UDK 539.3

HIGH-PRECISION MODELLING OF DEFORMATION OF SANDWICH STRUCTURES UNDER BILATERAL SYMMETRIC AND OBLIQUE-SYMMETRIC LOADING

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DOI: 10.32347/2410-2547.2024.112.258-267

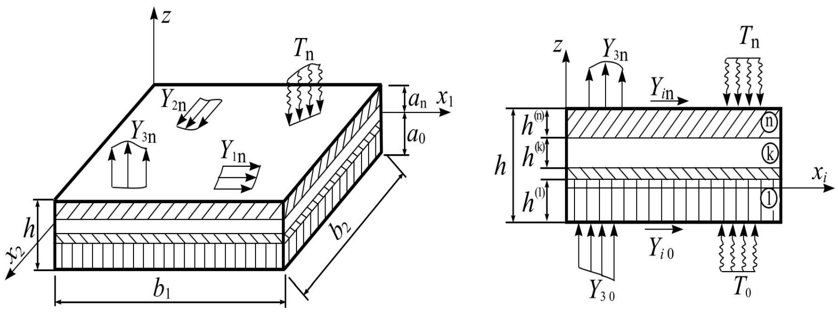
While building continual models of the deformation of layer plates with orthotropic layers, the specificiti of unflexural and flexural deformation from symmetric and oblique-symmetric components of bilateral loadings and temperature is considered. This made it possible to specify the known continuum models, which make it possible to find an improved solution when determining the stress-strain state of such plates. More accurate approximations for stresses and strains are introduced. A way of precise satisfaction of all defining correlations of the layers of material under keeping the conditions of their contact is found, while in the known continual models the dependence between the cross normal stress and cross deformation is only integral.

Keywords: refined continual model, multilayered plate, transverse shear, transverse compression.

Introduction. Analysis of exact three-dimensional solutions of particular problems of elastic deformation sandwich plates [1,2] shows the principal difference of stress-strained state (SSS) under bilateral symmetric and oblique-symmetric loadings of front surfaces that cause unflexural and flexural deformation. Therefore, while constructing approximate SSS models it is necessary to introduce the approximations of SSS along the transverse coordinate for symmetric components of loadings that would add to the approximations of flexural models [3, 4, 5 and others]. Under unflexural deformation taking into consideration of cross compression is necessary, because under symmetric cross loading the purely shear models [3] lead to simply-zero solution. Options for taking into account transverse shear and transverse compression in refined models for loadings that are symmetrical with respect to the middle surface of the plate are given in [5, 6, 7]. Below are refined continuum models of deformation of layered plates, in which the general order of differentiation of the solution equations does not depend on the number of layers and in the approximations of the transverse components of stresses and strains, the effect of symmetric and skew-symmetric loadings on both surfaces of the plate is separately taken into account.

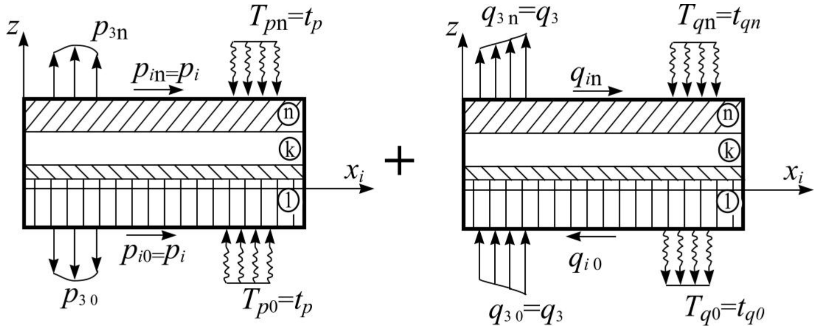
Materials and methods of research. SSS of rectangular plate having thickness h is being modelled in orthogonal system of co-ordinates x_α ($\alpha=\overline{1,3}$ $x_3=z$). Orthotropic axes coincide with axes x_α in rigidly connected orthotropic layers of arbitrary $h^{(k)}$ $(k=\overline{1,n})$ thickness. Axes z is orthogonal to facial surfaces of the plate $z=a_0, z=a_n$. Let's introduce the components of vectors of mechanic loading $Y_{\alpha 0}, Y_{\alpha n}$ in the directions of axes x_α and temperature T_0, T_n on the facial surfaces (Fig. 1) as a sum of symmetric $p_{\alpha 0}$, $p_{\alpha n}$, T_p and oblique-symmetric $q_{\alpha 0}$, $q_{\alpha n}$, T_{q0}, T_{qn} components (concerning the middle surface $z=a_m$) – Fig. 2.

To postulate "rational" hypotheses for stress distributions across the thicness of transverse shear $\sigma_{i3}^{(k)}$ $(i=\overline{1,2})$ and cross compression $\sigma_{33}^{(k)}$ while modelling is the most rational. In these hypotheses, the approximations for the transverse co-ordinate z are given in such way thet the static conditions of contact of layers and the conditions on the facial surfaces are satisfied.



(a) (b)

Fig. 1. Calculation diagram of a rectangular multilayer plate of arbitrary thickness:
(a) – scheme of power and thermal load; (b) – cross-section of the plate parallel to the plane zx_i



(a) (b)

Fig. 2. Schemes of reducing the load on the plate to the sum of two loads:
(a) – bilateral symmetrical load relative to the middle surface of the plate; (b) – bilateral oblique-symmetric load relative to the middle surface of the plate

Besides, in the thick-walled elements of sandwich structures the character of transverse deformation $e_{33}^{(k)}$ is siqnificantly influenced by not only the stress $\sigma_{33}^{(k)}$, but also by the deformation $e_{ij}^{(k)}$ in the plain of the "structure" because of Poisson's effect. This effect will be approximately accounted by means of the component $e_{v}^{(k)}$, taking $e_{33}^{(k)}$ in the form

$$e_{33}^{(k)} = \sigma_{33}^{(k)} / A_{3333}^{(k)} + e_{v}^{(k)} + (\alpha_{3}^{(k)} + \alpha_{i}^{(k)} A_{33ii}^{(k)} / A_{3333}^{(k)})T^{(k)} \quad (1)$$

and introducing an additional to $\sigma_{\alpha 3}^{(k)}$ hypotheses for $e_{v}^{(k)}$. Trying to meet Hooke's law precisely for $\sigma_{\alpha 3}^{(k)}$, $e_{33}^{(k)}$ and, at the same time, trying to meet all static conolition on the facial surfaces of layers for $\sigma_{33}^{(k)}$, such things as $(n+1)$ - differential dependences appear they don't let the continual model to be built.

