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## PARAMETRIC OPTIMIZATION OF BAR STRUCTURES WITH DISCRETE AND CONTINUOUS DESIGN VARIABLES USING IMPROVED GRADIENT PROJECTION METHOD

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The paper considers a parametric optimization problem for the bar structures formulated as nonlinear programming task, where the purpose function and non-linear constraints of the mathematical model are continuously differentiable functions. The method of the objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations has been used to solve the parametric optimization problem. A discretization technique for the design variables that should vary discretely has been proposed. The discretization of the optimal design solution obtained in the continuous space of the design variables is performed by the purposefully selecting discrete points around the point of the continuous optimum. The comparison of the optimization results presented by the paper demonstrates that improved gradient method together with proposed discretization technique for the discrete design variables converges to better solutions comparing to meta-heuristic algorithms.

**Keywords:** shape optimization, bar structures, nonlinear programming, design code constraints, gradient projection method, optimization software, finite element method.

**Introduction.** Over the past 50 years, numerical optimization and finite element method have individually made significant advances and have together been developed to make possible the emergence of structural optimization as a potential design tool [4, 6, 21]. In recent years, great efforts have been also devoted to integrate optimization procedures into the CAD facilities. With these new developments, lots of computer packages are now able to solve relatively complicated industrial design problems using different structural optimization techniques [9, 10].

Applied optimum design problems for the bar structures in some cases are formulated as parametric optimization problems, namely as searching problems for unknown structural parameters that provide an extreme value of the specified purpose function in the feasible region defined by the specified constraints [24]. In this case structural optimization performs by variation of the structural parameters when the structural topology, cross-section types and node type connections of the bars, the support conditions of the bar system, as well as loading patterns and load design values are prescribed and constants.

Mathematical model of the parametric optimization problem of the structures includes the set of design variables, the objective function, as well as constraints reflected in general case non-linear interdependences between them [5, 15].

Although many papers are published on the parametric optimization of the structures, the development of a general computer program for the design and optimization of building structures according to specified design codes remains an actual task. Therefore, the main *research goal* is the development of mathematical support and numerical algorithm to solve parametric optimization problems of the building structures with orientation on software implementation in a computer-aided design system.

One of the effective methods to solve parametric optimization problems for building structures is the gradient projection non-linear methods since the purpose function and non-linear constraints of the presented mathematical model are continuously differentiable functions, as well as the search space is smooth. Thus, the method of objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations has been successfully used for parametric optimization of cross-sectional dimensions for cold-formed steel structural members [1], steel trusses [18], as well as lattice portal frames [23].

When applying gradient projection methods, the search for the optimum point is performed in a continuous space of the design variables only. If a nonlinear programming problem is solved where some (or all) design variables vary discretely (for example, according to a defined set of the possible values), then after obtaining a continuous optimal solution, the question of its discretization arises. That is why, the following *research tasks* are states: to propose a discretization technique for the design variables that vary discretely allowing using the gradient projection methods to parametric optimization problems with mixed (discrete and continuous) design variables; to demonstrate the effectiveness of the proposed discretization technique by comparing obtained optimization, as well as the results presented by the literature and widely used for testing.

**Parametric optimization problem formulation for bar structures.** Let us consider a parametric optimization problem of a structure consists of the bar members, which can be formulated as presented below: to find optimum values for geometrical parameters of the structure, bar's cross-section sizes and initial pre-stressing forces introduced into the redundant members of the bar system, whose provide the extreme value of the determined optimality criterion and satisfy all load-bearing capacities and stiffness requirements. We assume, that the structural topology, cross-section types and node type connections of the bars, the support conditions of the bar system, as well as loading patterns and load design values are prescribed and constants.

The formulated parametric optimization problem can be stated as a non-linear programming task in the following mathematical terms: to find unknown structural parameters

$$\vec{X} = \{X_t\}^T, \quad t = \overline{1, N_X}, \quad (1)$$

providing the least value of the determined objective function:

$$f^* = f(\vec{X}^*) = \min_{\vec{X} \in \mathfrak{Z}} f(\vec{X}), \tag{2}$$

in feasible region (search space)  $\mathfrak{Z}$  defined by the following system of constraints:

$$\varphi(\vec{X}) = \{\varphi_\eta(\vec{X}) \leq 0 \mid \eta = \overline{1, N_{IC}}\}, \tag{3}$$

where  $\vec{X}$  – vector of the design variables (unknown structural parameters);  $N_X$  – total number of the design variables;  $f, \varphi_\eta$  – continuous functions of the vector argument;  $\vec{X}^*$  – optimum solution (the vector of optimum values of the structural parameters);  $f^*$  – optimum value of the optimum criterion (objective function);  $N_{IC}$  – number of constraints-inequalities  $\varphi_\eta(\vec{X})$ , which define a feasible region in the design space  $\mathfrak{Z}$ .

The vector of the design variables can include as components unknown geometrical parameters of the structure  $\vec{X}_G = \{X_{G,\alpha}\}^T, \alpha = \overline{1, N_{X,G}}$ , as well as unknown cross-sectional sizes of the structural members  $\vec{X}_{CS} = \{X_{CS,\beta}\}^T, \beta = \overline{1, N_{X,CS}}$ :

$$\vec{X} = \{\vec{X}_G, \vec{X}_{CS}\}^T, \quad t = \overline{1, N_X}, \tag{4}$$

where  $N_{X,CS}$  is the total number of the unknown cross-sectional sizes of the structural members,  $N_{X,G}$  is the total number of the unknown parameters of the structural geometry or shape,  $N_{X,G} + N_{X,CS} = N_X$ .

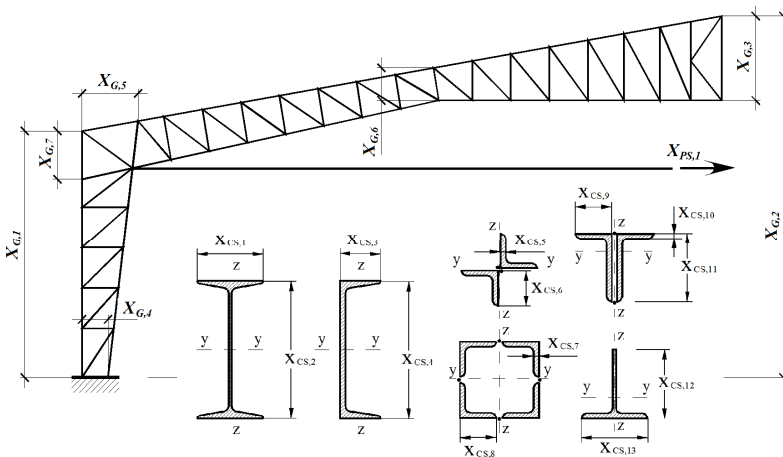


Fig. 1. Unknown parameters of a structure considered as design variables

The specific technical-and-economic index (material weight, material cost,

construction cost etc.) or another determined indicator can be considered as the objective function taking into account the ability to formulate analytical expression of the purpose function depending on the design variables  $\vec{X}$ . In some cases the material weight of the structure is considered as the objective function Eq. (2) of the optimization problem:

$$M^* = M(\vec{X}^*) = \min_{\vec{X} \in \mathfrak{S}_N} M(\vec{X}_G, \vec{X}_{CS}). \quad (5)$$

Load-bearing capacities constraints (strength and stability inequalities) for all design sections of the structural members subjected to all design load case combinations at the ultimate limit state as well as displacements constraints (stiffness inequalities) for the specified nodes of the bar system subjected to all design load case combinations at the serviceability limit state should be included into the system of constraints Eq. (3). Additional requirements, whose describe structural, technological and serviceability particularities of the building structure under consideration, can be also included into the system Eq. (3).

