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INFLUENCE OF INCOMPATIBILITY OF THERMOMECHANICAL PARAMETERS OF BEARING LAYERS OF A BRIDGE STRUCTURE ON ITS THERMO-STRESSED STATE

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The problem of the thermally stressed state of a two-layer fragment of a bridge structure under conditions of a change in the ambient temperature at different values of the coefficients of thermal linear expansion of the layers is considered. Using the finite element method, the fields of thermal stresses, deformations and displacements are constructed for various values of the thermomechanical characteristics of the layers. It is shown that with an increase in the incompatibility of these characteristics, the intensities of stresses and strains increase. The zones of concentration of these functions are found.

Key words: two-layer bridge structure, incompatibility of thermomechanical parameters, thermal stress fields, finite element analysis.

Introduction. In thermomechanics of substantially heterogeneous structures consisting of composite materials and layered massifs (for example, in road construction), the heterogeneity of thermal stress fields, their concentration and system strength largely depend on the inhomogeneity and incompatibility of their thermomechanical parameters (thermal conductivity coefficients, thermal linear expansion coefficients, moduli of elasticity, Poisson's ratios, etc.). For example, in [1] it is shown that an attempt to strengthen the asphalt concrete layer of the road surface with metal and non-metallic (fiberglass, basalt, polymer, etc.) reinforcing rods with different (incompatible) thermomechanical parameters can lead to unexpected negative effects associated with an increase in thermal stresses in the conjugation zones of contacting bodies.

A special case in the thermomechanics of layered media arises if the medium has a low coefficient of thermal conductivity, and the ambient temperature changes rapidly over time (i.e., the so-called "thermal shock" occurs). Then, since the temperature does not have time to quickly equalize, a high-gradient temperature boundary layer arises, leading to large shear stresses and contributing to the destruction of the structure [1]. At the household level, an example of this phenomenon can be the cracking of a faceted (thick-walled) glass when boiling...
water is poured into it. In this case, a thin-walled (so-called «tea») glass is not subject to destruction, since the temperature in it levels out faster.

Book [1] also drew attention to the negative thermomechanical phenomena that occur in the asphalt concrete layer laid on a metal or cement concrete base in bridge structures. It established that the incompatibility of the thermomechanical properties of the upper layer and the lower base leads to significant tangential stresses on the contact plane of dissimilar materials and can be the cause of thermal bending and delamination of the two-layer structure.

Moreover, the authors have shown [2-6] that an attempt to increase the strength of this structure by increasing the thickness of the upper layer only worsens the state of the system and leads to an increase in thermal stresses.

It is interesting to note that similar thermomechanical phenomena are used in the design of electrothermal relays for thermal action on a bimetallic plate with different coefficients of thermal linear expansion, so that, as a result of the bending movement of its elements, electrical switches on and switches out are activated.

In this article, the issues of thermal deformation of an inhomogeneous two-layer bridge structure are studied in more detail.

Below, on the basis of the finite element method, a computer study of the thermally deformed and stressed states of a two-layer bridge structure consisting of a lower metal plate and an upper asphalt concrete layer is carried out at different values of their thermo-mechanical parameters.

**Mathematical model of the thermally stressed state of a two-layer bridge structure.** The problem of the thermally stressed state of a two-layer bridge structure with different thermomechanical properties can be considered as a special case of the thermally stressed state of unidirectional layered composites. At different values of thermomechanical characteristics, they often exhibit significant shape distortions, local stratification, and general thermal destruction [7-8]. Moreover, attempts to reduce thermal stresses in such systems by increasing the thicknesses of different layers or their modulus of elasticity only lead to a worsening of the situation. Various formulations of such cases on the examples of road surfaces are considered in works [9-13]. The issues of mechanics of composite and laminated materials, as well as road structures, close to those discussed above, are analyzed in [14-24]. In this regard, the issue of reducing the level of thermal stresses in the asphalt-concrete layer of the top coating of the metal running track of the bridge structure is of scientific and practical interest. If the materials of the asphalt concrete pavement and the metal base have different values of the coefficients of thermal linear expansion, then with seasonal and daily changes in the ambient temperature, the elements of each of these materials expand and shorten in different ways, leading to their various incompatible deformations and displacements on their contact surface. To combine these deformations and displacements, significant tangential stresses are generated in the contact zone of these elements, ensuring the absence of their mutual slippage and delamination and joint deformation. In works [1-6], finite element modeling of the main features of these effects was carried out on the example of the construction of the South Bridge in Kiev within the framework of the theory of thermoelasticity. It is shown that the greatest shear stresses between
the layers of asphalt concrete and the metal base are concentrated in the edge zone of the system, and normal longitudinal stresses prevail in the central sections of the system. Note that similar features also take place in the mechanics of composite materials [7, 8, 15].