Then it is possible to define approximately hypothetical components $u_{\alpha}^{(k)}$ of the displacement vector in the layer k from the equations [5]:

$$u_{3}^{(k)} = v_3 + \int_{\delta_3}^{z} e_{33}^{(s)} dz ; \quad s = \overline{l_\alpha , k}; \quad i,j = \overline{1,2}; \quad u_i^{(k)} = v_i - \int_{\delta_i}^{z} u_{3,i}^{(s)} dz + \int_{\delta_i}^{z} a_{i3j3}^{(s)} \sigma_{j3}^{(s)} dz, \quad (2)$$

where $v_i(x_j)$ - unknown functions of tangential, and $v_3(x_j)$ - normal displacements on voluntary surfaces $z=\delta_\alpha$, accordingly, in the layers l_α ; $a_{\alpha\beta\gamma\delta}^{(k)}$ and $A_{\alpha\beta\gamma\delta}^{(k)}$ are coefficients of matrixes of pliability and rigidness of Hooke's law in axes x_α; $T^{(k)}$ is temperature, and $\alpha_{\alpha}^{(k)}$ - coefficients of the temperature widening of the layer k in the directions x_α (hypothese (1) also takes into consideration the transverse temperature "expansion").

Here and further the particular derivatives are substituted by lower indexes after a comma. Summarizing of repeated lower indexes is introduced; at that $i,j=\overline{1,2}; \alpha=\overline{1,3}$. The sum of integrals in the quantity of z from the non-continuous function is marked as one integral of the given function [3-5]. The upper index in brackets is the number of the layer.

Let's introduce the hypotheses for the stresses $\sigma_{\alpha 3}^{(k)}$ and for the component $e_v^{(k)}$ in deformation $e_{33}^{(k)}$ (1) in the symmetric structure looking like:

a) for oblique-symmetric normal loading $q_3 = 0{,}5(Y_{30} + Y_{3n})$:

$$\sigma_{i3(q_3)}^{(k)} = f_{i1}^{(k)}(z^2)\beta_{i1} + f_{i2}^{(k)}(z^4)\beta_{i2}; \quad \sigma_{33(q_3)}^{(k)} = F_1^{(k)}(z^3)q_3 + F_2^{(k)}(z^5)\gamma_2; \tag{3}$$

$$e_{v(q_3)}^{(k)} = \mu_1^{(k)}(z^3)\gamma_5 + \mu_2^{(k)}(z^5)\gamma_2 + \mu_7^{(k)}(z^1)\gamma_7;$$

b) for symmetric normal loading $p_{3n} = -p_{30} = Y_{3n} - q_3$:

$$\sigma_{i3(p_3)}^{(k)} = f_{i3}^{(k)}(z^3)\beta_{i3}; \quad \sigma_{33(p_3)}^{(k)} = p_{3n} + F_3^{(k)}(z^4)\gamma_3; \tag{4}$$

$$e_{v(p_3)}^{(k)} = \mu_3^{(k)}(z^4)\gamma_3 + \mu_6^{(k)}(z^0)\gamma_6;$$

c) for symmetric tangential loading $p_i = 0{,}5(Y_{i0} + Y_{in})$:

$$\sigma_{i3(p_i)}^{(k)} = f_{i4}^{(k)}(z^1)p_i + f_{i3}^{(k)}(z^3)\beta_{i3}; \quad \sigma_{33(p_i)}^{(k)} = F_4^{(k)}(z^2)\gamma_4 + F_3^{(k)}(z^4)\gamma_3; \tag{5}$$

$$e_{v(p_i)}^{(k)} = \mu_3^{(k)}(z^4)\gamma_3 + \mu_4^{(k)}(z^2)\gamma_4 + \mu_6^{(k)}(z^0)\gamma_6;$$

d) for oblique-symmetric tangential loading $q_{in} = -q_{i0} = Y_{in} - p_i$:

$$\sigma_{i3(q_i)}^{(k)} = q_{in} + \sigma_{i3(q_3)}^{(k)}; \quad \sigma_{33(q_i)}^{(k)} = F_5^{(k)}(z^3)\gamma_5 + F_2^{(k)}(z^5)\gamma_2; \quad e_{v(q_i)}^{(k)} = e_{v(q_3)}^{(k)}. \tag{6}$$

In brackets under functions $f_{is}^{(k)}(z)$, $F_s^{(k)}(z)$ and $\mu_s^{(k)}(z)$ the maximum stage of approximating grade polinom at z is given. Hypotheses contain six unknown functions of the transverse shear $\beta_{i\alpha}(x_j)$ and six unknown functions of the cross compression $\gamma_s(x_j)$.

In unsymmetric structure the unflexural and flexural deformations for tangential loading are interconnected. That is why the hypotheses (5) and (6) should not be separated. Symmetric tangential loading causes an essential flexural deformation in a non-symmetric package of layers.

Hypotheses (3)-(6) and the contents of the functions $f_{is}^{(k)}(z)$, $F_s^{(k)}(z)$ got as specifying of the next simpliest starting hypotheses for $\sigma_{\alpha 3}^{(k)}$: from the affect of loading q_3 - $\sigma_{i3} = \beta_{i\alpha}(x_j)$; $\sigma_{33} = f_0 q_3$; from p_{3n}, p_{30} - $\sigma_{i3} = 0$; $\sigma_{33} = p_{3n}$; from q_{in}, q_{i0} - $\sigma_{i3} = q_{in}$; $\sigma_{33} = 0$; from the affect p_i - $\sigma_{i3} = f_0 p_i$; $\sigma_{33} = F_0\gamma_0(x_j)$, where $f_0 = 2(z - a_0)/h - 1$; $F_0 = (z - a_0)[(z - a_0)/h - 1]$.