Design internal forces in the bar structural members used in the strength and stability inequalities of the system Eq. (3) are considered as *state variables* depending on design variables  $\vec{X}$  and can be calculated from the following linear equations system of the finite element method:

$$\mathbf{K}(\vec{X}_G, \vec{X}_{CS}) \times \vec{z}_{ULS,k} = \vec{p}_{ULS,k}(\vec{X}_G), \quad k = \overline{1, N_{LCC}^{ULS}}, \quad (6)$$

where  $\mathbf{K}(\vec{X}_G, \vec{X}_{CS})$  is the stiffness matrix of the finite element model of the bar structure, which should be formed depending on the design variable  $\vec{X} = (\vec{X}_G, \vec{X}_{CS})^T$  of the optimization problem Eqs. (1) – (3);  $\vec{p}_{ULS,k}(\vec{X}_G)$  is the column-vector of the node's external loads for  $k^{\text{th}}$  design load case combination corresponded to the ultimate limit state, which should be formed depending on unknown (variable) parameters of the geometrical scheme (shape)  $\vec{X}_G$  of the considered bar structures;  $\vec{z}_{ULS,k}$  is the result column-vector of the node displacements for  $k^{\text{th}}$  design load case combination corresponded to the ultimate limit state,  $\vec{z}_{ULS,k} = \mathbf{Z}_{FEM,k}^{ULS}(\vec{X}_{PS}, \vec{X}_{CS})$ ;  $N_{LCC}^{ULS}$  is the total number of the design ultimate load case combinations. In this way, for each  $i^{\text{th}}$  design section of  $j^{\text{th}}$  bar finite element subjected to  $k^{\text{th}}$  ultimate design load case combination the design internal forces (axial force  $N_{ijk}(\vec{X})$ , bending moments  $M_{y,ijk}(\vec{X})$ ,  $M_{z,ijk}(\vec{X})$  and corresponded shear forces  $Q_{z,ijk}(\vec{X})$ ,  $Q_{y,ijk}(\vec{X})$ ) can be calculated depending on node displacement column-vector  $\vec{z}_{ULS,k}$ .

Node displacements of the bar structure used in stiffness inequalities of the system Eq. (3) are also considered as state variables depending on design

variables  $\vec{X}$  and can be calculated from the following linear equations system of the finite element method:

$$\mathbf{K}(\vec{X}_G, \vec{X}_{CS}) \times \vec{z}_{SLS,k} = \vec{p}_{SLS,k}(\vec{X}_G), \quad k = \overline{1, N_{LCC}^{SLS}}, \quad (7)$$

where  $\vec{p}_{SLS,k}(\vec{X}_G)$  is the column-vector of the node's external loads for  $k^{\text{th}}$  design load case combination corresponded to the serviceability limit state, which should be formed depending on unknown (variable) parameters of the geometrical scheme (shape)  $\vec{X}_G$  of the considered bar structures;  $\vec{z}_{SLS,k}$  is the result column-vector of the node displacements for  $k^{\text{th}}$  serviceability design load case combination,  $\vec{z}_{SLS,k} = \mathbf{Z}_{FEM,k}^{SLS}(\vec{X}_G, \vec{X}_{CS})$ ;  $N_{LCC}^{SLS}$  is the total number of the design serviceability load case combinations. For each  $m^{\text{th}}$  node of the finite element model subjected to  $k^{\text{th}}$  serviceability design load case combination the design vertical  $\delta_{z,mk}(\vec{X})$  and horizontal  $\delta_{x,mk}(\vec{X})$  displacements can be calculated depending on node displacement column-vector  $\vec{z}_{SLS,k}$ .

The system of constraints Eq. (3) should cover *strength and stability constraints* formulated for all structural members of the bar structure subjected to all design load case combinations corresponded to the ultimate limit state. In case of parametric optimization of truss structures as particular sub-case of the bar structures the following *normal stresses verifications* should be included in the system of constraints:

$$\frac{N_{t,jk}(\vec{X})}{A_j(\vec{X}_{CS})\sigma_{t,ult}} - 1 \leq 0; \quad \forall j = \overline{1, N_B}, \quad \forall k = \overline{1, N_{LCC}^{ULS}}, \quad (8)$$

$$\frac{N_{c,jk}(\vec{X})}{A_j(\vec{X}_{CS})\sigma_{c,ult}} - 1 \leq 0; \quad \forall j = \overline{1, N_B}, \quad \forall k = \overline{1, N_{LCC}^{ULS}}, \quad (9)$$

where  $N_B$  is the total number of the truss structural members;  $A_j(\vec{X}_{CS})$  is the cross-section area of  $j^{\text{th}}$  structural member of the truss structure;  $\sigma_{t,ult}$  and  $\sigma_{c,ult}$  are the allowable tension and compression normal stresses respectively;  $N_{t,jk}(\vec{X})$  and  $N_{c,jk}(\vec{X})$  are the tension and compression axial force respectively acting in  $j^{\text{th}}$  structural member of the truss structure subjected to  $k^{\text{th}}$  ultimate load case combination calculated from the linear equations system of the finite element method Eq. (6). In case of statically indeterminate truss structure the value of the axial force should be calculated depending on the variable geometrical parameters of the structure  $\vec{X}_G$  and variable cross-sectional dimensions of the structural members  $\vec{X}_{CS}$ .