This effect is one of the factors that helps to establish the reason for the intense detachment of the asphalt concrete layer from the metal base in the winter-spring period. It can be argued that the intensity of the indicated interlayer tangential stresses is primarily determined by the difference in the values of the coefficients of thermal linear expansion and the thickness of the asphalt concrete layer, which affect the incompatibility of deformations and displacements of contacting structural fragments that are subjected to alignment. In this case, the thickness of the metal layer of the bridge structure obviously plays a lesser role due to the low elastic deformability of steel.

To check the influence of the factor of incompatibility of the thermomechanical parameters of asphalt concrete and steel, finite element calculations of the thermally stressed state of a fragment of the section of the bridge structure, the cross section of which is shown in Fig. 1, were performed. For this case, the thickness of the steel base was \( h_m = 0.014 \text{ m} \), and the thickness of the asphalt concrete layer was \( h_a = 0.07 \text{ m} \).

At the bottom, the structure is reinforced with vertical ribs with a step of \( 0.3 \text{ m} \), which practically do not affect the thermally stressed state of the system.

![Fig. 1. Dimensions of structural elements](image)

The thermomechanical characteristics of the steel were chosen unchanged: modulus of elasticity \( E = 2.1 \cdot 10^{11} \text{ Pa} \), Poisson's ratio \( \nu = 0.3 \), coefficient of thermal linear expansion \( \alpha_T = 1.3 \cdot 10^{-5} \text{ K}^{-1} \); for asphalt concrete, the parameters \( E = 5 \cdot 10^9 \text{ Pa} \), \( \nu = 0.2 \) were fixed, the coefficient of thermal linear expansion (in order to study its effect) was assigned the values \( \alpha_T = 1.9 \cdot 10^{-5} \text{ K}^{-1} \), \( 2.4 \cdot 10^{-5} \text{ K}^{-1} \), \( 3 \cdot 10^{-5} \text{ K}^{-1} \). The influence of these values on the change in the general thermally stressed state of the structure at these values of \( \alpha_T \) was investigated. It was assumed that the initial ambient temperature was \( T_0 = 0 \) and then dropped to \( T = -25^\circ \text{C} \).

In the general case, the evolution of the temperature field in each fragments of the system is determined by the equation [25-26]
\[ \nabla^2 T - \frac{1}{a} \frac{\partial T}{\partial t} = 0. \tag{1} \]

Here \( a = \lambda_q / c_{ob} \) – the thermal diffusivity coefficient, \( \lambda_q \) – the thermal conductivity coefficient, \( c_{ob} \) – the specific volumetric heat capacity, the term \( \nabla^2 T \) is equivalent to the expression \( \partial^2 T / \partial x^2 + \partial^2 T / \partial y^2 + \partial^2 T / \partial z^2 \).

We believe that in the case of thermoelastic deformation of the entire massif, inertial forces can be neglected. Then the field of elastic displacements \( u(x, y, z) \) is described by the vector equation [25-26]

\[ \mu \nabla^2 u + (\lambda + \mu) \text{grad} \text{div} u - (3\lambda + 2\mu)\alpha_T \text{grad}(T) = 0, \tag{2} \]

where \( \lambda \) and \( \mu \) are the Lamé isothermal parameters.