In the procedure of the specifying the stresses $\sigma_{\alpha 3}^{(k)}$ were filled into (1), (2) by $e_v^{(k)} = 0$ and got $u_\alpha^{(k)}$ - into Cauchy's correlations and the law of Hooke. Further the specified expressions $\sigma_{i3}^{(k)}$ were defined from the equations of steadiness $\sigma_{i\alpha}^{(k)},_\alpha = 0$ by means of integrating (mind, as for two beams with elastic properties of the matter in the direction of axes x_i), and with $\sigma_{i3}^{(k)}$ were integrated with the new unknown functions $\beta_{i\alpha}, \gamma_s$ (like in [4-7]) - derivatives from loading and from functions v_α, and also all static conditions were satisfied on the in-plane surfaces of the layers and of the plate. Hypotheses for $e_v^{(k)}$ are obtained from the correlation $e_v^{(k)} \approx e_{ii}^{(k)} A_{ii33}^{(k)} / A_{3333}^{(k)}$ where deformations $e_{ii}^{(k)}$ were defined from (2) with the consideratoin of hypotheses of the shear (3)-(6) for $\sigma_{i3}^{(k)}$ under $e_{33}^{(k)} = 0$ where the transition from $\beta_{i\alpha}$ to the functions γ_s was used, by the analogy to that used while putting down the hypotheses for $\sigma_{33}^{(k)}$. To decrease the quantity of approximating functions the hypothesis about the similarity of different degree functions $f_{is}^{(k)}(z)$, $F_s^{(k)}(z)$, $\mu_s^{(k)}(z)$ with the same highest degree of z is introduced, if during the transition to the homogeneous plate these functions are becoming linearly dependent.

As a result, the functions $f_{is}^{(k)}(z)$, $F_s^{(k)}(z)$, $\mu_s^{(k)}(z)$ in expressions (3)-(6) look like:

$$f_{i1}^{(k)} = \int_{a_0}^{z} A_{iiii}^{(r)}(z-\zeta_i)dz; \quad k,l=\overline{1,n}; \quad r=\overline{1,k}; \quad s=\overline{1,r}; \quad p=\overline{1,l}; \quad (7)$$

$$f_{i2}^{(k)} = \int_{a_0}^{z} A_{iiii}^{(r)} \int_{a_m}^{z} f_{i1}^{(s)} a_{i3i3}^{(s)} dz^2 - \varphi_i^{(k)} \int_{a_0}^{a_n} A_{iiii}^{(l)} \int_{a_m}^{z} f_{i1}^{(p)} a_{i3i3}^{(p)} dz^2;$$

$$f_{i3}^{(k)} = \theta_i^{(k)} - \theta_i^{(n)}(a_n)\varphi_i^{(k)}; \qquad f_{i4}^{(k)} = 2\varphi_i^{(k)} - 1;$$

$$F_1^{(k)} = 2\eta_1^{(k)} / \eta_1^{(n)}(a_n) - 1; \qquad F_2^{(k)} = 0{,}5(F_1^{(k)}+1)\eta_2^{(n)}(a_n) - \eta_2^{(k)};$$

$$F_3^{(k)} = \eta_3^{(k)}; \qquad F_4^{(k)} = \int_{a_0}^{z} \varphi_i^{(r)} dz - \int_{a_0}^{a_n} \varphi_i^{(l)} dz (z-a_0)/h;$$

$$F_5^{(k)} = \eta_1^{(k)} - \eta_1^{(n)}(a_n)(z-a_0)/h; \quad \mu_\varepsilon^{(k)} = A_{33jj}^{(k)} / A_{3333}^{(k)} \mu_{\varepsilon jj}^{(k)}; \quad \mu_{tjj}^{(k)} = \int_{\zeta_j}^{z} a_{j3j3}^{(r)} f_{jt}^{(r)} dz; \quad t=\overline{1,4};$$

$$\mu_{7jj}^{(k)} = \int_{\zeta_j}^{z} dz; \qquad \mu_{6jj}^{(k)} = 1,$$

where $f^{(n)}(a_n)$ - function at $z=a_n$, δ_{ii} - Kroneker's symbol and

$$\zeta_i = \int_{a_0}^{a_n} A_{iiii}^{(l)} z dz \Big/ \int_{a_0}^{a_n} A_{iiii}^{(l)} dz; \quad \varphi_i^{(k)} = \int_{a_0}^{z} A_{iiii}^{(r)} dz \Big/ \int_{a_0}^{a_n} A_{iiii}^{(l)} dz;$$

$$\zeta_0 = [\delta_{ii} \int_{a_0}^{a_n} \varphi_{3i}^{(l)} dz - \delta_{ii} \varphi_{3i}^{(n)}(a_n) \int_{a_0}^{a_n} \varphi_i^{(l)} dz] / [\delta_{ii} \varphi_{4i}^{(n)}(a_n) \int_{a_0}^{a_n} \varphi_i^{(l)} dz - \delta_{ii} \int_{a_0}^{a_n} \varphi_{4i}^{(l)} dz];$$

$$\eta_\alpha^{(k)}(z) = \int_{a_0}^{z} (f_{1\alpha}^{(r)} + f_{2\alpha}^{(r)}) dz; \quad \theta_i^{(k)} = \varphi_{3i}^{(k)} + \zeta_0 \varphi_{4i}^{(k)};$$

$$\varphi_{3i}^{(k)}(z) = \int_{a_0}^{z} A_{iiii}^{(r)} \varphi_0^{(r)} dz; \quad \varphi_{4i}^{(k)}(z) = \int_{a_0}^{z} A_{iiii}^{(r)} \int_{a_m}^{z} dz^2; \quad \varphi_0^{(k)}(z) = \int_{a_m}^{z} \int_{a_0}^{z} a_{3333}^{(r)} dz^2.$$

On the facial surfaces $z=a_0$ and $z=a_n$ functions $f_{is}^{(k)}(z)$, $F_s^{(k)}(z)$ $(s=\overline{2,5})$ have zero meaning and the remaining ones are equal to 1 and -1. Functions $f_{is}^{(k)}(z)$, $F_s^{(k)}(z)$ were used in [3-5 and others], while $f_{i2}^{(k)}(z)$ are introduced in [3] and the remaining - are proposed here. Let's mark that introduced approximations $f_{i3}^{(k)}(z)$, $F_3^{(k)}(z)$ coincide in quality with the given in [1] exact solutions for $\sigma_{i3}^{(k)}$.

