle, \langle\vec{\zeta}(0)\rangle = \vec{\zeta}_0. \tag{22}$$

System (22) is open with respect to the variables $\langle\vec{\zeta}(t)\rangle = (\langle\zeta_1(t)\rangle, \langle\zeta_2(t)\rangle, ..., \langle\zeta_{2m}(t)\rangle)^T$, as contains new unknown features $\langle q(t)\vec{\zeta}(t)\rangle = (\langle q(t)\zeta_1(t)\rangle, \langle q(t)\zeta_2(t)\rangle, .., \langle q(t)\zeta_{2m}(t)\rangle)^T$, which are the correlations at the instant of time t of the random process $q(t)$ with the solution of system (21), the components of which are functionals of the random process $q(t)$ in the interval $[0, t]$. When considering system (22), the approach of splitting the average product of two functionals is used. If $R[q(t)]$ is a functional from a Gaussian centered process, then the Furuttsu-Novikov formula holds for the average product $\langle q(t)R[q(t)]\rangle$ [15, 23]

$$\langle q(t)R[q(t)]\rangle = \int_0^t d\tau K(t-\tau)\langle\frac{\delta R[q]}{\delta q(\tau)}\rangle. \tag{23}$$

Here $K(t)$ – the correlation function of the process $q(t)$, $\langle\frac{\delta R[q]}{\delta q(\tau)}\rangle$ – the average of the variational derivative of the functional $R[q]$ at q the point τ.

As a result of the given approach, system (22) takes the form of a connected infinite sequence of equations for moment functions of the second order

$$\frac{d}{dt}\langle\frac{\delta\left(\vec{\zeta}(t)\vec{\zeta}^T(t)\right)}{\delta z(\tau)}\rangle = A\langle\frac{\delta\left(\vec{\zeta}(t)\vec{\zeta}^T(t)\right)}{\delta z(\tau)}\rangle + B\int_0^t d\tau_1 K(t-\tau_1) + \langle\frac{\delta^2\left(\vec{\zeta}(t)\vec{\zeta}^T(t)\right)}{\delta z(\tau)\delta z(\tau_1)}\rangle +$$

$$+\langle\frac{\delta\left(\vec{\zeta}(t)\vec{\zeta}^T(t)\right)}{\delta z(\tau)}\rangle A^T + \int_0^t d\tau_1 K(t-\tau_1)\langle\frac{\delta^2\left(\vec{\zeta}(t)\vec{\zeta}^T(t)\right)}{\delta z(\tau)\delta z(\tau_1)}\rangle B^T \quad (24)$$

with initial conditions

$$\langle\frac{\delta^2\left(\vec{\zeta}(\tau_1)\vec{\zeta}^T(\tau_1)\right)}{\delta z(\tau)\delta z(\tau_1)}\rangle = B\langle\frac{\delta\left(\vec{\zeta}(t)\vec{\zeta}^T(t)\right)}{\delta z(\tau)}\rangle + \langle\frac{\delta\left(\vec{\zeta}(t)\vec{\zeta}^T(t)\right)}{\delta z(\tau)}\rangle B^T, \quad \langle\frac{\delta^2\left(\vec{\zeta}(t)\vec{\zeta}^T(t)\right)}{\delta z(\tau)\delta z(\tau_1)}\rangle = 0, \quad t < \tau. \quad (25)$$

The question of stochastic stability of system (22) is reduced to the study of stability of trivial solutions of integral-differential equations (25). Taking into account the parametric load, which is applied to the hollow shell, in the form of a delta-correlated random process with correlation function (19), finite correlation time (20) and variance of stochastic influence, the matrix differential equation for second-order moments (24) becomes closed, and other equations of the infinite system become redundant. The resulting system of equations will represent the equations of the first Markov approximation for the second moment functions

$$\frac{d}{dt}\langle\vec{\zeta}(t)\vec{\zeta}^T(t)\rangle = \left(A + DB^2\right)\langle\vec{\zeta}(t)\vec{\zeta}^T(t)\rangle + \langle\vec{\zeta}(t)\vec{\zeta}^T(t)\rangle\left(A + DB^2\right)^T + 2\sqrt{D}B\langle\vec{\zeta}(t)\vec{\zeta}^T(t)\rangle B^T \quad (26)$$

with initial conditions $\langle\vec{\zeta}(0)\vec{\zeta}^T(0)\rangle = \left(\vec{\zeta}_0\vec{\zeta}_0^T\right)$.