The following *flexural and torsional-flexural buckling verifications* should be included in the system of constraints Eq. (3) formulated for all structural members of the truss structure subjected to all ultimate load case combination, namely:

$$\frac{N_{c,jk}(\bar{X})}{\varphi_{y,j}(\bar{X}_G, \bar{X}_{CS}) A_j(\bar{X}_{CS}) \sigma_{c,ult}} - 1 \leq 0; \quad \forall j = \overline{1, N_B}, \quad \forall k = \overline{1, N_{LCC}^{ULS}}, \quad (10)$$

$$\frac{N_{c,jk}(\bar{X})}{\varphi_{z,j}(\bar{X}_G, \bar{X}_{CS}) A_j(\bar{X}_{CS}) \sigma_{c,ult}} - 1 \leq 0; \quad \forall j = \overline{1, N_B}, \quad \forall k = \overline{1, N_{LCC}^{ULS}}, \quad (11)$$

$$\frac{N_{c,jk}(\bar{X})}{\varphi_{c,j}(\bar{X}_G, \bar{X}_{CS}) A_j(\bar{X}_{CS}) \sigma_{c,ult}} - 1 \leq 0; \quad \forall j = \overline{1, N_B}, \quad \forall k = \overline{1, N_{LCC}^{ULS}}, \quad (12)$$

where  $\varphi_{y,j}(\bar{X}_G, \bar{X}_{CS})$  and  $\varphi_{z,j}(\bar{X}_G, \bar{X}_{CS})$  are column's stability factors corresponded to flexural buckling relative to main axes of inertia and calculated according to the specified design code depending on the design lengths  $l_{ef,y,j}$ ,  $l_{ef,z,j}$ , cross-section type and cross-section geometrical properties for the  $j^{\text{th}}$  structural member;  $\varphi_{c,j}(\bar{X}_G, \bar{X}_{CS})$  is the column's stability factor corresponded to torsional-flexural buckling and calculated according to the specified design code depending on the design lengths  $l_{ef,y,j}(\bar{X}_G)$ ,  $l_{ef,z,j}(\bar{X}_G)$ ,  $l_{ef,T,j}(\bar{X}_G)$ , cross-section type and cross-section geometrical properties for  $j^{\text{th}}$  structural member. The flexural buckling factors  $\varphi_{y,j}(\bar{X}_G, \bar{X}_{CS})$  and  $\varphi_{z,j}(\bar{X}_G, \bar{X}_{CS})$ , as well as torsional-flexural buckling factor  $\varphi_{c,j}(\bar{X}_G, \bar{X}_{CS})$  should be calculated depending on the variable geometrical parameters of the structure  $\bar{X}_G$  and variable cross-sectional dimensions of the structural members  $\bar{X}_{CS}$ .

In some cases the system of stability constraints Eqs. (10)–(12) can be simplified by considering the flexural Euler's buckling verifications only. The following flexural Euler's buckling verifications can be included in the system of constraints Eq. (3) formulated for all structural members of the truss structure subjected to all ultimate load case combinations, namely:

$$\frac{N_{c,jk}(\bar{X})}{A_j(\bar{X}_{CS}) \sigma_{cr,\min,j}(\bar{X}_G, \bar{X}_{CS})} - 1 \leq 0; \quad \forall j = \overline{1, N_B}, \quad \forall k = \overline{1, N_{LCC}^{ULS}}, \quad (13)$$

where  $\sigma_{cr,\min,j}$  is the minimum Euler's buckling critical stresses calculated as presented below:

$$\sigma_{cr,\min,j}(\bar{X}_G, \bar{X}_{CS}) = \min \left\{ \sigma_{cr,y,j}(\bar{X}_G, \bar{X}_{CS}), \sigma_{cr,z,j}(\bar{X}_G, \bar{X}_{CS}) \right\}, \quad (14)$$

$$\sigma_{cr,z,j}(\vec{X}_G, \vec{X}_{CS}) = \frac{\pi^2 E \cdot i_{y,j}^2(\vec{X}_{CS})}{l_{ef,y,j}^2(\vec{X}_G)}, \quad (15)$$

$$\sigma_{cr,z,j}(\vec{X}_G, \vec{X}_{CS}) = \frac{\pi^2 E \cdot i_{z,j}^2(\vec{X}_{CS})}{l_{ef,z,j}^2(\vec{X}_G)}, \quad (16)$$

where  $E$  is the modulus of material elasticity;  $i_{y,j}(\vec{X}_{CS})$  and  $i_{z,j}(\vec{X}_{CS})$  are the radiuses of inertia in the main planes of inertia calculated depending on the variable cross-sectional dimensions of the structural members  $\vec{X}_{CS}$ .

In case of equal design lengths in the main planes of inertia  $l_{ef,j}(\vec{X}_G) = l_{ef,y,j}(\vec{X}_G) = l_{ef,z,j}(\vec{X}_G)$ , the minimum Euler's buckling critical stresses can be determined by the following equation:

$$\sigma_{cr,min,j}(\vec{X}_G, \vec{X}_{CS}) = \frac{\pi^2 E \cdot i_{min,j}^2(\vec{X}_{CS})}{l_{ef,j}^2(\vec{X}_G)}, \quad (17)$$

where  $i_{min,j}(\vec{X}_{CS})$  is the minimum radius of inertia calculated depending on the variable cross-sectional dimensions of the structural members  $\vec{X}_{CS}$ . Taking into account Eq. (17) the Euler's buckling constraint-inequality Eq. (13) can be rewritten as follow:

$$\frac{N_{c,jk}(\vec{X}) l_{ef,j}^2(\vec{X}_G)}{\pi^2 E \cdot A_j(\vec{X}_{CS}) i_{min,j}^2(\vec{X}_{CS})} - 1 \leq 0 \quad (18)$$

or

$$\frac{N_{c,jk}(\vec{X})}{\kappa E} \cdot \frac{l_{ef,j}^2(\vec{X}_G)}{A_j^2(\vec{X}_{CS})} - 1 \leq 0, \quad (19)$$

where  $\kappa$  is the factor determined depending on the cross-sectional type.

The system of constraints Eq. (3) should also cover the *displacements constraints* (stiffness inequalities) for the specified nodes of the truss structure subjected to all design load case combinations at the serviceability limit state. The following horizontal and vertical displacements constraints should be included into the system of constraints Eq. (3) formulated for nodes of the truss structure subjected to all serviceability load case combination, namely:

$$\frac{\delta_{x,mk}(\vec{X})}{\delta_{ux,m}} - 1 \leq 0; \quad \forall m = \overline{1, N_N}; \quad \forall k = \overline{1, N_{LCC}^{SLS}}, \quad (20)$$

$$\frac{\delta_{z,mk}(\vec{X})}{\delta_{uz,m}} - 1 \leq 0; \quad \forall m = \overline{1, N_N}; \quad \forall k = \overline{1, N_{LCC}^{SLS}}, \quad (21)$$

where  $\delta_{x,mk}(\vec{X})$  and  $\delta_{z,mk}(\vec{X})$  are the horizontal and vertical displacements respectively for  $m^{\text{th}}$  node of the truss structure subjected to  $k^{\text{th}}$  serviceability load case combination calculated from the linear equations system of the finite element method Eq. (7);  $\delta_{ux,m}$  and  $\delta_{uz,m}$  are the allowable horizontal and vertical displacements for  $m^{\text{th}}$  structural node;  $N_N$  is the total node number in structure.