Under the arbitrary directed vector of loading $Y_{\alpha 0}$, $Y_{\alpha n}$ the hypotheses will look like:

$$\sigma_{i3}^{(k)} = f_{i\alpha}^{(k)} \beta_{i\alpha} + (\varphi_i^{(k)} - 1)Y_{i0} + \varphi_i^{(k)} Y_{in}; \quad \alpha=\overline{1,3}; \quad i=\overline{1,2}; \qquad (8)$$

$$\sigma_{33}^{(k)} = F_p^{(k)} \gamma_p + 0{,}5(F_1^{(k)}-1)Y_{30} + 0{,}5(F_1^{(k)}+1)Y_{3n}; \; p=\overline{2,5};$$

$$e_v^{(k)} = \mu_\varepsilon^{(k)} \gamma_\varepsilon; \quad \varepsilon=\overline{2,7}.$$

In the linear elastic problem instead of (8) it is possible to consider 4 simplier problems (3)-(6) under symmetric structure and 3 problems - under any structure of the package of layers.

In case of stationary facial thermal affect with the known temperature T_0, T_n on the facial surfaces $z=a_0$, $z=a_n$ the distribution of the temperature $T^{(k)}$ along the thickness of the non-thick plate can be introduced as a piece-linear law [8] with approximating function $\psi_\lambda^{(k)}$. Using (3),(4) and dividing the temperature into symmetric $T_p(z) = (T_0+T_n)/2 = t_p$ and oblique-symmetric $T_q(z) = (2\psi_\lambda^{(k)} - 1)t_{qn}$, $t_{qn} = (T_n - T_0)/2$ components, let's take hypotheses in the form:

$$\sigma_{i3}^{(k)} = S_{i1}^{(k)}(z^1) t_{p},_i + S_{i2}^{(k)}(z^2) t_{qn},_i + f_{i\alpha}^{(k)} \beta_{i\alpha}; \quad \sigma_{33}^{(k)} = F_p^{(k)} \gamma_p; \; p=\overline{2,5}; \qquad (9)$$

where the division into unflexural ($\beta_{ij} = \gamma_2 = \gamma_5 = \gamma_7 = 0$ under $t_{qn}=0$) and flexural ($\beta_{i3} = \gamma_3 = \gamma_4 = \gamma_6 = 0$ under $t_p = 0$) deformation is possible only in the case of symmetry of the structure of the plate in its thickness.

Functions $S_{ij}^{(k)}$, $\psi_{\lambda}^{(k)}$ look like:

$$S_{i1}^{(k)} = \int_{a_0}^{z} A_{iijj}^{(r)}\alpha_j^{(r)}dz - \overline{\alpha}_j \int_{a_0}^{z} A_{iijj}^{(r)}dz; \quad \psi_{\lambda}^{(k)} = \int_{a_0}^{z} (\lambda_3^{(r)})^{-1}dz / \int_{a_0}^{a_n} (\lambda_3^{(l)})^{-1}dz;$$

$$S_{i2}^{(k)} = \int_{a_0}^{z} A_{iijj}^{(r)}\alpha_j^{(r)}(2\psi_{\lambda}^{(r)} - 1)dz - (2\overline{\alpha_j}/h)\int_{a_0}^{z} A_{iijj}^{(r)}(z - \zeta_j^t)dz,$$

where $\overline{\alpha_j}$ are average in the thickness of the plate coefficients of the temperature widening in the directions x_j, and $\lambda_3^{(k)}$ is the coefficient of thermal conductivites in the direction of axes z.

The values of $\overline{\alpha_j}$ are defined from two equations $S_{i1}^{(n)}(a_n) = 0$ and then ζ_j^t from $S_{i2}^{(n)}(a_n) = 0$. They insure self-balance in the thickness of transverse shear stresses in the "free" plate with the piece-linear dependence of $T(z)$ from z.

In the thick plates, the distribution of the temperature along the thickness is recommended to depict in the form of non-linear function [9] (namely, $T_p(z)$ - in the form quadratic and $T_q(z)$ - cubical functions). Under it hypotheses (9) will not change, because the influence of non-linear (according to z) component of the temperature will be taken into consideration in $\sigma_{i3}^{(k)}$ by the functions $f_{i\alpha}^{(k)}(z)\beta_{i\alpha}$.

Let's introduce the kinematic model (2) corresponding to hypotheses (3)-(6) or (8), (9) in a summarized look (by analogy with [4, 5]):

$$u_3^{(k)} = v_3 + \psi_p^{(k)},_3(z)\gamma_p + U_3; \;\; p = \overline{2,5}; \;\; u_i^{(k)} = v_i - \psi^{(k)}(z)v_{3},_i - \psi_p^{(k)}(z)\gamma_{p},_i - \psi_{i\alpha}^{(k)}(z)\beta_{i\alpha} + U_i, \quad (10)$$

where U_α are addings, containing components of facial loadings, temperatures and their derivatives.

The expressions for the SSS components may be obtained by filling (10) into the Cauchy's correlation and then into the law of Hooke. This leads to the unfulfilling in $\sigma_{33}^{(k)}$ of static conditions on the facial surfaces of layers, which in the problems of the curve has a little influence on the accuracy of computation. However, the definition $\sigma_{33}^{(k)}$ from the law of Hooke and not from the hypothes, allows to get a symmetric matrix of coefficients in the solutable system of differential equations of the linear elastic problem:

$$L_{\varepsilon\alpha}(v_\alpha) + L_{\varepsilon p}(\gamma_p) + L_{\varepsilon i\alpha}(\beta_{i\alpha}) = Z_\varepsilon; \qquad \varepsilon = \overline{1,15}. \quad (11)$$

Here $L_{\varepsilon s}(...)$ are differential operators of a not higher that the fourth level, and Z_ε are expressions for loading and temperature. System (11) with the correspondent boundary conditions is gotten according to the methodics [3-7] from the variational equation of Lagrange. The general order of differentiation (11) does not depend on the quantity of layers (continual model), and the part of each of the functions v_i, $\beta_{i\alpha}$ in it makes 2, and functions v_3, γ_s - 4. That is why for each of the hypotheses separately (1), (3)-(6) the general order of differentiation (11) will be given below.

