OPTIMIZATION OF MANIPULATOR'S MOTION MODE ON ELASTIC BASE ACCORDING TO THE CRITERIA OF THE MINIMUM CENTRAL SQUARE VALUE OF DRIVE TORQUE

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This article presents the results of research on optimizing the motion mode of the manipulator's jib that is established on an elastic base with known rigidity. The results of this work show ways to significantly reduce the oscillations of the loaded power elements of the manipulator's boom system. Since the main external factor of oscillation perturbation in the metal structure of the manipulator is the driving moment of the drive, here we use the target optimization function in the form of the root mean square value of the driving moment of the drive mechanism. The dependence of the driving moment of the drive was determined from the dynamic equations of motion of the manipulator, which are built based on Lagrange equations of the second kind. The selected criterion for optimization mode of movement the manipulator is presented in the form of an integral functional, and the search for its minimum value had carried out using the methods of variation calculus.

The results of the study make it possible to develop a control system for the drive during the design of the manipulator and during its operation, as well as to assess the dynamics of oscillations of such structural elements of boom systems of cranes. Implementation of the obtained optimal modes of movement can be carried out using a hydraulic drive.

Keywords: manipulator, boom system, optimization, criterion, root mean square moment, elastic base, oscillation minimization.

Introduction. Manipulation systems have a wide range of potential applications in the construction industry. Such solutions are dominant in the technological processes of logistics of cargo movement and in hoisting and transport machines. Manipulators use as actuators to move in the space of special working bodies. General purpose hydraulic manipulators have become widely used [1, 2]. In the development and implementation of modern models of such systems, the main indicator of their choice is competitiveness, which depends on design solutions, reliability and low cost of operation.

In the existing studies, the modeling of dynamics the manipulation systems based on the assumption that all parts of the manipulator's mechanical system and its support mechanism are absolute rigid, and the support base are horizontal [3–5]. In general, this assumption is true for many stationary systems, but in
mobile transport technology machines, which actually work on different support surfaces, the mechanical properties of which are not known in advance or difficult to predict, there are cases of loss of machine stability or increase the dynamics of rolling motion. This is due to the action of significant unbalanced contact forces from the mobile mechanical system of the machine on the support mechanism and the supporting base and their resistance to external loads and can be cause by spatial movements of the chassis of the manipulator's mobile platform during its operation [6]. Important aspect of modeling manipulation systems is to take into account the elasticity of the manipulator, which will affect the accuracy of technological and transport work [7, 8].

Therefore, in order to increase the reliability of modeling of kinematics, dynamics and strength of manipulation systems, it is necessary to perform modeling of manipulators taking into account the influence of elasticity of links and backlashes in kinematic pairs. For the manufacturers of industrial works, taking into account the elasticity of the hinges and legs is important in terms of providing the given position error, and for mobile systems, it is important in terms of forming a picture of dynamic loading of the supporting mechanism [2].

Manipulators usually have a complex hematic structure, which is essentially a broken kinematic chain with a given number of degrees of movement. Each link of the manipulator’s kinematic chain has its own mass and geometric configuration and kinematic pairs of the fifth class make their connection - cylindrical or prismatic. Cylindrical kinematic pairs provide rotational movement relative to the longitudinal or transverse axis of the link and prismatic provide translational movement of one link along the other. This creates telescopically connection links of the kinematic chain. This complexity of the configuration of the manipulator, taking into account the elasticity of the links imposes significant difficulties in its mathematical model. Therefore, at the initial stage of research on the dynamics of manipulators, it is desirable to use simplified calculation schemes, which can it will be to use to assess the relationship between control parameters and external perturbations in the system.

**Analysis of publications.** The problem of unbalance of the jib system of the manipulator on an elastic basis was considered by the authors in [9, 10], but qualitative and quantitative assessment of the impact of driving force created by the hydraulic drive system on the dynamics of the elastic support mechanism is not considered. In [9], the method of synthesis of the optimal mode of motion of the manipulator on an elastic basis by kinematic characteristics have presented, but optimization by the function of the driving moment was not considered.

In [11] which proposed a method of balancing the potential energy of deformation of the metal structure of the elastic boom of the manipulator by the amount of kinetic energy that occurs during oscillations of unbalanced masses of the boom system. The authors consider the stability conditions of a mechanical system using the Lyapunov function.