The system of equations (26) can be presented in the form of a system of three deterministic differential equations of the first Markov approximation with respect to moment functions of the second order in the phase variables $\zeta_1(t) = y_i(t)$, $\zeta_2(t) = \dot{y}_i(t)$ for each frequency of natural oscillations of the shallow shell [8] with initial conditions $\zeta_1(0) = y_{i0}$, $\zeta_2(0) = \dot{y}_{i0}$

$$\frac{d}{dt}\langle\zeta_1^2(t)\rangle = 2\langle\zeta_1(t)\zeta_2(t)\rangle,$$

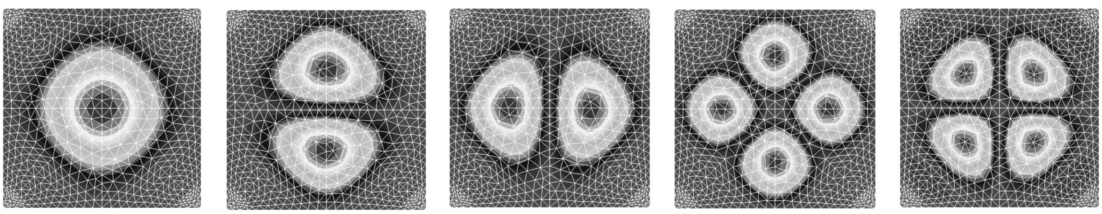
$$\frac{d}{dt}\langle\zeta_1(t)\zeta_2(t)\rangle = \langle\zeta_2^2(t)\rangle - \omega_i^2\langle\zeta_1^2(t)\rangle - 2\varepsilon_i\omega_i\langle\zeta_1(t)\zeta_2(t)\rangle,$$

$$\frac{d}{dt}\langle\zeta_2^2(t)\rangle = -4\varepsilon_i\omega_i\langle\zeta_2^2(t)\rangle - 2\omega_i^2\langle\zeta_1(t)\zeta_2(t)\rangle + a_i^2\omega_i^4 q_0\langle\zeta_1^2(t)\rangle + a_i^2\omega_i^4\sigma_0^2\tau_0\langle\zeta_1^2(t)\rangle, \quad (27)$$

Here ω_i – the shell natural frequencies, ε_i – damping parameter, $a_i = g_{ii}^* / \omega_i^2$ – coefficients that characterize the influence of the constant component of the parametric load on the shell stiffness characteristics, which is determined using the approach [15]; τ_0 – the finite correlation time, which is determined by formula (20), $D = \sigma_0^2\tau_0$ – the variance of the stochastic parametric influence.

The modes (Fig. 3) and frequencies of the shell natural oscillations were determined using the NASTRAN software:

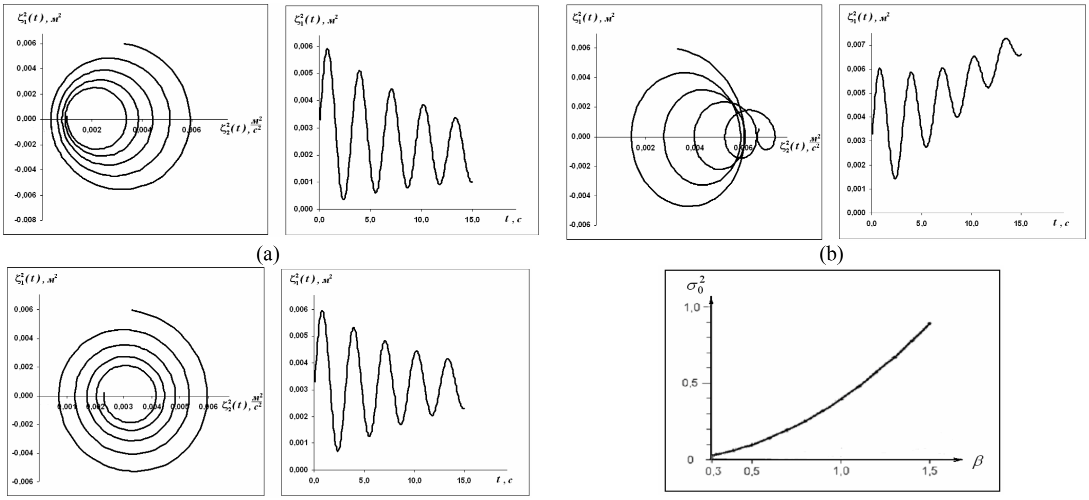
$$\omega_{i=1,\ldots,5} = [1243,4;2486,8;2493,2;3906,2;4402,3] \text{ rad/s}.$$