Neglecting in hypotheses (1), (3)-(6) and in equations (11) the component $e_v^{(k)}$, that is, the influence of the Poisson effect on transverse deformations $e_{33}^{(k)}$, it is possible to significantly simplify the model and reduce the general order of differentiation of the system of resolving equations (11).

Let's indicate the possibilily of executing of all conditions of the contact of layers, of the conditions on the facial surfaces and of the correlations of Hooke's laws in any continious kinematik model $u_\alpha^{(k)}$ of the type (2), (10). For this, we ohould add to $u_3^{(k)}$ in (10) $(2n+1)$ - unknown functions $\chi_i^{(k)}(x_j)$, $\chi_3^{(k)}(x_j)$ of co-ordinate surface

$$u_3^{(k)*} = u_3^{(k)}(x_\alpha) + \varphi_i^{(k)}(z)\chi_i^{(k)}(x_j) + \varphi_3^{(k)}(z)\chi_3(x_j). \quad (12)$$

For functions that are qiven in each layer with the facial surfaces $z = a_k$, $z = a_{k-1}$, the requirements of their linear dependence are put on, as well as

$$\int_{a_{k-1}}^{a_k} \varphi_\alpha^{(k)} dz = 0; \quad k = \overline{1,n}.$$

This turns the differential dependences between $\chi_i^{(k)}(x_j)$, $\chi_3^{(k)}(x_j)$, followed from the kinematic and static conditions on the surfaces of layers for $u_3^{(k)*}$ and $\sigma_{33}^{(k)*}$, into the algebraic ones. It is the most convenient to choose $\varphi_\alpha^{(k)}$ in the form of power functions of z, for example:

$\varphi_\alpha^{(k)} = z^\alpha - b_\alpha^{(k)}; \ \alpha = 1,2,3; \ b_1^{(k)} = (a_k + a_{k-1})/2;$

$b_2^{(k)} = (a_k^2 + a_k a_{k-1} + a_{k-1}^2)/3;$

$b_3^{(k)} = (a_k^3 + a_k a_{k-1}^2 + a_k^2 a_{k-1} + a_{k-1}^3)/4.$

However, in this case the expression (12) for $u_3^{(k)*}$ will also contain the first derivatives of the functions v_i, shear functions $\beta_{i\alpha}$ and the second derivatives of the functions v_3, γ_p:

$$u_3^{(k)*} = v_3 + \psi_{0ii}^{(k)*},_3(z) v_3,_{ii} + \psi_p^{(k)*},_3(z)\gamma_p + \psi_{pii}^{(k)*},_3(z)\gamma_p,_{ii} + \eta_{ji}^{(k)*},_3 v_j,_i + \eta_{j\alpha ii}^{(k)*},_3 \beta_{j\alpha},_{ii} + f^*(U_\alpha,_\alpha),$$

and displacements $u_i^{(k)*}$ obtained from (2) - are their second and third derivatives, accordinqly, which as a result will essentially increase the general order of differentiation of the resultant equation system. That's why this specification of the model has more of a theoretical interest.

Numerical results and their analysis. Numerical solutions, obtained according to the proposed model (3)-(11) for maximum meanings of stresses $\sigma_{i\alpha}^{\#} = 10\sigma_{i\alpha}^{(k)}/g_0$ and of displacement $u_3^{\#} = 10 u_3^{(k)} G_{12}^{(k)}/(g_0 h)$ in square $b_1 \times b_2 \times h$ sandwich plates (meanings are in brackets in table 1 and marked CS_2 in table 2) under boundary Navier-type conditoins practically coincide with the exact three-dimensional solutions obtained according to methodics [1]. According to the results of the calculations of a two-layer plate ($b_i/h = 3$), affected by the symmetric and oblique-symmetric loadings q_3, p_3 (where $p_3 = p_{30} + p_{3n}$), $\delta_{ii} q_i$ (where $q_i = q_{i0} + q_{in}$), $\delta_{ii} p_i$, the quality picture of SSS, confirming suggested approximations taken in co-ordinate z in hypotheses is seen. Components of loadings: $[q_3; p_{3n}] = g_0 \sin(\pi x_1/b_1)\sin(\pi x_2/b_2)$; $[q_{in}; p_i] = g_0 \cos(\pi x_i/b_i)\sin(\pi x_j/b_j)$; $i \neq j$. Characteristics of transversal-isotropic layers: $h^{(1)} = 2h/3$; $h^{(2)} = h/3$; $G_{12}^{(k)} = 10^4 MPa$; $E_i^{(k)} = 2{,}6 G_{12}^{(k)}$; $E_3^{(1)} = 0{,}1 E_i^{(k)}$; $E_3^{(2)} = 0{,}5 E_i^{(k)}$; $G_{i3}^{(1)} = 0{,}5 G_{12}^{(k)}$; $G_{i3}^{(2)} = 0{,}2 G_{12}^{(k)}$; $\nu_{12}^{(k)} = \nu_{21}^{(k)} = \nu_{3i}^{(k)} = 0{,}3$; $\nu_{i3}^{(1)} = 0{,}03$; $\nu_{i3}^{(2)} = 0{,}15$; $i,k = 1,2$.

The influence on the general SSS in symmetric component p_3 of the loading rises with the decrese of the relative transverse normal rigidness of the layer ($E_3^{(k)}/E_i^{(k)} \le 0{,}1$), and the influence of symmetric loading p_i - with the decrese of the relative transverse shear rigidness of the layer ($G_{i3}^{(k)}/E_i^{(k)} \le 0{,}1$ by $b_i/h \le 3$ and $G_{i3}^{(k)}/E_i^{(k)} \le 0{,}02$ by $b_i/h \le 5$) rises.