As a criterion for optimization in the study of boom systems of manipulators with flexible links also use the functions of energy storage by the
elastic element during the spatial movement of the load [12, 14], the value of which is proposed to be minimized.

The problem of stability of construction hoisting machines related to the deformation of the bearing surface is shown in [15], where, in particular, the limit pressure for different bearing surfaces, which is in the range from 100 kN/m$^2$ to 2000 kN/m$^2$.

In [16] the problem of overloading and overheating of the power drive of the manipulator boom, which arises due to dynamic loads, which are known to be formed during start-up and braking due to vibrations of elastic parts of the boom and imbalance of masses [1, 2, 8].

Therefore, it should be noted that the issue of manipulator dynamics taking an elastic characteristic of links and support base is relevant because a number of authors in different variations considers it, but such studies are not enough to solve this problem. In the particular, it is not known how the mode of operation of the boom system of the manipulator will affect the dynamic loads in the structural elements, as well as how to move the boom of the manipulator with minimal power consumption of the drive. In this work is propose to consider the optimization problem on the criterion of minimum average-weighted value of the driving moment for the simplified scheme of the manipulation with spring support chain, which characterizes flexibility not only of the supporting surface, but also of metal construction of the machine.

Purpose of the paper. The purpose of the study is to increase the efficiency of the manipulator on an elastic basis by optimizing the mode of movement of the drive mechanism in the process of changing the departure of the load and study the behavior of the boom system of the manipulator when working on the developed mode.

Research results. For research, we use a simplified dynamic model of the boom system of the manipulator with the load in the plane of change of departure (Fig. 1) [10].

The adopted dynamic model consists of rigid boom links in length $l_1$ and a support frame in length $l_2$. The support frame has two support points, one of which is common to the boom support joint and is assume rigid and the other replaced by a movable elm, which reflects the elastic properties of the deformed support mechanism and support surface and have characterized by a reduced coefficient of elasticity of $c$. The drive mechanism in the form of a hydraulic cylinder is placed between the boom and the support frame and is attached to the boom at a distance $a_1$ and to the support frame at a distance $a_2$ measured from the rigid boom base.

The dynamic model provides the following assumptions:

- the cargo concentrated weight $m$ is rigidly fastened to the end of the shooting of the manipulator;

- the centers of mass of the boom $m_1$ and the support frame $m_2$ are located in the middle of the respective elements and regardless of the geometry of these parts are at distances $\frac{1}{2}l_1$ and $\frac{1}{2}l_2$ from the rigid support hinge of the boom;
- the friction and backlash in kinematic pairs did not taken into account;
- the mechanical boom system was placed on a horizontal support base.

![Dynamic model of the boom system of the manipulator with hydraulic drive mounted on an elastic base](image)

The generalized independent coordinates are the angle $\phi$ of rotation of the support frame relative to the horizontal surface, which will occur due to elastic deformation of the support joint and the angle of rotation of the boom $\alpha$ relative to the horizontal, which change will be determined by moving the drive cylinder rod.

The change in the departure of the load is performed by the manipulator in a completely constant space and will be determined by the angular coordinate of rotation of the boom of the manipulator $\alpha$ relative to the horizontal base. This angular coordinate depends on the joint movement of the system due to the oscillations of the support frame and the rotation of the manipulator boom relative to the support frame, that is

$$\alpha = \psi + \phi.$$  \hspace{1cm} (1)

In [1, 17, 18] it is shown that the use of D'Alembert principle for Lagrange equations allows to obtain explicit mathematical models that reflect the influence of rotational and translational motion of the boom system elements of a complex structure on its dynamics and are best suited for solving problems management. Therefore, in the framework of Newton's theory on the application of Lagrange equations of the second kind, a mathematical model of
the considered holozoic boom system is defined, which is written as next system of recurrent differential equations [9]

\[
\begin{aligned}
(J_1 + ml_1^2) \ddot{\alpha} &= M - (m + \frac{m_1}{2})gl_1 \cos \alpha; \\
J_2 \ddot{\varphi} &= -M - m_2 gl_2 \cos \varphi - cl_2^2 (\varphi - \varphi_0),
\end{aligned}
\]

where \( J_1 \) and \( J_2 \) – moments of inertia of the boom and the support frame relative to the point of their rotation; \( \ddot{\alpha} \) and \( \ddot{\varphi} \) – angular accelerations of the boom and the supporting surface; \( M \) – external driving moment of the drive; \( \varphi_0 \) – the initial deviation of the reference link (hereinafter accepted = 0).