Additional requirements describing structural, technological and serviceability particularities of the considered structure can be also included into the system Eq. (3). In particular these requirements can be presented in the form of constraints on lower and upper bound values for the design variables, namely:

$$1 - \frac{X_t}{X_t^L} \leq 0; \quad \forall t = \overline{1, N_X}, \quad (22)$$

$$\frac{X_t}{X_t^U} - 1 \leq 0; \quad \forall t = \overline{1, N_X}, \quad (23)$$

where  $X_t^L$  and  $X_t^U$  are the lower and upper bounds for the  $t^{\text{th}}$  design variable  $X_t$ .

**Parametric optimization algorithm based on the gradient projection method.** The parametric optimization problem stated as non-linear programming task by Eq. (4), Eq. (5), Eq. (8), Eq. (9), Eqs. (10) – (12) or Eq. (19), Eqs. (20) – (23) can be successfully solved using gradient projection non-linear methods since the purpose function and non-linear constraints of the presented mathematical model are continuously differentiable functions, as well as the search space is smooth. The method of *objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations* ensures effective searching for solution of the non-linear programming tasks [19]. The gradient projection method operates with the first derivatives or gradients only of both the objective function Eq. (5) and constraints Eq. (8), Eq. (9), Eqs. (10) – (12) or Eq. (19), Eqs. (20) – (23). The method is based on the iterative construction of such sequence Eq. (24) of the approximations of the design variables  $\vec{X} = \{X_t\}^T$ ,  $t = \overline{1, N_X}$ , that provides the convergence to the optimum solution (optimum values of the structural parameters) [22]:

$$\vec{X}_{t+1} = \vec{X}_t + \Delta\vec{X}_t, \quad (24)$$

where  $\vec{X}_t = \{X_t\}^T$ ,  $t = \overline{1, N_X}$  is the current approximation to the optimum solution  $\vec{X}^*$  that satisfies constraints-inequalities Eq. (8), Eq. (9), Eqs. (10) – (12) or Eq. (19), Eqs. (20) – (23) with the extreme value of the objective function Eq. (5);  $\Delta\vec{X}_t = \{\Delta X_t\}^T$ ,  $t = \overline{1, N_X}$ , is the increment vector for the current values of the design variables  $\vec{X}_t$ ;  $t$  is the iteration's index.

Let present the following numerical algorithm to solve the parametric



optimization problem for truss structures formulated above.

*Step 1.* Describing an initial design (a set of design variables) and initial data for structural optimization.

The design variable vector  $\vec{X}_k = (\vec{X}_G, \vec{X}_{CS})_k^T$  has been specified, where  $k$  is the iteration index,  $k = 0$ . The structural topology, cross-section types and node type connections of the bars, the support conditions of the bar system, as well as loading patterns, load case combinations and load design values are prescribed and constants.

Initial data for optimization of the considered steel structure are design strength for steel member (allowable stresses taken into account safety factors), factors to define flexural design lengths  $l_{ef,y,j}$ ,  $l_{ef,z,j}$  for all column structural members; allowable values for horizontal and vertical displacements  $\delta_{uz,l}$  and  $\delta_{uz,l}$  of the specified nodes of the considered steel structure; lower  $\vec{X}^L$  and upper  $\vec{X}^U$  bounds for the design variables; as well as specified objective function  $f(\vec{X}_k)$ .

*Step 2.* Calculation of the geometrical and design lengths for all structural members.

The geometrical lengths  $l_j$  of all structural members are calculated based on the node coordinates of the considered steel structure. The latter depend on the unknown (variable) geometrical parameters of the structure  $\vec{X}_G$ . The design lengths  $l_{ef,y,j}$ ,  $l_{ef,z,j}$  of all column structural members are calculated using calculated geometrical lengths  $l_j$  and initial data relating to the design length factors. The latter are constant during the iteration process presented below. Variation of the geometrical lengths  $l_j$  and corresponded design lengths  $l_{ef,y,j}$ ,  $l_{ef,z,j}$  on the further iterations has been performed based on the current values of the variable (unknown) parameters  $\vec{X}_G$  of the geometrical scheme.

*Step 3.* Calculation of the cross-section dimensions and geometrical properties for all design cross-sections.

Geometrical properties of the design cross-sections (areas, moments of inertia, elastic section moments, radiuses of inertia, etc.) have been calculated depending on the current values of the unknown (variable) cross-section dimensions  $\vec{X}_{CS}$ .

*Step 4.* Linear structural analysis of the considered truss structure.

For each  $m^{\text{th}}$  node of the finite element model subjected to  $k^{\text{th}}$  serviceability load case combination the displacements and rotations, as well as the design horizontal  $\delta_{x,mk}(\vec{X})$  and vertical  $\delta_{z,lk}(\vec{X})$  displacements can be calculated using the linear equations system of the finite element method presented by Eq. (7).

For each  $i^{\text{th}}$  design section of  $j^{\text{th}}$  structural member subjected to  $k^{\text{th}}$  ultimate load case combination the design internal forces can be calculated using the linear equations system of the finite element method presented by Eq. (6).

*Step 5.* Calculation of the state variables (normal stresses, buckling factors or buckling stresses etc.).

The value of the normal  $\sigma_{x,ijk}(\bar{X})$  stresses at the specified cross-section point has been calculated depending on the axial force acting in  $i^{\text{th}}$  design section of  $j^{\text{th}}$  structural member subjected to  $k^{\text{th}}$  ultimate load case combination as presented by the design code.

The flexural buckling factors  $\varphi_{y,j}(\bar{X}_G, \bar{X}_{CS})$ ,  $\varphi_{z,j}(\bar{X}_G, \bar{X}_{CS})$  have been calculated depending on the corresponded design lengths, cross-section type and cross-section geometrical properties for the structural members according to the considered design code.

*Step 6.* Verifications of the constraints and construction the set of active constraints numbers  $\mathbf{A}$ .

Verification the constraints Eq. (8), Eq. (9), Eqs. (10)–(12) or Eq. (19) has been performed for all ultimate load case combinations and all design cross-sections of all structural members. Verification the constraints Eqs. (20)–(21) have been also conducted for all serviceability load case combinations and all design structural nodes. Additional requirements Eqs. (22)–(23) on the lower and upper bounds for the design variables have been also verified.

*Step 7.* Calculation the increment vector for the current design variables and determination the improved approximation to the optimum solution. The increment vector  $\Delta\bar{X}_k$  for the current design variables values  $\bar{X}_k$  has been calculated according to resolving equations of the method of *objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations* described by the paper [17]. The improved approximation  $\bar{X}_{k+1}$  to the optimum solution has been determined according to Eq. (24).

*Step 8.* Stop criteria verification of iterative searching for the optimum solution. If all constraints are satisfied with appropriate accuracy, as well as one of the stop criteria described by the paper [17] is also satisfied, then transition to the step 9 has been performed. In contrary case return to the step 1 has been conducted with  $k \leftarrow k + 1$ .