Functions $\beta_{i2}(x_j)$ and $\gamma_2(x_j)$ in many problems can be neglected. However, in the places of the localization of loading [5] and also in the relationship between coefficients of pliability of the material $a_{\alpha\beta\gamma\delta}^{(k)}$ in the different layers of more then $10^2 \div 10^3$ [3], or under highly weak relative transverse shear rigidness of the layer ($G_{i3}^{(k)}/E_i^{(k)} \le 0{,}02$ by $b_i/h \le 3$ and $G_{i3}^{(k)}/E_i^{(k)} \le 0{,}005$ by $b_i/h \le 5$), they specify SSS essentially.

Analysis of maximum stresses $\sigma_{11}^{\#} = 10\sigma_{11}^{(k)}/g_0$ in three-layer square plate ($b_i/h = 2$), given in table 2 demonstrates it, where the solutions are compared to the exact solution ($3D$ [1]) of the solution of the given (SC_2) and simplified purely-shear flexural models S_1, S_2 (in S_1 - only $\beta_{i1} \neq 0$, and in S_2 - only

$\beta_{ij} \neq 0$), and also according to the model considering cross compression SC_1 (only $\beta_{i1} \neq 0$; $\beta_{i3} \neq 0$; $\gamma_6 \neq 0$; $\gamma_7 \neq 0$).

Table 1

Maximum stresses $\sigma_{i\alpha}^{\#}$ and displacement $u_3^{\#}$ in two-layer plate ($b_i / h = 3$)

z/h	Y_α	$u_3^{\#}$	$\sigma_{13}^{\#}$, $\sigma_{23}^{\#}$	$\sigma_{33}^{\#}$	$\sigma_{11}^{\#}$, $\sigma_{22}^{\#}$	$\sigma_{12}^{\#}$
1		26,02 (25,91)	-10,00 (10,00)	0 (0,02)	46,71 (46,80)	-25,15 (-25,20)
5/6		26,61 (26,53)	-0,90 (-0,88)	1,72 (1,69)	23,10 (23,14)	-12,04 (-12,06)
2/3	q_i	26,15 (26,08)	2,94 (2,91)	1,23 (1,20)	6,75 (6,80)	-3,35 (-3,35)
1/3		26,13 (26,06)	2,78 (2,79)	-1,17 (-1,19)	-7,47 (-7,43)	3,75 (3,76)
0		24,61 (24,56)	-10,00 (-10,00)	0 (0,01)	-52,70 (-52,96)	28,38 (28,44)
1		67,25 (66,94)	0 (0,00)	10,00 (9,96)	61,18 (60,86)	-30,64 (-29,86)
5/6		62,93 (62,65)	10,74 (10,81)	7,84 (7,83)	24,09 (23,97)	-11,16 (-10,92)
2/3	q_3	59,65 (59,30)	13,22 (13,19)	3,45 (3,45)	-3,00 (-2,89)	2,41 (2,35)
1/3		59,55 (59,20)	10,93 (10,87)	-5,09 (-5,07)	-8,06 (-7,96)	3,17 (3,06)
0		59,98 (59,61)	0 (0,00)	-10,00 (-10,02)	-45,86 (-45,62)	22,39 (22,01)
1		4,61 (4,52)	-10,00 (-10,00)	0 (0,03)	26,56 (26,70)	-14,30 (-13,99)
5/6		4,51 (4,46)	-4,58 (-4,50)	2,45 (2,45)	15,29 (15,30)	-7,67 (-7,70)
2/3	p_i	3,01 (2,89)	-1,49 (-1,42)	3,47 (3,40)	9,37 (9,34)	-4,24 (-4,41)
1/3		2,60 (2,60)	1,72 (1,77)	3,31 (3,38)	6,88 (6,80)	-2,94 (-2,98)
0		3,27 (3,24)	10,00 (10,00)	0 (0,00)	32,56 (32,60)	-17,53 (-17,56)
1		-12,30 (-12,22)	0 (0.000)	-10,00 (-9,98)	-6,96 (-6,84)	1,44 (1,44)
5/6		-6,06 (-6,04)	-0,587 (-0,596)	-9,85 (-9,88)	-1,20 (-1,14)	-1,62 (-1,63)
2/3	p_3	0,19 (0,10)	-0,232 (-0,244)	-9,70 (-9,76)	0,28 (0,22)	-2,39 (-2,41)
1/3		2,65 (2,62)	0,358 (0,369)	-9,78 (-9,80)	-0,95 (-1,02)	-1,75 (-1,80)
0		5,03 (4,99)	0 (0,000)	-10,00 (-9,98)	-4,22 (-4,36)	-0,04 (-0,12)

Table 2

Maximum stress $\sigma_{11}^{\#}$ in three-layer plate ($b_i / h = 2$)

	S_1	S_2 [2]	SC_1	SC_2	$3D$ [1]	S_1	S_2	SC_1	SC_2	$3D$ [1]
z/h	q_3					p_3				
1,0	73,13	361,4	392,7	308,9	306,1	0	0	-12,61	-13,40	-13,66
0,95	-123,3	-171,2	-159,4	-147,1	-149,2	0	0	20,94	16,45	16,21
0,95	-0,965	-0,009	3,783	2,906	2,944	0	0	-2,914	-2,700	-2,630
0,65	1,701	-2,721	-8,565	0,350	0,302	0	0	-1,817	-1.207	-1,140
0,3	8,407	4,742	5,481	3,821	3,795	0	0	-2,847	-2,752	-2,734
0,3	470,2	265.5	245,2	230,5	228,2	0	0	13,76	13,12	13,08
0	-569,1	-495,4	-506,9	-373,8	-368,9	0	0	-17,39	-12,03	-11,76
z/h	q_1					p_1				
1,0	229,7	288,8	290,1	250,2	247,1	-449,4	-275,3	-251,2	-238,7	-234,4
0,95	76,74	67,93	71,09	68,11	67,62	-34,30	-57,94	-49,56	-77,12	-73,41
0,95	1,087	1,376	1,362	1,250	1,243	-2,183	-1,085	-0,954	-1,289	-1,234
0,65	-2,622	-3,646	-4,508	0,101	0,083	5,500	2,215	-1,426	-0,206	-0,121
0,3	-0,443	-1,063	-0,705	0,160	0,142	0,480	-1,049	-1,096	-0,120	-0,085
0,3	-22,01	-65,15	-65,23	11,06	9,081	66,52	-63,30	-70,38	-3,141	1,348
0,15	83,78	82,03	81,04	-7,033	-3,452	155,59	68,61	76,09	-8,222	-3,301
0	-393,0	-376,3	-380,3	-239,0	-238,1	-394,8	-342,0	356,7	-231,6	-225,5