At the conditional ends of the selected area, it is assumed that there are no heat fluxes in the normal directions, therefore, the derivative of \( T \) in the normal \( n \) direction is equal to zero,

\[ \partial T / \partial n = 0. \tag{3} \]

When formulating the boundary conditions for the vector function \( u(x, y, z) \), we assume that on all free surfaces the normal and tangential stresses are equal to zero, and on the plane of contact of the asphalt concrete layer with the metal base, the conditions for the compatibility of displacements are set.

The accepted formulation of the problem of thermoelastic deformation of a selected two-layer massif made it possible to use an algorithm for its solution, in which the problem of unsteady heat conductivity for equation (1) is first solved in a time \( t \) range equal to 12 hours (43200 s). Then, at the right moments in time \( t_i \), using the temperature fields \( T(x, y, z, t_i) \), using equations (2), the fields of displacements, deformations and stresses are found.

The solution of these equations is carried out by passing to the finite element model [27]

\[ [K_T] \{T\} - [A] \{T\} = \{T_f(t)\}, \tag{4} \]

\[ [K_u] \{u\} = [L] \{T(t_i)\}. \]

Here \([K_T]\) – is the matrix of coefficients of the finite element model of the heat equation, \([A]\) – is the matrix of the coefficients of the model at the derivative \( \dot{T} \), \( \{T_f(t)\} \) – is the vector of specified temperature \( T \) values on the coating surface, \([K_u]\) – is the stiffness matrix for the finite element model of an elastic massif, \([L]\) – is the matrix reflecting the effect of temperature on the displacements of the elements of the massif.

After calculating the values of the displacement vector components \( \{u\} \) at the nodes of the finite element model, the components of the strain \( \varepsilon_{jk} \) and stress \( \sigma_{jk} \) tensors are calculated. They are determined using the equalities [25]
\[ \varepsilon_{jk} = \frac{1}{2} (u_{j,k} + u_{k,j}) , \]
\[ \sigma_{jk} = 2\mu \varepsilon_{jk} + [\lambda \varepsilon_{ll} - (3\lambda + 2\mu)\alpha_T \cdot T] \delta_{jk} , \]
discretized at each node of the model.

In these equalities, the indices \( j, k, l \) run through the values 1, 2, 3; the directions \( x_1, x_2, x_3 \) correspond to the directions \( x, y, z \); \( u_{j,k} = \partial u_j / \partial x_k \)
and \( \varepsilon_{ll} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} \); \( \delta_{jk} \) – is the Kronecker symbol equal to 0 at \( j \neq k \)
and equal to 1 at \( j = k \).

In our case, the considered structure has the property associated with the fact that it is in free contact with the air environment. Therefore, for example, at night (in the absence of solar thermal radiation), the temperature of all its elements has time to equalize and instead of the initial value \( T_0 = 0 \) takes the same value \( T_0 = -25^\circ C \).

In contrast to work [1], we believe that the coefficient of thermal linear expansion \( \alpha_T \) of the asphalt concrete layer can take on the values \( \alpha_T = 1.9 \cdot 10^{-5}, 2.4 \cdot 10^{-5}, 3 \cdot 10^{-5} \) (K\(^{-1}\)).

For these cases, Fig. 2 shows the graphs of the functions of shear stresses \( \tau_{xy}(x) \)
on the left section of the contact plane of the coating and the base.

As you can see, these functions have a noticeable concentration at the edge \( x = 0 \), and quickly subside with distance from the edge. In the places where the vertical reinforcing ribs are located, they have small bursts. This kind of these functions contributes to the delamination of the structure in the marginal zone.

Calculations have confirmed the opinion that with an increase in the coefficient \( \alpha_T \) of asphalt concrete, the maximum values of shear stresses

Fig. 2. Diagrams of shear stresses \( \tau_{xy}(x) \) on the interface plane between the asphalt and steel layers for values \( \alpha_T = 1.9 \cdot 10^{-5} \) K\(^{-1}\) (a), \( \alpha_T = 2.4 \cdot 10^{-5} \) K\(^{-1}\) (b) and \( \alpha_T = 3 \cdot 10^{-5} \) K\(^{-1}\) (c)
increase significantly. They also extend over a large contact area. Their values for the considered cases are given in the table of the values of the most characteristic parameters of the system.