Mode 1 Mode 2 Mode 3 Mode 4 Mode 5

Fig. 3. The modes of natural oscillations of the shallow shell

Using the Runge-Kutta direct method, the system of differential equations (27) was integrated under the action of a parametric load with different values of its intensity σ_0^2. In Fig. 4 presents the phase trajectories and dynamic behavior of the solution $\zeta_1^2(t)$: in stable (Fig. 4, a), unstable (Fig. 4, b) and at the border of the stability region (Fig. 3, c) regimes of oscillations of the shell according to the first form of natural oscillations. In a stable regime of stochastic oscillations, the solution $\zeta_1^2(t)$ of system (27) decreases over time, in an unstable regime, it increases. In Fig. 4, d presents the critical values of the intensity of the stochastic component of the load σ_0^2 according to the first form of shell natural oscillations with a variable coefficient $\beta = \theta_\alpha / \omega_1$. The region of instability of the parametric oscillations of the shallow shell lies above the curve.



(a) (b) (c) (d)

Fig. 4. Stable (a), unstable (b), at the boundary of the instability region (c) modes of shell oscillations; critical values of the intensity of the stochastic component of the parametric disturbance (d)

If we rewrite the system of deterministic differential equations with respect to moment functions of the second order (27) in the form of a linear autonomous system

$$\frac{d}{dt}\begin{Bmatrix} \langle\zeta_1^2(t)\rangle \\ \langle\zeta_1(t)\zeta_2(t)\rangle \\ \langle\zeta_2^2(t)\rangle \end{Bmatrix} = G(t)\begin{Bmatrix} \langle\zeta_1^2(t)\rangle \\ \langle\zeta_1(t)\zeta_2(t)\rangle \\ \langle\zeta_2^2(t)\rangle \end{Bmatrix}, \tag{28}$$

where $G(t)$ – matrix whose coefficients are $2\pi/\omega_i$ – periodic functions

$$G(t) = \begin{vmatrix} 0 & 2 & 0 \\ -\omega_i^2 & -2\varepsilon_i\omega_i & 1 \\ a_i^2\omega_i^4\sigma_0^2\tau_0 & -2\omega_i^2 & -4\varepsilon_i\omega_i \end{vmatrix}, \tag{29}$$

then the analysis of the shell stability is reduced to the problem of the stability of trivial solutions of system (28). With the help of the method of generalized Hill determinants, when solving an algebraic problem for eigenvalues, the characteristic indicators are determined and the boundaries of the region of shell instability are constructed. In Fig. 5 presents the behavior of the characteristic Hill indicators of the system (29) under the influence of stochastic load with the frequency of the hidden periodicity $\theta_\alpha = \omega_1$, the correlation parameter $\alpha = \varepsilon_1\omega_1 = 0{,}0276\omega_1$, and the correlation radius $\tau_0 = 0{,}0552/\omega_1$. The real parts of the characteristic indicators are shown by a solid line, the complex ones by a dashed line. The positive real parts of the characteristic indicators, which correspond to the unstable mode of oscillations, lie in the upper half-plane. The points of intersection of the solid curve of the coordinate axis correspond to the critical values of the stochastic component of parametric fluctuations.