Characteristics of the external orthotropic layers ($k = 1,3$): $h^{(1)} = 0,3h$; $h^{(3)} = 0,05h$; $E_2^{(k)} = E_3^{(k)} = 10^6 MPa$; $E_1^{(k)} = 25E_2^{(k)}$; $G_{12}^{(k)} = 0,05E_2^{(k)} = G_{13}^{(k)}$; $G_{23}^{(k)} = 0,02E_2^{(k)}$; $\nu_{21}^{(k)} = \nu_{31}^{(k)} = \nu_{23}^{(k)} = \nu_{32}^{(k)} = 0,25$ and of the internal layer ($k = 2$): $E_\alpha^{(2)} = 4\times10^5$ MPa; $G_{12}^{(2)} = 1,6\times10^5$ MPa; $G_{i3}^{(2)} = 1,6\times10^4$ MPa; $\nu_{i\alpha}^{(2)} = 0,25$. Increasing in 10 times in the given problem in all the layers of all the modes of the shear $G_{i\alpha}^{(k)}$ considerably approaches to $3D$ [1] the solutions according S_1 and S_2 and the solutions according SC_1 becomes practically identical to the exact.

Conclusions. As can be seen above from the results of the test problems calculations, the constructed mathematical model allows us to obtain results that are qualitatively and quantitatively close to three-dimensional solutions. The model can be used to calculate the SSS of significantly thick plates ($a / h = 2$), with a wide range of changes to the parameters of the relative transtropy in the layer ($1 \le E_i / E_3 \le 500$, $1 \le G_{12} / G_{i3} \le 500$; $i = 1,2$) and significant differences in the stiffness of the individual layers ($E^{(k)} / E^{(k+1)} = 10^3 \div 10^5$).

REFERENCES

1. *Piskunov V.G., Sipetov V.S., Tuymetov Sh.Sh.* Resheniye zadach statiki dlya sloistykh ortotropnykh plit v prostranstvennoy postanovke [Solution of statics problems for layered orthotropic plates in a spatial setting (in Russian)] // Applied Mechanics. - 1990. - Vol. 26, N. 2. - P.41–49.
2. *Prusakov A.P.* K teorii izgiba sloistykh plastin [On the theory of bending of laminated plates (in Russian)]// Applied Mechanics. - 1997. - Vol.33, N. 3. – P. 64-70.
3. *Rasskazov A.O., Burygina A.V.* K utochneniiu teorii sdviga sloistykh pologikh obolochek [To the specification of shift theory of orthotropic hoping shells (in Russian)] // Applied Mechanics. – 1988. - Vol.24, N. 4. - P. 32-37.
4. *Piskunov V.G. Rasskazov A.A.* Teoriya sdviga vtorogo priblizheniya dlya mnogosloynykh pologikh obolochek i plastin [Shear theory of the second approximation for multilayer shallow shells and plates (in Russian)] // Mechanics of Composite Materials. – 1998. – Vol. 34, N.3. – P. 363–370.
5. *Gurtovyi O.G., Piskunov V.G.* Novyye raschetnyie modeli i sravneniye priblizhennykh utochnennykh s tochnymi trekhmernymi resheniyami zadach izgiba mnogosloynykh anizotropnykh plastin [New account patterns and comparison of approximate specified with exact three-dimensional solutions problems of the flexure laminated anisotropic plates (in Russian)] // Mechanics of Composite Materials. – 1988. – N. 1, P. 93-101.
6. *Gurtovyi O.G.* Vysokotochnoye modelirovaniye deformirovaniya sloistykh struktur [High-precision modeling of deformation of layered structures (in Russian)] // Mechanics of composite materials. – 1999. - V. 35, N. 1. - P. 13–28.

7. *Gurtovyi O.G., Tynchuk S.O.* Bezyzgibnaya utochnennaya model deformirovaniya mnogosloynykh plit na nedeformiruyemom osnovanii [An unflexural refined model of deformation of multilayer plates on an undeformable foundation (in Russian)] // Mechanics of composite materials. - 2006. - V. 42, № 5. - P. 643–654.
8. *Piskunov V.G., Sipetov V.S.* Raschet sloistykh pologikh obolochek iz anizotropnykh kompozitov na staticheskoye i temperaturnoye vozdeystviye [Calculation of laminated hoping shells of anisotropic composits for static and themperature action (in Russian)] // Problemy Prochnosti. – 1987. – N. 10, P. 79-82.
9. *Piskunov V.G., Sipetov V.S.* Utochnennaya model' raspredeleniya temperaturnogo rezhima dlya resheniya zadach termouprugosti mnogosloynykh sistem [A refined model of the temperature pattern distribution for solving the problems on thermoelasticity of the multilayer systems (in Russian) // Doklady Akademii Nauk Ukrainskoy SSR. – 1987. – N.5, P.49-52.

Стаття надійшла 06.12.2023

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ВИСОКОТОЧНЕ МОДЕЛЮВАННЯ ДЕФОРМАЦІЙ СЕНДВІЧ-КОНСТРУКЦІЙ ПРИ ДВОСТОРОННЬОМУ СИМЕТРИЧНОМУ ТА КОСИСИМЕТРИЧНОМУ НАВАНТАЖЕННІ