*Step 9.* Discretization the optimum solution  $\bar{X}_k$  obtained in the continuum space of the design variables.

*Step 10.* Optimum parameters of the truss structure is  $\bar{X}_k$  with the optimum value of the objective function  $f(\bar{X}_k)$ .

**Discretization technique for the design variables that vary discretely.** When moving along the direction of searching for the optimum point, hitting

to the nodes of a discrete grid is associated with significant complications of the optimization algorithm and may lead to deterioration of the optimization process convergence. Therefore, when applying gradient methods, the search for the optimum point is performed in a continuous space of the design variables only. If a nonlinear programming problem is solved where some (or all) design variables vary discretely (for example, according to a defined set of the possible values), then after obtaining a continuous optimal solution, the question of its discretization arises. The discretization of the optimal design solution obtained in the continuous space of the design variables can be performed by purposefully selecting discrete points around the point of the continuous optimum.

*Step 1.* Let  $\vec{X}^*$  is the optimum structural design obtained in the continuous space of the design variables, moreover the design variable set covers as continuous design variables  $\vec{X}_C^* = \{X_{C,\chi}^*\}^T$ ,  $\chi = \overline{1, N_{XC}}$ , (for example, variable parameters of the geometrical scheme of the truss structure), as well as design variables which should vary discretely  $\vec{X}_D^* = \{X_{D,\delta}^*\}^T$ ,  $\delta = \overline{1, N_{XD}}$  (for example, variable size of the cross-section sizes of the structural members):  $\vec{X}^* = \{\vec{X}_C^*, \vec{X}_D^*\}$ , where  $N_{XC}$  is the total number of the continuous design variables;  $N_{XD}$  is the total number of the design variables which should vary discretely.

*Step 2.* For each design variable  $X_i^*$ ,  $X_i^* \in \vec{X}_D^*$ , two neighbor values can be specified from the predefined set of the possible discrete values:  $X_{i,D}^{*L} < X_i^*$  and  $X_i^* > X_{i,D}^{*U}$ , where  $X_{i,D}^{*L}$  is the neighbor value on the left (lower) and  $X_{i,D}^{*U}$  is the neighbor value on the right (upper).

*Step 3.* Among all design variables  $\vec{X}_D^* = \{X_{D,\delta}^*\}^T$ ,  $\delta = \overline{1, N_{XD}}$ , which should vary discretely, the one  $X_p^*$ ,  $X_p^* \in \vec{X}_D^*$ , with the largest length of the purpose function gradient is selected:

$$\ell_{\nabla,p} = \max \left\{ \frac{\partial f}{\partial X_{D,\delta}^*} \right\}, \forall \delta = \overline{1, N_{XD}}; \quad (25)$$

and further discretized at the level of neighbor lower discrete value:  $X_p^* \leftarrow X_{p,D}^{*L}$ . The design variables vector should be truncated accordingly:  $\vec{X}^* \leftarrow \vec{X}^* - \{X_p^*\}$ . Total number  $N_{XD}$  of the design variables which should vary discretely is decreased respectively:  $N_{XD} \leftarrow N_{XD} - 1$ .

*Step 4.* Searching for the optimum point with truncated design variable vector  $\vec{X}^*$  is performed. If at the same time (when  $X_p^* \leftarrow X_{p,D}^{*L}$ ) the optimum solution does not exist, then the discretization of such variable should be

performed at the level of neighbor upper discrete value ( $X_p^* \leftarrow X_{p,D}^{*U}$ ) with the further searching for the optimum point.

Third and fourth steps are carried out until the design variables which should vary discretely  $\vec{X}_D^* = \{X_{D,\delta}^*\}^T$ ,  $\delta = \overline{1, N_{XD}}$ , will be fully discretized, namely while  $N_{XD} > 0$ .

### **Geometry and cross-sectional optimization of a 18-bar cantilever truss.**

A parametric optimization methodology presented above has been realized in software OptCAD [18, 23]. This software provides solutions to a wide range of problems, namely: (i) linear static analysis of bar structures; (ii) verification of the load-bearing capacity of the structural members according to the specified design code; (iii) searching for values of the structural parameters when structure complies with design code requirements and designer's criterions; (iv) parametric optimization of the steel bar structures by the determined criterion.

In order to estimate an efficiency of the new methods, techniques or algorithms, a comparison with alternative methods or algorithms presented by other authors using different optimization techniques should be performed. Criteria to implement such comparison are described, e.g. by Haug & Arora [13] and Crowder et al. [3]. Many of these criteria, such as robustness, amount of functions calculations, requirements to the computer memory, numbers of iterations etc. cannot be used due to lack of corresponded information in the technical literature. Therefore, an efficiency estimation of the proposed methodology for solving parametric optimization problems presented above will be based on the comparison of the optimization results obtained using the proposed numerical algorithm, as well as of the results presented by the literature and widely used for testing. The initial data and mathematical models of the parametric optimization problems considered below were assumed as the same as described in the literature.

Figure 2 shows a 18-bar cantilever truss designed for the vertical loads  $P = 20$  kips = 88.9644 kN (only one design load case combination). Initial data for truss optimum design are: material density  $\rho = 0.1$  lb/inch<sup>3</sup> = 27.8014 ton/m<sup>3</sup>, coefficient of elasticity  $E = 10^4$  ksi = 6894.76 kN/cm<sup>2</sup>. The allowable displacements in the horizontal and vertical direction for all nodes are limited by  $\delta_{ux} = \delta_{uz} = \pm 10$  inch =  $\pm 254$  mm. The absolute value of the allowable normal stresses in tension and compression are  $\sigma_{t,ult} = \sigma_{c,ult} = 20$  ksi = 137.895 N/mm<sup>2</sup>.

The geometry and cross-sectional optimization problem for 18-bar cantilever truss has been formulated as searching for optimum values of the coordinates for all nodes of the truss lower chord, as well as for optimum values of the cross sectional areas for all truss members. Variable unknown cross-sectional areas  $\vec{X}_{CS} = \{A_1, A_2, A_3, A_4\}$ , as well as unknown horizontal and vertical coordinates for all truss lower chord nodes  $\vec{X}_G = \{x_3, z_3, x_5, z_5, x_7, z_7, x_9, z_9\}$ , were considered as design variables Eq. (4). Table 1 presents the

lower and upper bounds for the design variable and member grouping for the variable cross-section areas.