4. Stochastic stability relative to moment functions of different order from phase variables. Let us consider the results of determining the boundaries of the regions of stochastic stability of shell parametric oscillations with respect to moment functions of different orders (7). A circular cylindrical shell had the characteristics, which was consider bellow in item 2. Each equation of the system of the differential equation that describes the shell parametric oscillations at each natural frequency $\omega_{m_1 m_2}^{(i)}$ with the corresponding number of half-waves in the longitudinal and circular directions [3] is the stochastic analogue of the Mathieu-Hill equation

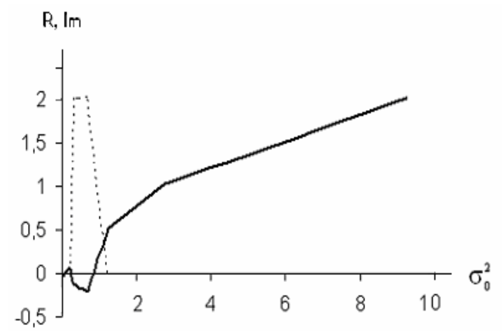
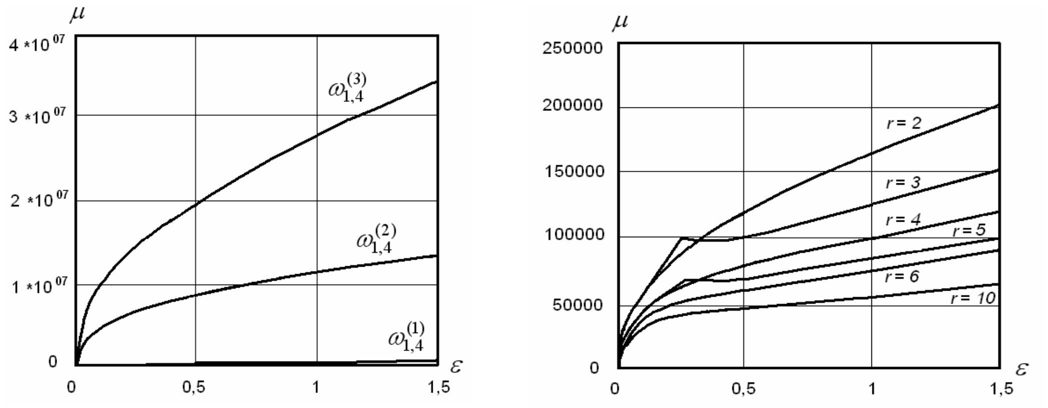


Fig. 5. Dependence of Hill's characteristic indicators from the intensity of stochastic influence

$$\ddot{y}(t) + 2\varepsilon\omega_0 \dot{y}(t) + \omega_0^2 \left(1 + \mu f(t)\right) y = 0, \tag{30}$$

where $y = y_{m_1 m_2}^{(i)}$ − the generalized coordinate, $\omega_0 = \omega_{m_1 m_2}^{(i)}$ − the frequency of the shell natural oscillations, ε − the damping parameter, which is determined using the logarithmic decrement of the oscillations $\delta = 0,05$; $f(t)$ − stationary normal process of white noise type, μ − intensity of random parametric disturbance.

The stochastic stability of a cylindrical shell was investigated numerically by the continuation by parameter method [14]. The boundaries of the regions of shell dynamic stability with respect to the second moments under the parametric influence of the type of white noise for various natural modes are presented in Fig. 6, a. The graphs corresponding to frequency $\omega_0 = \omega_1 = 4764,205$ rad/s for moment functions of various orders are shown in Fig. 6, b. The dependence of critical values μ on the order of moments is such that, when considering moments of even and odd orders, as the order of moments increases, the regions of stability become narrower. The curves corresponding to moments of different parity intersect at certain values $f(t)$.



(a) (b)

Fig. 6. Boundaries of the regions of shell dynamic stability with relative to different moment functions: r=2 (a) and r=2-10 (b)

Analysis of the results shown in Fig. 6, b demonstrates the following: as the frequency decreases, the region of dynamic stability narrows and for all ε the critical values μ are the smallest for the lowest frequency ω_1.