The first and second equations of system (2) are interrelated, which is determined by the next dependence

\[
\cos(\alpha - \varphi) \cos(\alpha + \varphi) = a_1 a_2 a_3 a_4 a_5 a_6 M, \quad (3)
\]

where \( F_1 \) – the force on the rod of the hydraulic cylinder.

From the first equation of system (2) we express the driving moment \( M \), which depends on the coordinate \( \alpha \) and has the next form

\[
M = (J_1 + ml_1^2) \dot{\alpha} + (m + \frac{m_1}{2})gl_1 \cos \alpha. \quad (4)
\]

The criterion for optimizing the mode of change of the departure of the manipulator boom was taken as the root mean square value of the driving torque, because this component will significantly affect the oscillations of the mechanical system of the manipulator on the bearing base

\[
M_{cr} = \left[ \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} M^2 dt \right]^{1/2}, \quad (5)
\]

where \( t \) – the time; \( t_0, t_1 \) – the initial and final value of the time at which you want to determine the optimal law of motion.

Minimize criterion (5) want to reduce the undesirable power properties of the system. Practical implementation will have at least a square of the moment, then for the considered system the objective function will be the following expression

\[
I = \int_{t_0}^{t_1} \left[(J_1 + ml_1^2) \dot{\alpha} + (m + \frac{m_1}{2})gl_1 \cos \alpha \right]^2 dt \to \min. \quad (6)
\]

The result of substituting the sub integral expression of the functional (6) into the Euler-Poisson equation, we obtain a differential equation, which is a condition of its minimum

\[
-\frac{1}{2} M_{cr} \sin \alpha \left(M_{cr} \cos \alpha + 2J_{ip} \dot{\alpha} \right) + \\
+2J_{ip} \left(-\frac{1}{2} M_{cr} \left(\dot{\alpha}^2 \cos \alpha + \ddot{\alpha} \sin \alpha \right) + \alpha J_{ip} \right) = 0 \quad (7)
\]
or

\[ \alpha - \ddot{\alpha} \frac{M_c}{J_{np}} \sin \alpha - \frac{\ddot{\alpha}^2}{2} \frac{M_c}{J_{np}} \cos \alpha - \frac{1}{4} \left( \frac{M_c}{J_{np}} \right)^2 \sin \alpha \cos \alpha = 0, \quad (8) \]

where \( J_{np} = J_1 + mL_1^2 \); \( M_c = gL_1(2m + m_1) \).

Equation (8) is a nonlinear differential equation of the fourth order, which cannot be analytically integrated. In the Mathematica program applied an approximate numerical method for solving the differential equation (8) under boundary conditions (9).

The manipulator jib system on the elastic base has the following parameters: \( m_1 = 300 \text{ kg} \); \( m_2 = 100 \text{ kg} \); \( m = 900 \text{ kg} \); \( l_1 = 4 \text{ m} \); \( l_2 = 2 \text{ m} \); \( J_1 = \frac{1}{3} m_1 l_1^2 \); \( J_2 = \frac{1}{3} m_2 l_2^2 \); \( g = 9.81 \text{ m/s}^2 \); \( \varphi(0) = 0 \); \( \psi(0) = 45^\circ \); \( \alpha(0) = 45^\circ \); \( \alpha(t_1) = 85^\circ \); \( c = 5 \times 10^5 \text{ N/m} \).

To minimize criterion (6) taking into account expression (8) it is necessary to provide boundary conditions of motion. For example, for the mode of movement of a mechanical system with sections of full run, the boundary conditions are as follows:

\[
\begin{cases}
  t = 0, \alpha = \alpha_n, \dot{\alpha} = 0; \\
  t = t_1, \alpha = \alpha_k, \dot{\alpha} = 0,
\end{cases}
\]

where \( \alpha_n = \alpha(0) \) – the boom initial position; \( \alpha_k = \alpha_n + \omega t_1/2 \) – the boom position at the time \( t_1 \); \( \omega \) – the boom speed of rotation.