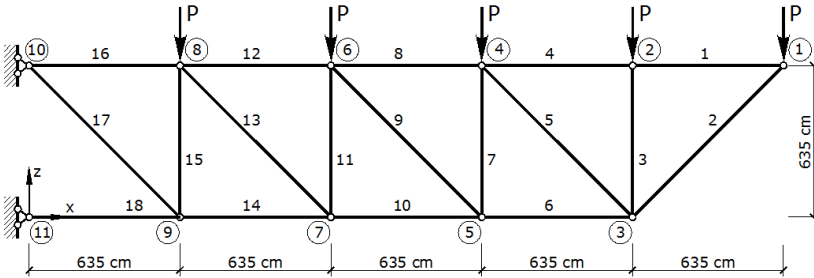


Fig. 2. Design scheme of the 18-bar cantilever truss (with specified numbers of nodes and numbers of bars)

Table 1

The lower and upper bounds for the design variable and member grouping for the variable cross-section areas of the 18-bar cantilever truss problem

Design variable	Unit	Lower bound	Upper bound	Truss member's number	Type of design variable	
$\vec{X}_{CS}$	$A_1$	cm <sup>2</sup>	12.9032	140.3223	1,4,8,12,16	discrete with step 1.6129 cm <sup>2</sup>
	$A_2$	cm <sup>2</sup>	12.9032	140.3223	2,6,10,14,18	discrete with step 1.6129 cm <sup>2</sup>
	$A_3$	cm <sup>2</sup>	12.9032	140.3223	3,7,11,15	discrete with step 1.6129 cm <sup>2</sup>
	$A_4$	cm <sup>2</sup>	12.9032	140.3223	5,9,13,17	discrete with step 1.6129 cm <sup>2</sup>
$\vec{X}_G$	$x_3$	cm	1968.5	3111.5	–	continuous
	$z_3$	cm	-571.5	622.3	–	continuous
	$x_5$	cm	1333.5	2476.5	–	continuous
	$z_5$	cm	-571.5	622.3	–	continuous
	$x_7$	cm	698.5	1841.5	–	continuous
	$z_7$	cm	571.5	622.3	–	continuous
	$x_9$	cm	63.5	1206.5	–	continuous
	$z_9$	cm	571.5	622.3	–	continuous

The optimum values of the design variables are presented by Table 2 (see column 2). It should be noted that searching for the optimum point has been firstly realized in the continuous design space.

The optimum continuous values for coordinates of the truss lower chord are presented by Fig. 3. The optimum structural weight for the considered 18-bar cantilever truss is 2043.852 kg. There are 10 active constraints in the optimum point, namely the flexural Euler's buckling verification Eq. (19) for 2<sup>nd</sup>, 6<sup>th</sup>, 7<sup>th</sup>,

10<sup>th</sup>, 11<sup>th</sup>, 14<sup>th</sup>, 15<sup>th</sup> and 18<sup>th</sup> truss members (see Fig. 2), as well as tension stress verifications Eq. (8) for 16<sup>th</sup> and 17<sup>th</sup> truss members.

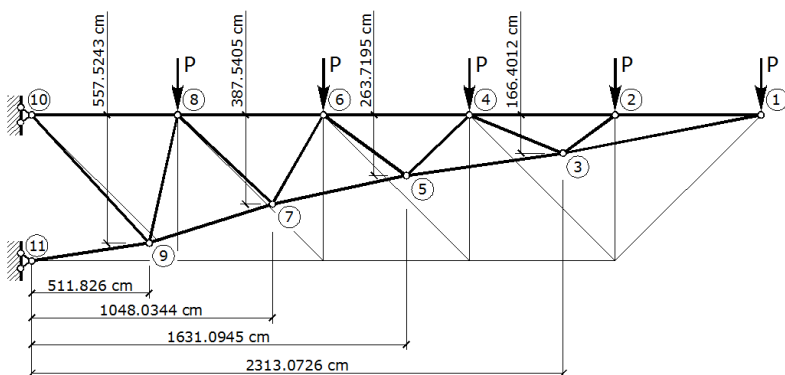


Fig. 3. Optimum coordinates values for all nodes of the 18-bar cantilever truss lower chord

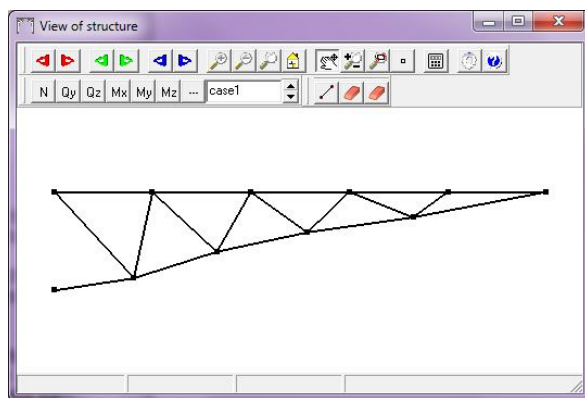


Fig. 4. Optimum truss layout for 18-bar cantilever truss (OptCAD screenshot)

The variable cross-sectional areas of the truss members have been further discretized based on the proposed discretization technique presented above. At *the first step* the purpose function gradient relative to the variable cross-section areas of the truss structural members was  $\{0.710440, 1.031737, 0.157964, 0.143727\}$ , where the component corresponded to the variable  $A_2$  had the maximum value. That is why, the variable  $A_2$  has been discretized firstly at the level of neighbor value on the left (lower)  $A_2 = 114.5159 \text{ cm}^2$ . Then searching for the optimum point in the design space of the variable cross-sectional areas of the truss structural members  $\{A_1, A_3, A_4\}$ , as well as variable parameters of the truss geometry  $\{x_3, z_3, x_5, z_5, x_7, z_7, x_9, z_9\}$  has been performed. As a result the optimum truss design with structural weight 2043.905 kg has been obtained (see 3<sup>rd</sup> column of Table 2).

Table 2

Optimum truss solution in continuous design space, as well as mixed design space

Design variable	Optimum solution in continuous space	Optimum solution in the mixed design space			
		Step 1	Step 2	Step 3	Step 4
1	2	3	4	5	6
$A_1, \text{cm}^2$	80.59012	80.58439	<b>80.645</b>	<b>80.645</b>	<b>80.645</b>
$A_2, \text{cm}^2$	114.95256	<b>114.5159</b>	<b>114.5159</b>	<b>114.5159</b>	<b>114.5159</b>
$A_3, \text{cm}^2$	34.11417	34.68031	34.70766	<b>35.4838</b>	<b>35.4838</b>
$A_4, \text{cm}^2$	23.86916	23.94215	23.84594	23.84583	<b>24.1935</b>
$x_3, \text{cm}$	2315.42400	2313.12283	2312.99463	2313.27053	2311.89259
$z_3, \text{cm}$	471.46299	468.63030	468.55013	468.72269	467.85908
$x_5, \text{cm}$	1634.69376	1631.16983	1630.97784	1630.06180	1628.59515
$z_5, \text{cm}$	374.30071	371.29859	371.25254	371.24689	369.69831
$x_7, \text{cm}$	1051.21702	1048.09826	1047.93536	1046.4583	1045.74314
$z_7, \text{cm}$	249.80907	247.41659	247.52573	247.72279	250.77424
$x_9, \text{cm}$	513.70153	511.86057	511.77234	511.77234	511.77235
$z_9, \text{cm}$	78.10165	77.32506	77.70894	77.70894	77.70894
<b>Weight, kg</b>	<b>2043.85212</b>	<b>2043.90517</b>	<b>2043.90744</b>	<b>2047.48540</b>	<b>2049.60990</b>
Count of active constraints	10	11	12	11	11
The maximum constraint violation	$2.073 \times 10^{-9}$	$1.039 \times 10^{-8}$	$1.980 \times 10^{-6}$	$5.372 \times 10^{-10}$	$3.753 \times 10^{-14}$