Conclusion. A review of studies of parametric oscillations of shells and their stability showed that most of them concern parametric oscillations of shells due to the action of periodic disturbances. The problem of studying stochastic parametric oscillations of shell structures remains relevant due to the complexity of forming calculation models of parametric oscillations and solving the problem of their stochastic stability. Currently, calculation models of parametric oscillations of shells can be formed on the basis of the asymptotic and functional approaches using the calculation procedures of finite element analysis software. Stochastic stability of elastic shells can be formulated as stability in probability, on average and with respect to the moment functions of different order. The critical values of stochastic load intensity and the regions of stochastic stability of shells can be obtained by Runge-Kutta method of the fourth order and the continuation by parameter method.

REFERENCES

1. *Andronov O.O , Pontryagin L.S. and Vitt O.A.* O statystycheskom rassmotrenyy dynamycheskykh system (On statistical consideration of dynamic systems). ZhTEF, T.3, №3, 1933.
2. *Bogolyubov N.N., Mitropolskyi Yu.A.* Asymptotycheskye metody v teoryy nelyneinykh kolebanyi (Asymptotic methods in the theory of nonlinear oscillations). M.: Nauka, 1974.
3. *Bolotin V.V.* Dynamycheskaia ustoichyvost upruhykh system (Dynamic stability of elastic systems). M.: Hosudarstvennoe yzdatelstvo tekhnyko-teoretycheskoi lyteratury, 1956.
4. *Bolotin V.V.* Sluchainye kolebanyia upruhykh system (Random vibrations of elastic systems). M.: Nauka, 1979.
5. *Gikhman I.I. and Skorokhodov A.V.* Teoryia sluchainykh protsessov (Theory of random processes). Tom 3, M.: Nauka, 1975.
6. *Dimentberg M.F.* Sluchainye protsessy v dynamycheskykh systemakh s peremennymy parametramy (Random processes in dynamic systems with variable parameters). M.: Nauka, 1989.
7. *Klosek-Dygas M.M., Matkowsky BJ., and Schuss Z.* Stochastic stability of nonlinear oscillators // SIAM J. Appl. Math., 1988, 48, 5, 1115-1127.
8. *Klyatskin V.I.* Stokhastycheskye uravnenyia y volny v sluchaino-neodnorodnykh sredakh (Stochastic equations and waves in randomy-inhomogeneous areas). M.: Nauka, 1980.
9. *Labou M.* On stability of parametrically excited linear stochastic systems // Int. Appl. Mech., 2011, 47(10), 1440-1453.
10. *Lyapunov A.M.* Obshchaia zadacha ob ustoichyvosty dvyzhenyia (General problem of motion stability) . M.-L.: Gosyzdat, 1950.
11. *Stratonovich R.L.* Uslovnye markovskye protsessy (Conditional Markov processes). M.: MGU, 1966.
12. *Khasminski R.Z.* Ustoichyvost system dyfferentsyalnykh uravnenyi pry sluchainykh vozmushchenyiakh ykh parametrov (Stability of systems of differential equations under random disturbances of their parameters). M.: Nauka, 1969.
13. *Yakubovich V.A., Starzynskyi V.M.* Parametrycheskyi rezonans v lyneinykh systemakh (Parametric resonance in linear systems). M.: Nauka, 1987.
14. *Bazhenov V.A., Busetta M., Dekhtyaruk E.S., Otrashevska V.V.* Dynamichna stiikist pruzhnykh system pry stokhastychnomu parametrychnomu zbudzhenni (Dynamic stability of the elastic systems under stochastic parametric influence) // Strength of Materials and Theory of Structures: Scientificand-technical collected articles. − K.: KNUBA, 2000. − Issue 67. − P. 51-59 (ukr).
15. *Bazhenov V.A., Dekhtyaruk E.S., Lukianchenko O.O, Kostina O.V.* Chyslennoe postroenye redutsyrovannыkh modelei stokhastycheskykh parametrycheskykh kolebanyi polohykh obolochek (Numerical construction of reduce models of parametric vibrations of the shallow shells) // Strength of Materials and Theory of Structures: Scientificand-technical collected articles. − K.: KNUBA, 2011. − Issue 72. − P. 51-59.
16. *Bazhenov V.A., Lukianchenko O.O., Vorona Yu.V., Kostina O.V.* Dynamichna stiikist parametrychnykh kolyvan pruzhnykh system (Dynamic stability of parametric vibrations of the elastic systems) // Strength of Materials and Theory of Structures: Scientificand-technical collected articles. − K.: KNUBA, 2015. − Issue 95. − P. 159-185 (ukr).
17. *Bazhenov V.A., Luk'yanchenko O.A., Vorona Yu.V., Kostina E.V.* On Stability of Parametric Oscillations of a Shell in the Form of Hyperbolic Paraboloid // International Applied Mechanics. –2018, Volume 54, №3. – P. 36-49.274-286.
18. *Dekhtyaruk E.S., Lukianchenko O.O., Otrashevska V.V.* Dynamichna stiikist pruzhnykh system pry kombinovanomu stokhastychnomu navantazhenni (Dynamic stability of the elastic systems at combained stochastic load) // Strength of Materials and Theory of Structures: Scientificand-technical collected articles. − K.: KNUBA, 2003. − Issue 72. − P. 51-59 (ukr).
19. *Goncharenko M.V., Poshyvach D.V.* Doslidzhennia umov dynamichnoi stiikosti pruzhnykh system pry stokhastychnomu parametrychnomu vplyvi (Study of conditions of dynamic stability of elastic systems under stochastic parametric influence) // Vibratsii v tekhnytsi ta tekhnolohiiakh. − 2006. − Issue 2(44). − P. 14-19. (ukr)
20. *Gotsulyak E.A., Dekhtyaruk E.S., Lukianchenko O.O., Poshyvach D.V.* Doslidzhennia dynamichnoi stiikosti ploskoi formy zghynu pruzhnykh system pry kombinovanomu rezonansi (Research of dynamic stability of the flat bending shape of the elastic systems at combined resonance)// Strength of Materials and Theory of Structures: Scientificand-technical collected articles. − K.: KNUBA, 2007. − Issue 81. − P. 33-42 (ukr).
21. *Gotsulyak E.A., Dekhtyaruk E.S., Lukianchenko O.O.* Pobudova redukovanoi modeli parametrychnykh kolyvan tsylindrychnoi obolonky pry chystomu zghyni (A construction of reduce model of parametric oscillations of cylindrical shell bending) // Strength of Materials and Theory of Structures: Scientificand-technical collected articles. − K.: KNUBA, 2009. − Issue 84. − P. 11-19 (ukr).
22. *Lukianchenko O.O., Paliy O.M.* Chyselne modeliuvannia stiikosti parametrychnykh kolyvan tonkostinnoi obolonky vidiemnoi hausovoi kryvyzny (Numeral design of vibrations stability of the thin-walled shell with negative gaussian curvature) // Strength