On Fig. 2–4 shows graphs of the kinematic characteristics of the boom rotation, which provide the implementation of equation (8) under boundary conditions (9). After substituting the found numerical solution of equation (8) into the dependence (4), the law of change of the driving moment of the drive is found, which is obtained because of the minimization of criterion (8) under boundary conditions (9).

![Fig. 2. Graphs of change of angular coordinate of the manipulator’s boom at duration of movement: (a) – 1 sec.; (b) - 3 sec](image-url)
In Fig. 3 shows graphs of change of absolute angular speed of the manipulator’s boom at duration of movement: (a) - 1 sec; (b) - 3 sec

In Fig. 4 shows graphs of change of absolute angular acceleration of the manipulator’s boom at duration of movement: (a) – 1 sec.; (b) – 3 sec

In Fig. 5 shows graphical dependences of the change of the driving moment of the drive in the area of movement of the boom from 45° to 85° for different durations.

Substituting the optimal law of change of the driving moment in the second equation of the system (2) and solving it under the initial conditions: \( t = 0, \phi = \phi_0, \dot{\phi} = 0 \), the law of oscillations of the support mechanism at the
minimum action of the driving moment is obtained. On Fig. 6 and Fig. 7 shows a graphical dependence of the simulation results.

![Graphs of oscillations of the angle of the support frame of the manipulator during the duration of movement](image)

Fig. 6. Graphs of oscillations of the angle of the support frame of the manipulator during the duration of movement: (a) - 1 sec.; (b) - 3 sec

![Graphs of change in the relative angle of the manipulator boom rotation during the duration of movement](image)

Fig. 7. Graphs of change in the relative angle of the manipulator boom rotation during the duration of movement: (a) – 1 sec.; (b) – 3 sec

**Discussion of results.** The results of optimizing the mode of movement of the manipulator mounted on an elastic basis by the criterion of the minimum mean square value of the driving moment of the drive show that when implementing this approach at the beginning of the drive the drive want create a significant acceleration of the mechanical system (Fig. 4). There will also be a significant power load on the drive (Fig. 5). When the average angular velocity of the load has reduced by increasing the travel time in a given area of change of the boom angle, there is a decrease in oscillations. From the graphs in Fig. 2 (b) - Fig. 5 (b) as we can see there may be cases when at a very low average angular velocity will move in the opposite direction. This is not due to the possibility of the drive system in the start-up process to compensate for external static load, and since the developed mathematical model does not provide for restrictions on changing the angle, later in the calculations it leads to errors due to symmetry of trigonometric functions.

This article presents an approach to optimization by the function of torque similar to that considered in [19] for the mechanism of rotation of the tower crane. Note that in the mechanism of rotation of the crane at the beginning of the movement, there is no significant external load, in contrast to the
mechanism of lifting the boom, and therefore to compensate for such a moment at the beginning of the movement is not required. In the considered scheme, the optimization model shows that at creation of the considerable driving moment, that considerably exceeds value of the static moment visible at movement for 1 with (fig. 5, and see), the system further has to react quickly and create the moment in the end of movement with the opposite effect. In further research, it is desirable to justify the value of the average angular velocity for this method of calculation and to consider other limiting conditions of movement of the boom of the manipulator, in particular the mode of start and braking.

The dependence of the change in force in the drive hydraulic cylinder during a given period of movement of the manipulator can be determined from expression (3) in which it is necessary to specify certain characteristics of the torque and angles of rotation. On Fig. 8 shows, the graphs of change force on the rod of the drive cylinder at different values of $a_1$, $a_2$ the size of the installation of the cylinder in the studied kinematic scheme of the manipulator.

The graphs on Fig. 8 show that the nature of the change in force on the rod of the drive cylinder practically repeats the dependence of the change in drive torque.

**Conclusions.** As a result, the mode of movement of the boom system of the manipulator with elastic base optimized. The optimization criterion is the root mean square value of the driving torque of the manipulator boom. The obtained optimal mode of movement of the manipulator made it possible to minimize the root mean square value of the driving moment of the drive mechanism during
movement and reduce the oscillations of the boom and the elastic support part. The obtained optimal mode of movement does not completely eliminate the oscillations of the boom system of the manipulator. This can be explain by the fact that the paper does not consider the feedback on the control and does not take into account the impact of changes in the coefficient of elasticity of the flexible element. To eliminate oscillations and practical implementation, it is necessary to determine a set of optimal motion modes according to different criteria for each specific case and establish criteria that lead to the elimination of oscillations of the manipulator boom system.