At the second step the purpose function gradient relative to the variable cross-section areas of the truss structural members was  $\{0.708205, 0.161196, 0.146550\}$ , where the component corresponded to the variable  $A_1$  had the maximum value. That is why, the variable  $A_1$  has been discretized secondly at the level of nearest discrete value  $A_1 = 80.645 \text{ cm}^2$ . Then searching for the optimum point in the design space of the variable cross-sectional areas of the truss structural members  $\{A_3, A_4\}$ , as well as variable parameters of the truss geometry  $\{x_3, z_3, x_5, z_5, x_7, z_7, x_9, z_9\}$  has been performed. As a result the optimum truss design with structural weight 2043.907 kg has been obtained (see 4<sup>th</sup> column of Table 2).

At the third step the purpose function gradient relative to the variable cross-section areas of the truss structural members was  $\{0.161319, 0.145911\}$ , where the component corresponded to the variable  $A_3$  had the maximum value. That is why, the variable  $A_3$  has been further discretized at the level of nearest discrete value  $A_3 = 35.4838 \text{ cm}^2$ . Then searching for the optimum point in the design space of the variable cross-sectional area of the truss structural members  $\{A_4\}$ , as well as variable parameters of the truss geometry  $\{x_3, z_3, x_5, z_5, x_7, z_7, x_9, z_9\}$  has been performed. As a result the optimum truss design with

structural weight 2047.485 kg has been obtained (see 5<sup>th</sup> column of Table 2).

Finally, *at the fourth step* the variable  $A_4$  has been further discretized at the level of nearest discrete value  $A_4 = 24.1935 \text{ cm}^2$ . Than searching for the optimum point in the design space of variable parameters of the truss geometry only  $\{x_3, z_3, x_5, z_5, x_7, z_7, x_9, z_9\}$  has been performed.

Table 3

Optimization results for the 18-bar cantilever truss problem

Design variable	Rajeev & Krishna-moorthy [20]	Gholizadeh [8]		Cheng et al. [2]	Farqad et al. [7]	This study
	GA	PSO	SCPSO	TLBO	ABC	GPM
$A_1, \text{cm}^2$	80.645	77.4192	77.4192	80.6450	80.6450	80.6450
$A_2, \text{cm}^2$	104.8385	119.3546	111.2901	116.1290	114.5159	114.5159
$A_3, \text{cm}^2$	51.6128	33.8709	40.3225	33.8709	37.0967	35.4838
$A_4, \text{cm}^2$	25.8064	29.0322	30.6451	24.1935	24.1935	24.1935
$x_3, \text{cm}$	2265.426	2296.111	2293.402	2322.8910	2319.013	2311.893
$z_3, \text{cm}$	369.062	471.883	443.789	479.5342	466.5487	467.8591
$x_5, \text{cm}$	1550.924	1638.089	1607.091	1644.2715	1632.4943	1628.595
$z_5, \text{cm}$	300.228	368.222	358.891	380.1948	365.4859	369.6983
$x_7, \text{cm}$	978.916	1087.678	1034.116	1058.7507	1045.697	1045.743
$z_7, \text{cm}$	184.15	255.428	218.270	257.3833	246.7550	250.7742
$x_9, \text{cm}$	468.376	532.236	502.087	518.5791	510.3081	511.7724
$z_9, \text{cm}$	59.436	61.912	50.316	80.4215	76.7564	77.7089
<b>Weight, kg</b>	<b>2094.145</b>	<b>2090.608</b>	<b>2068.894</b>	<b>2053.280</b>	<b>2057.98</b>	<b>2049.61</b>
$A_1, \text{cm}^2$	80.6450	83.8708	79.0321	79.0321	80.6450	80.6450
$A_2, \text{cm}^2$	117.7417	117.742	117.7417	117.7417	116.1290	114.5159
$A_3, \text{cm}^2$	35.4838	35.4838	30.6451	30.6451	33.8709	35.4838
$A_4, \text{cm}^2$	24.1935	19.3548	27.4193	27.4193	24.1935	24.1935
$x_3, \text{cm}$	2369.82	2319.02	2328.926	2338.862	2324.5920	2311.893
$z_3, \text{cm}$	477.52	462.28	487.6063	434.1165	478.6536	467.8591
$x_5, \text{cm}$	1671.32	1645.92	1661.729	1621.885	1645.1308	1628.595
$z_5, \text{cm}$	375.92	386.08	396.494	355.270	378.2959	369.6983
$x_7, \text{cm}$	1071.88	1059.18	1075.69	1039.917	1058.3781	1045.743
$z_7, \text{cm}$	254	261.62	260.5303	233.106	255.5695	250.7742
$x_9, \text{cm}$	520.7	518.16	527.0983	504.889	517.7244	511.7724
$z_9, \text{cm}$	81.28	99.06	72.5907	74.940	79.5078	77.7089
<b>Weight, kg</b>	<b>2074.859</b>	<b>2062.89</b>	<b>2058.750</b>	<b>2065.259</b>	<b>2053.798</b>	<b>2049.61</b>

As a result the optimum truss design with structural weight 2049.61 kg has been obtained (see 6<sup>th</sup> column of Table 2).

The considered geometry and cross-sectional optimization problem for 18-bar cantilever truss has been also solved in the papers [2, 7, 8, 11, 12, 14, 16, 20] using the different optimization methods and calculation techniques such



as genetic algorithm (GA), improved genetic algorithm (IGA), Jaya algorithm (JA), group search optimization (GSO), particle swarm optimization (PSO), sequential cellular particle swarm optimization (SCPSO) as well as force method and genetic algorithm (FMGA) and teaching learning-based optimization (TLBO). Table 3 presents the results of the performed optimization for the 18-bar cantilever truss. As you can see, the optimum truss design obtained using the gradient projection method is better than the cited references [2, 7, 8, 11, 12, 14, 16, 20].

**Conclusion.** The results of the presented study can be formulated as follow:

1. The paper considers geometry and cross-sectional sizes optimization problems for the truss structures formulated as nonlinear programming task with discrete and continuous design variables.

2. The method of the objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations has been used to solve the presented parametric optimization problem in the continuous design space.