of Materials and Theory of Structures: Scientificand-technical collected articles. − K.: KNUBA, 2018. − Issue 101. − P. 45-59 (ukr).
23. *Lukianchenko O.O., Kostina O.V.* Kolyvannia parametrychno zbudzenyh pruznyh obolonok (Oscillations of parametric excited elastic shells). – Kyiv: Vyd-vo "Karavela", 2022. – 164 s (ukr).
24. *Poshyvach D.V.* Chyselynyi analiz dynamichnoi stiikosti kruhovoi tsylindrychnoi obolonky pry vypadkovomu parametrychnomy zbudzhenni (Numerical analysis of the dynamic stability of a circular cylindrical shell under random parametrical excitation) // Strength of Materials and Theory of Structures: Scientificand-technical collected articles. − K.: KNUBA, 2002. − Issue 71. − P. 115-124. (ukr)
25. *Poshyvach D.V., Lukianchenko O.O.* Doslidzhennia stokhastychnoi stiikosti parametrychnykh kolyvan konstruktsii metodom Monte-Karlo (Research of stochastic stability of constructions parametric vibrations by the Monte-Carlo method) // Strength of Materials and Theory of Structures: Scientificand-technical collected articles. − K.: KNUBA, 2024. – Issue. 112. – P. 327-331. (ukr)
26. *Vorona Yu.V., Lukianchenko O.O., Kostina O.V.* Stokhastychni parametrychni kolyvannia pruzhnykh system z urakhuvanniam yikh poperednikh staniv (Stochastic parametric vibrations of elastic systems with regard to their previous states)// Strength of Materials and Theory of Structures: Scientificand-technical collected articles. − K.: KNUBA, 2014. − Issue 92. − P. 102-113 (ukr).
27. *Jaecheol Koh.* Siemens NX Nastran: Tutorials for Beginners and Advanced Users. ASIN: B0B19ZBZCM, 2022. – 566 p.