To completely exclude oscillatory processes from the operation of the boom system of the manipulator in the future it is necessary to investigate the optimization function with a comprehensive criterion [20], which would take into account not only the magnitude of the driving moment of the drive, but also the intensity of its change over time.

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ОПТИМІЗАЦІЯ РЕЖИМУ РУХУ МАНІПУЛЯТОРА НА ПРУЖНІЙ ОПОРІ ЗА КРІТЕРІЄМ МІНІМУМУ СЕРЕДНЬОКВАДРАТИЧНОГО ЗНАЧЕННЯ РУШІЙНОГО МОМЕНТУ ПРИВОДУ

В роботі представлено результати досліджень оптимізації режиму руху стрілі маніпулятора, який встановлено на пружну опору з відомою жорсткістю. Мета даного дослідження полягає в зменшенні коливань стрілової системи маніпулятора, що забезпечить підвищення загальної ефективності маніпулятора, довгоносостійкості та надійності елементів його металоконструкції. Досягнення поставленої мети здійснено шляхом застосування керованого режиму роботи приводу з динамічним врівноваженням привідного механізму.

Застосовуючи рівняння Лагранжа другого роду складено рівняння руху стріли маніпулятора та визначено вираз узагальненого рушійного моменту привідного механізму стрілової системи маніпулятора. У даному дослідженні розглядається лише кутове переміщення стріли маніпулятора. Незрівноважений привідний рушійний момент стріли маніпулятора оцінюється складовою сумарного інерційного моменту рухомої маси стріли і вантажу та статичним відносним моментом від маси стріли і вантажу на привідний механізм.

Пружна опора маніпулятора представлена у вигляді пружини з відомими характеристиками з бруцірованя коливань в металоконструкції маніпулятора. Рушійний момент приводу, розрахований на основі вищезазначених умов, залежить від частоти коливань і частоти руху стріли маніпулятора. На основі вибраного критерію визначається оптимальний режим руху стріли маніпулятора.

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The results of studies of optimizing the mode of movement of the manipulator boom, mounted on an elastic base with a known stiffness the paper presents. The purpose of this scientific research is to reduce the oscillations of the manipulator turnout system, which will increase the overall efficiency of the manipulator, durability and reliability of the metal structure elements. The implementation of this goal have achieved by applying a controlled mode of operation of the drive with dynamic balancing of the drive mechanism. Using the Lagrange equation of the second kind, the equation of motion of the manipulator boom was compile and the expression for the generalized driving moment of the drive mechanism of the manipulator boom system was determined. This study considers only the angular displacement of the manipulator boom. The unbalanced drive driving moment of the manipulator boom had estimated by the component of the total inertial moment of the moving mass of the boom and load and the static load from the mass of the boom and load on the drive mechanism. The elastic base of the manipulator was present in the form of a linear spring with a given coefficient of elasticity. Since the main external factor of oscillation, perturbation in the metal structure of the manipulator is the driving moment of the drive, so we used the target optimization function, which estimates the root mean square value of the driving moment of the drive mechanism. The main criterion for optimizing the mode of motion was present in the form of an integral functional, and the search for its minimum value is carried out using the methods of calculus of variations.

The results of this work can used by the drive control system at the design stage of the manipulator and during its operation. The dynamics of oscillations of such structural elements of boom systems of cranes is also estimated. The implementation of the obtained optimal modes of movement can be carried out using a hydraulic drive.

Keywords: manipulator, boom system, optimization, criterion, root mean square moment, elastic base, oscillation minimization.
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The dynamic balancing of the drive mechanism is considered for the boom system of a manipulator mounted on an elastic support with a known stiffness. The unbalanced driving moment of the manipulator boom is estimated by the component of the total inertial moment of the moving mass of the boom and the cargo, the static load from the mass of the boom, and the load on the drive mechanism. The elastic support of the manipulator is presented in the form of a linear spring with a given elasticity coefficient.
Table -. Fig. 8. Ref. 20.