### Table 1

<table>
<thead>
<tr>
<th>$\alpha_T$ (K$^{-1}$)</th>
<th>$\tau_{xy}$, max (kPa)</th>
<th>$P(X)$ (N/m)</th>
<th>$\sigma_x$, asph (MPa)</th>
<th>$\sigma_x$, asph (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.9 - 10$^{-5}$</td>
<td>571.2</td>
<td>11871</td>
<td>0.6604</td>
</tr>
<tr>
<td>2</td>
<td>2.4 - 10$^{-5}$</td>
<td>1221.7</td>
<td>25392</td>
<td>1.4127</td>
</tr>
<tr>
<td>3</td>
<td>3 - 10$^{-5}$</td>
<td>1849.1</td>
<td>38431</td>
<td>2.1380</td>
</tr>
</tbody>
</table>

It also gives the values of the integrals

$$P(X) = \int_{0}^{X} \tau_{xy} dx$$

from these stresses in the sections $0 \leq x \leq X$ corresponding to the shaded areas in Fig. 2. As you can see, they have concentration on the left edge and then quickly decrease.

Analyzing the graphs of the forces $\tau_{xy}(x)$ and the values of the integrals $P(X)$, it can be concluded that they are $\tau_{xy}(x)$ concentrated only in the near-edge zone, and then quickly acquire almost zero values. At the same time, the integrals $P(X)$ of the stresses $\tau_{xy}(x)$ are the resultant limiting stresses $\sigma_x(y)$ in both the upper and lower layers, and since the integrals $P(X)$ practically do not change with increasing $X$, the longitudinal forces $P(X)$ also change little.

Calculations also confirmed the position that with an increase $\alpha_T$ for asphalt concrete, stresses $\tau_{xy}(x)$ and forces $P(X)$ rapidly increase.

Fig. 3 shows graphs of changes in the thickness of the structure of forces $\sigma_x(y)$ in its central section for the values of the coefficient $\alpha_T$ of asphalt concrete. The upper parts of the graphs correspond to the asphalt concrete layer, the lower parts correspond to the steel base.

Within each layer, these functions are distributed linearly, which is consistent with the theory of bent plates, and in the upper layer the stresses of a positive sign (it is stretched bent) prevail, in the lower layer of stress of a negative sign (it is compressed bent). The maximum modulus values of stresses are shown in the graphs and are summarized in the table. Note that the resultant of these forces over the entire thickness of the structure is equal to zero, therefore, the condition is valid

$$\int_{(h_{asph})} \sigma_{xx}(y)dy + \int_{(h_u)} \sigma_{xx}(y)dy = 0.$$
Analyzing the values of the power characteristics shown in the graphs and in the table, it can be noted that for the same temperature value in the considered inhomogeneous two-layer structure, the stress fields significantly depend on the difference in the coefficients of thermal linear expansion of its components. Indeed, for example, with an increase $\alpha_T$ in asphalt concrete by about one and a half times, the stresses $\tau_{xy}(x)$ increased by about three times, and the stresses $\sigma_{xx}$ – more than three times.

The general picture of the thermally deformed state of a system of ten sections for the three considered cases is shown in Fig. 4. Qualitatively, these cases are the same and differ only in the deflection arrows $H$, which are equal to the difference between the vertical displacements of the edges of the system and its middle. These values were $H_1 = 0.0011$ m, $H_2 = 0.0029$ m, $H_3 = 0.0068$ m. Naturally, with an increase in the coefficient $\alpha_T$, the value $H$ increases markedly.