Стаття надійшла 30.09.2024

Лук'янченко О.О., Пошивач Д.В., Кара І.Д.

СТОХАСТИЧНА СТІЙКІСТЬ ПАРАМЕТРИЧНИХ КОЛИВАНЬ ПРУЖНИХ ОБОЛОНОК

На теперішній час існує достатньо потужний математичний апарат, який виник на основі броунівського руху теорії Марківських процесів і процесів дифузійного типу. Він дозволяє розв'язувати складні динамічні задачі з урахуванням флуктуаційних процесів. Класичним результатом в цій області можна вважати статтю О.О. Андронова, Л.С. Понтрягіна та О.А. Витта, в якій вперше методи теорії Марківських процесів було застосовано до дослідження задач статистичної динаміки нелінійних систем. Пізніше була розроблена строга математична теорія стохастичних диференціальних рівнянь Іто, яка представлена в роботі І.І. Гіхмана та А.В. Скорохода. Важливим кроком в застосуванні цієї теорії до дослідження задач динаміки пружних систем стали роботи Р.Л. Стратоновича. Його дослідження базувались на поєднанні методу усереднення Крилова-Боголюбова з методом теорії марківських процесів. Строге обґрунтування цього підходу було зроблено Р.З. Хасьмінським. Значний внесок у розвиток теорії стохастичних систем та впровадженню імовірнісних методів до розрахунку конструкцій було зроблено В.В. Болотіним та його послідовниками. Також важливими були дослідження стохастичних параметричних вібрацій різних систем В.І. Дименберга, В.І.. Кляцкіна та інших. З початку 80-х років минулого століття вчені кафедр будівельної і теоретичної механіки Київського національного університету будівництва і архітектури розробляли і удосконалювали чисельні методи дослідження стохастичної стійкості параметричних коливань пружних систем. В статті представлені результати числових досліджень стійкості параметричних коливань циліндричної і пологої оболонок при різних стохастичних впливах. Розрахункові моделі параметричних коливань оболонок сформовані за допомогою асимптотичного або функціонального підходу, методу Монте-Карло із застосуванням обчислювальних процедур програмного комплексу скінченно-елементного аналізу. Стохастична стійкість оболонок сформульована за імовірністю, у середньому і відносно моментних функцій різного порядку. Критичні значення інтенсивності стохастичного навантаження і області нестабільності оболонок отримані із застосуванням методів Рунге-Кутти четвертого порядку і продовження за параметром.

Ключові слова: оболонка, випадкове параметричне навантаження, стохастична стійкість, імовірність, моментні функції.

Lukianchenko O.O., Poshyvach D.V., Kara I.D.

STOCHASTIC STABILITY OF PARAMETRIC OSCILLATIONS OF ELASTIC SHELLS

There is a powerful mathematical apparatus which appeared up on the basis of the theory of Brownian motion of Markov processes and processes of diffusional type nowadays. It allows deciding intricate dynamic problems taking into account fluctuation processes. A classic result in this area is the article of O.O. Andronov, L.S. Pontryagin and O.A. Vitt, in which firstly the methods of the theory of Markov processes were applied to research of problems of statistical dynamics of the nonlinear systems. Later, the strict mathematical theory of stochastic differential equations of Ito was presented in an article of I.I. Gikhman and A.V. Skorokhodov. An important step in application of this theory to research of dynamic problems of the elastic systems was become researches of R.L. Stratonovich. These researches were based on combination of Krilov-Bogolyubov method of averaging with the method of theory of Markov processes. The strict ground of this approach was done by R.Z. Khasminski. A significant contribution to the development of the theory of stochastic systems and the introduction of probabilistic methods for the calculation of structures was made by V.V. Bolotin and his followers. V.V. Bolotin performed significant work on the application of probabilistic methods to the calculation of structures. Also important are studies of stochastic parametric oscillations of various systems by Dimentberg, V.I. Klyatskin and others. From the beginning of 80-th of the last century the scientists of Structural and Theoretical mechanics department of the Kyiv National University of Construction and Architecture were engaged in development of the numeral research of stochastic stability of elastic systems. Results of numerical researches of stability of parametric oscillations of the cylindrical and shallow shells under different stochastic influences were presented at this article. Parametric oscillations models of the shells were formed on the basis of the asymptotic or functional approaches and Monte-Carlo method using the calculation procedures of finite element analysis software. Stochastic stability of elastic shells was formulated as stability in probability, on average and with respect to the moment functions of different order. The critical values of stochastic load intensity and the regions of stochastic stability of shells were obtained by Runge-Kutta method of the fourth order and the continuation by parameter method.