3. A discretization technique for the design variables that vary discretely has been proposed for parametric optimization problems stated as non-linear programming task where the purpose function and non-linear constraints of the mathematical model are continuously differentiable functions. The discretization of the optimal design solution obtained in the continuous space of the design variables is performed by the purposefully selecting discrete points around the point of the continuous optimum.

4. The comparison of the optimization results presented by the paper demonstrates that improved gradient method together with proposed discretization technique for the discrete design variables converges to better solutions comparing to metaheuristic algorithms (such as genetic algorithms, improved genetic algorithms, Jaya algorithm, group search optimization, particle swarm optimization, sequential cellular particle swarm optimization as well as force method and genetic algorithm and teaching learning-based optimization).

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### **ПАРАМЕТРИЧНА ОПТИМІЗАЦІЯ СТЕРЖНЕВИХ КОНСТРУКЦІЙ ЗА НАЯВНОСТІ ДИСКРЕТНИХ ТА НЕПЕРЕРВНИХ ЗМІННИХ ПРОЄКТУВАННЯ З ВИКОРИСТАННЯМ ПОКРАЩЕНОГО МЕТОДУ ПРОЄКЦІЇ ГРАДІЄНТА**

У статті розглядається задача параметричної оптимізації стержневих конструкцій, сформульована як задача нелінійного програмування, у якій функція мети та нелінійні обмеження математичної моделі є неперервно диференційованими функціями змінних проєктування. Для розв'язку сформульованої задачі оптимізації використовується метод проєкції градієнта функції мети на поверхню активних обмежень з одночасною ліквідацією нев'язок у порушених обмеженнях. Запропонована методика дискретизації змінних проєктування, які повинні варіювати дискретно. Дискретизація оптимального розв'язку, отриманого у неперервному просторі змінних проєктування, виконується за допомогою цілеспрямованого відбору дискретних точок в околі точки неперервного оптимуму. У статті представлено порівняння результатів оптимізаційних розрахунків, яке засвідчило, що покращений метод проєкції градієнта разом із запропонованою методикою дискретизації змінних проєктування, що повинні варіювати дискретно, забезпечує збіжність до кращого розв'язку задачі порівняно до мета-евристичних алгоритмів.

**Ключові слова:** оптимізація форми, стержневі конструкції, нелінійне програмування, нормативні обмеження, метод проєкції градієнта, програмне забезпечення, метод скінченних елементів.

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### **PARAMETRIC OPTIMIZATION OF BAR STRUCTURES WITH DISCRETE AND CONTINUOUS DESIGN VARIABLES USING IMPROVED GRADIENT PROJECTION METHOD**

The paper considers a parametric optimization problem for the bar structures formulated as nonlinear programming task, where the purpose function and non-linear constraints of the mathematical model are continuously differentiable functions of the design variables. The method of the objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations has been used to solve the parametric optimization problem. A discretization technique for the design variables that should vary discretely has been proposed. The discretization of the optimal design solution obtained in the continuous space of the design variables is performed by the purposefully selecting discrete points around the point of the continuous optimum. The comparison of the optimization results presented by the paper demonstrates that improved gradient method together with proposed discretization technique for the discrete design variables converges to better solutions of the problem comparing to the meta-heuristic algorithms.

**Keywords:** shape optimization, bar structures, nonlinear programming, design code constraints, gradient projection method, optimization software, finite element method.

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### **ПАРАМЕТРИЧЕСКАЯ ОПТИМИЗАЦИЯ СТЕРЖНЕВЫХ КОНСТРУКЦИЙ ПРИ НАЛИЧИИ ДИСКРЕТНЫХ И НЕПРЕРЫВНЫХ ПЕРЕМЕННЫХ ПРОЕКТИРОВАНИЯ С ИСПОЛЬЗОВАНИЕМ УЛУЧШЕННОГО МЕТОДА ПРОЕКЦИИ ГРАДИЕНТА**

В статье рассматривается задача параметрической оптимизации стержневых конструкций, сформулированная как задача нелинейного программирования, в которой функция цели и нелинейные ограничения математической модели являются непрерывно дифференцируемыми функциями переменных проектирования. Для решения сформулированной задачи оптимизации используется метод проекции градиента функции цели на поверхность активных ограничений при одновременной ликвидации невязок в нарушенных ограничениях. Предложена методика дискретизации переменных проектирования, которые должны варьировать дискретно. Дискретизация оптимального решения, полученного в непрерывном пространстве переменных проектирования, выполняется при помощи целенаправленного отбора дискретных точек в окрестности точки непрерывного оптимума. В статье представлено сравнение результатов оптимизационных расчетов, показывающее, что улучшенный метод проекции градиента вместе с предложенной методикой дискретизации переменных проектирования, которые должны варьировать дискретно, обеспечивает сходимость к лучшему решению задачи по сравнению с мета-эвристическими алгоритмами.

**Ключевые слова:** оптимизация формы, стержневые конструкции, нелинейное программирование, нормативные ограничения, метод проекции градиента, программное обеспечение, метод конечных элементов.

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*У статті розглядається задача параметричної оптимізації стержневих конструкцій, сформульована як задача нелінійного програмування, у якій функція мети та нелінійні обмеження математичної моделі є неперервно диференційованими функціями змінних оптимізації. Для розв'язку сформульованої задачі оптимізації використовується метод проєкції градієнта функції мети на поверхню активних обмежень з одночасною ліквідацією нев'язок у порушених обмеженнях. Запропонована методика дискретизації змінних проєктування, які повинні варіювати дискретно. Представлено порівняння результатів оптимізаційних розрахунків, яке засвідчило, що покращений метод проєкції градієнта разом із запропонованою методикою дискретизації змінних проєктування, що повинні варіювати дискретно, забезпечує збіжність до кращого розв'язку задачі порівняно до мета-евристичних алгоритмів.*

Табл. 3. Іл. 3. Бібліог. 24 назв.

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*The paper considers a parametric optimization problem for the bar structures formulated as nonlinear programming task, where the purpose function and non-linear constraints of the mathematical model are continuously differentiable functions of the design variables. The method of the objective function gradient projection onto the active constraints surface with simultaneous correction of the constraints violations has been used to solve the parametric optimization problem. A discretization technique for the design variables that should vary discretely has been proposed. Presented comparison of the optimization results demonstrates that improved gradient method together with proposed discretization technique for the discrete design variables converges to better solutions of the problem comparing to the meta-heuristic algorithms.*

Табл. 3. Figs. 3. Refs. 24.

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*Пелешко І. Д., Юрченко В. В. Параметрическая оптимизация стержневых конструкций при наличии дискретных и непрерывных переменных проектирования с использованием улучшенного метода проекции градиента // Сопrotивление материалов и теория сооружений: науч.-тех. сборн. – К.: КНУСА, 2023. – Вып. 110. – С. 178-198.*

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Табл. 3. Ил. 3. Библиог. 24 назв.

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