Conclusions
1. As shown by numerical studies, under the considered thermal perturbations, the functions of deflections and longitudinal displacements of the system are smooth and have relatively small values, however, the deformation and stress fields caused by them are significantly inhomogeneous and in places of concentration their values are significant.
2. The values of thermal stresses increase significantly with an increase in the difference in the

Fig. 3. Diagrams of function $\sigma_x(y)$ distribution in the vertical central cross-section for cases for cases $\alpha_T = 1.9 \cdot 10^{-5}$ K$^{-1}$ (a), $\alpha_T = 2.4 \cdot 10^{-5}$ K$^{-1}$ (b), and $\alpha_T = 3 \cdot 10^{-5}$ K$^{-1}$ (c)
values of the coefficients of thermal linear expansion of the asphalt concrete pavement and the metal base of the bridge. Moreover, the most noticeable is the concentration of shear stresses in the edge zone of the contact plane of the layers. This effect can be the reason for the phenomenon of the initial and subsequent stratification of the system, which is often observed in practice.

Fig.4. Sectional diagram of the bridge structure in a thermally deformed state

3. In this regard, it is possible to recommend that bridge designers, when choosing construction materials, avoid their combination with large differences in the values of their thermal expansion coefficients.

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Gaidaichuk V.V., Shevchuk L.V., Bilobrytska O.I. 
**INFLUENCE OF INCOMPATIBILITY OF THERMOMECHANICAL PARAMETERS OF BEARING LAYERS OF A BRIDGE STRUCTURE ON ITS THERMO-STRESSED STATE**

On the basis of the theory of thermoelasticity, the problem of the thermally stressed state of a two-layer fragment of a bridge structure, consisting of a metal base and an asphalt-concrete upper layer, under conditions of a change in the ambient temperature at different values of the coefficients of thermal linear expansion of the layers is considered. Using the finite element method, the fields of thermal stresses, deformations and displacements are constructed for various values of the thermomechanical characteristics of the layers. The analysis of the influence of the values of thermomechanical parameters on the stress-strain state of the system is carried out. It is shown that with an increase in the incompatibility of these characteristics, the intensities of stresses and strains increase. The zones of concentration of these functions are found. It is recommended to use materials with close values of their thermomechanical parameters when designing bridges to avoid their premature destruction.

**Key words:** two-layer bridge structure, incompatibility of thermomechanical parameters, thermal stress fields, finite element analysis.

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**ВЛИЯНИЕ НЕСОВМЕСТИМОСТИ ТЕРМОМЕХАНИЧЕСКИХ ПАРАМЕТРОВ НЕСУЩИХ СЛОЕВ МОСТОВОЙ КОНСТРУКЦИИ НА ЕЁ ТЕРМОНАПРЯЖЕННОЕ СОСТОЯНИЕ**

На базе теории термоупругости рассмотрена задача о термонапряженном состоянии двухслойного фрагмента мостовой конструкции, состоящей из металлической основы и асфальтобетонного верхнего слоя, в условиях изменения температуры окружающей среды при различных значениях коэффициентов температурного линейного расширения слоев. С помощью метода конечных элементов построены поля температурных напряжений, деформаций и перемещений при различных значениях термомеханических характеристик слоев. Выполнен анализ влияния значений термомеханических параметров на напряженно-деформированное состояние системы. Показано, что с увеличением несовместимости этих характеристик интенсивности напряжений и деформаций возрастают. Найдены зоны концентрации этих функций. Рекомендовано при проектировании мостов для избегания их преждевременной деструкции использовать материалы с близкими значениями их термомеханических параметров.

**Ключевые слова:** двухслойная мостовая конструкция, несовместимость термомеханических параметров, поля термонапряжений, конечноэлементный анализ.

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В статті наведені результати комп'ютерного аналізу напружено-деформованого стану багатошарового асфальтобетонного дорожнього покриття під дією транспортних навантажень.

Таб. 1. Рис. 4. Бібліогр. 27 назв.

UDC 539.3


The article presents the results of a computer analysis of the stress-strain state of a multilayer asphalt pavement under the influence of traffic loads.

Tab. 1. Fig. 4. Ref. 27.
В статті приведені результати комп'ютерного аналізу напружено-деформованого стання многоповерхового асфальтобетонного дорожнього покриття під дією транспортних навантажень.

Таб. 1. Рис. 4. Бібліогр. 27 назв.

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