Keywords: shell, random parametric load, stochastic stability, probability, moment functions.

УДК 539.3
Лук'янченко О.О., Пошивач Д.В., Кара І.Д. **Стохастична стійкість параметричних коливань пружних оболонок** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2024. – Вип. 113. – С. 63-74.
Представлені результати числових досліджень стійкості параметричних коливань циліндричної і пологої оболонок при різних стохастичних впливах. Розрахункові моделі параметричних коливань оболонок сформовані за допомогою асимптотичного і функціонального підходу із застосуванням обчислювальних процедур програмного комплексу скінченно-елементного аналізу. Стохастична стійкість оболонок сформульована за імовірністю, у середньому і відносно моментних функцій високого порядку. Критичні значення інтенсивності стохастичного навантаження і області нестабільності оболонок отримані із застосуванням методів Монте-Карло, Рунге-Кутти четвертого порядку і методу продовження за параметром.
Табл. 0. Іл. 6. Бібліогр. 27 назв.

UDC 539.3
Lukianchenko O.O., Poshyvach D.V., Kara I.D. **Stochastic stability of parametric oscillations of elastic shells** // Strength of Materials and Theory of Structures: Scientificand-technical collected articles. − K.: KNUBA, 2024. – Issue. 113. – P. 63-74.
Results of numerical researches of stability of parametric oscillations of the cylindrical and shallow shells under different stochastic influences were presented. Parametric oscillations models of the shells were formed on the basis of the asymptotic and functional approaches using the calculation procedures of finite element analysis software. Stochastic stability of elastic shells was formulated as stability in probability, on average and with respect to the moment functions of different order. The critical values of stochastic load intensity and the regions of stochastic stability of shells were obtained by Monte-Carlo method, Runge-Kutta method of the fourth order and the continuation by parameter method.
Tab. 0. Figs. 6. Refs. 27.

Автор (вчена ступень, вчене звання, посада): доктор технічних наук, професор, провідний науковий співробітник НДІ будівельної механіки, професор кафедри будівельної механіки КНУБА, ЛУК'ЯНЧЕНКО Ольга Олексіївна.
Адреса робоча: 03037 Україна, м. Київ, проспект Повітряних сил, 31, Київський національний університет будівництва і архітектури
Робочий тел.: +38(044) 241-54-20
E-mail: lukianchenko.oo@knuba.edu.ua, lukianch0907@meta.ua
ORCID ID: https://orcid.org/0000-0003-1794-6030

Автор (вчена ступень, вчене звання, посада): старший викладач кафедри опору матеріалів КНУБА, ПОШИВАЧ Дмитро Володимирович.
Адреса робоча: 03037 Україна, м. Київ, проспект Повітряних сил, 31, Київський національний університет будівництва і архітектури.
Робочий тел.: +38(044) 241-54-21
Мобільний тел.: +38(067) 290-99-39
E-mail: poshyvach.dv@knuba.edu.ua
ORCID ID: https://orcid.org/0000-0002-8273-0298

Автор (науковий ступінь, вчене звання, посада): кандидат технічних наук, доцент, доцент кафедри будівельної механіки КАРА Ірина Дмитрівна.
Адреса робоча: 03037 Україна, м. Київ, проспект Повітряних сил, 31, Київський національний університет будівництва і архітектури.
Роб. тел.:+ 38(044) 2415412
E-mail: karaidknuba@tutanota.com
ORCIDID: https://orcid.org/0000-0003-4700-997X