Аналіз та оцінка напружено-деформованого стану (НДС) багатошарових плит з ортотропними шарами при дії стаціонарного поперечного та дотичного навантаження є актуальною задачею. Він включає в себе розрахунки на міцність і деформативність різних однорідних і багатошарових пластин з шарами постійної товшини але довільної будови за товщиною пластини. Об'єднання матеріалів з ізотропними та трансверсально-ізотропними фізичними характеристиками в багатошаровий пакет дозволяє створювати багатофункціональні конструкції. НДС таких конструкцій, зважаючи на їх структурну неоднорідність та відносно низьку поперечну жорсткість окремих шарів, суттєво пов'язаний з впливом деформацій поперечного зсуву та деформацій поперечного обтиснення. Тому актуальною є задача уточненого моделювання НДС плит, яка б враховувала ці види деформацій. Ґрунтуючись на розкладанні НДС плити на згинові та беззгинові складові, пропонується оптимізація розрахункової схеми деформування прямокутної багатошарової плити. Це суттєво спрощує його моделювання. Для беззгинового та виключно згинового НДС побудовані в пружній постановці двовимірні, високого ступеня ітераційного наближення, але тривимірні за характером відображення НДС моделі деформування багатошарових прямокутних плит з ізотропними, трансверсально-ізотропними та ортотропними шарами, які достатньо повно враховують деформації поперечного зсуву та поперечного обтиснення при поперечному та дотичному навантаженні пластини. Модель – континуальна, тобто кількість рівнянь та порядок диференціювання розрахункової системи рівнянь не залежить від кількості шарів в плиті. Цей порядок диференціювання і кількість розрахункових рівнянь може залежати лише від порядку ітераційного наближення моделі. Запропоновано також спосіб точного виконання всіх визначальних співвідношень для шарів матеріалу при дотриманні умов їх контакту, у той час як у відомих континуальних моделях залежність між поперечними нормальними напругами та поперечними деформаціями є тільки інтегральною. Приведено результати аналітичного розв'язку задачі деформування прямокутної пластини при граничних умовах типу Нав'є під дією поперечного та дотичного навантаження. Розв'язанням тестових задач деформування двошарової з трансверсально-ізотропними шарами та тришарової з ортотропними шарами пластин та порівнянням розв'язків з отриманими за відомими методиками точними тривимірними розв'язками цих задач, дано оцінку точності запропонованих уточнених моделей. Встановлено межі допустимих параметрів пружних характеристик трансверсально-ізотропних і ортотропних плит для застосування запропонованих моделей.

Ключові слова: уточнена континуальна модель, плита багатошарова, поперечний зсув, поперечне обтиснення.

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HIGH-PRECISION MODELLING OF DEFORMATION OF SANDWICH STRUCTURES UNDER BILATERAL SYMMETRIC AND OBLIQUE-SYMMETRIC LOADING

The analysis and assessment of the stress-strain state (STS) of multilayer plates with orthotropic layers under the action of stationary transverse and tangential loads is an urgent task. It includes calculations on the strength and deformability of various homogeneous and multi-layered plates with layers of constant thickness but arbitrary structure according to the thickness of the plate. Combining materials with isotropic and transversely isotropic physical characteristics into a multilayer package allows you to create multifunctional structures. The SSS of such structures, due to their structural heterogeneity and relatively low transverse stiffness of the individual layers, is significantly associated with the influence of transverse shear deformations and transverse compression deformations. Therefore, the problem of refined modeling of SSS plates, which would take into account these types of deformations, is urgent. Based on the decomposition of the SSS plate into flexural and unflexural components, it is proposed to optimize the design scheme of deformation of a rectangular multilayer plate. This significantly simplifies its modeling. For unflexural and exclusively flexural SSS, a two-dimensional, high-degree iterative approximation, but three-dimensional models of deformation of multilayer rectangular plate on a rigid foundation with isotropic, transverse-isotropic and orthotropic layers are constructed in an elastic formulation. That models takes full account deformation of transverse shear and transverse compression at transverse and the tangential loading of a plate. The model is continuous, that is, the number of equations and the order of differentiation of the calculation system of equations does not depend on the number of layers in the slab. This order of differentiation and the number of calculation equations can depend only on the order of iterative approximation of the model. A way of precise satisfaction of all defining correlations of the layers of material under keeping the conditions of their contact is found, while in the known continual models the dependence between the cross normal stress and cross deformation is only integral.The results of the analytical solution of the problem of deformation of a rectangular plate under boundary conditions of the Navier type under the action of transverse and tangential loads are given. By solving the test problems of the deformation of two-layer plates with transversally isotropic layers and three-layer plates with orthotropic layers and comparing the solutions with the exact three-dimensional solutions of these problems obtained by known methods, an assessment of the accuracy of the proposed refined models is given. The limits of admissible parameters of elastic characteristics of transversely isotropic and orthotropic plates for application of the offered models are established.

Keywords: refined continual model, multilayered plate, transverse shear, transverse compression.

УДК 539.3
Гуртовий О.Г., Пискунов С.О. **Високоточне моделювання деформацій сендвіч-конструкцій при двосторонньому симетричному та косисиметричному навантаженні** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2024. – Вип. 112. – С. 258-267. – Англ.
При побудові континуальних моделей деформування багатошарових пластин з ортотропними шарами враховується специфіка беззгинальної та чисто-згинальної деформації від симетричної та кососиметричної складових двосторонніх навантажень та температури. Це дозволило уточнити відомі континуальні моделі, що дозволяють знаходити уточнене рішення щодо напружено-деформованого стану таких пластин. Введені точніші апроксимації для напруг та деформацій. Запропоновано також спосіб точного виконання всіх визначальних співвідношень для шарів матеріалу при дотриманні умов їх контакту, у той час як у відомих континуальних моделях залежність між поперечними нормальними напругами та поперечними деформаціями є тільки інтегральною.
Табл. 2. Бібліогр. 9 назв.

UDK 539.3
Gurtovyi O.G., Pyskunov S.O. **High-precision modelling of deformation of sandwich structures under bilateral symmetric and oblique-symmetric loading** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles – Kyiv: KNUBA, 2024. – Issue 112. – P. 258-267.
While building continual models of the deformation of layer plates with orthotropic layers, the specificiti of unflexural and flexural deformation from symmetric and oblique-symmetric components of bilateral loadings and temperature is considered. This made it possible to specify the known continuum models, which make it possible to find an improved solution when determining the stress-strain state of such plates. More accurate approximations for stresses and strains are introduced. A way of precise satisfaction of all defining correlations of the layers of material under keeping the conditions of their contact is found, while in the known continual models the dependence between the cross normal stress and cross deformation is only integral.
Tabl. 2. Ref. 9.

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