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TRANSIENT RESPONSES IN PIEZOCERAMIC MULTILAYER ACTUATORS TAKING INTO ACCOUNT EXTERNAL VISCOELASTIC LAYERS

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The work develops a generalized approach to the study of thickness (radial) vibrations arising in the piezoceramic plates, cylinders, spheres under electrical loads. The state of the problem and the main approaches, used in the problems of studying the oscillations of electroelastic bodies, are described. The use of multilayer elements with electroded interface surfaces and variable direction of polarization of the layers increases the conversion efficiency of electrical energy into mechanical energy, so multilayer piezoceramic plates, cylinders, spheres with changing polarization directions with electroded interfaces are considered. Because of piezoelectric elements are often embedded in the housing and supplemented with matching layers to protect against mechanical damage, it is necessary to study their effect on the oscillations of the element. The proposed approach makes it possible to study the vibrations of plane, cylindrical and spherical bodies with layers made of various electroelastic and elastic materials. Numerical implementation is carried out using finite differences.

Nonstationary oscillations of PZT-4 ceramic elements at zero initial conditions are investigated. Oscillations of multilayer plates, cylinders and spheres with and without an external elastic or viscoelastic reinforcing layer under impulse and harmonic unsteady loads are investigated and compared. There are found own frequencies for 5-layer bodies of different geometry with and without an external layer. The first natural frequency for cylinder and sphere corresponds to the radial mode of oscillations, while the second natural frequency for cylinders and spheres and the first for flat bodies are almost equal and correspond to thickness mode. The transient processes in the elements under impulse loads and the influence of the outer elastic layer (housing or matching layer) are studied, taking into account the Rayleigh attenuation. It is established that for a flat layer the outer layer increases the amplitude and the period of free vibrations after removing the load, and for cylinders and spheres it decreases. The presence of an elastic layer enhances the third and dampens the fourth natural frequency of the transducer, thereby expanding the frequency range of its operation.

Key words: piezoceramic multilayer transducers; non-stationary oscillations; electrical disturbance; piezoelectric actuators; elastic, viscoelastic layers; thickness, radial vibrations.

Piezoelectric transducers are used for exciting (actuators) and receiving (sensors) acoustic (low-frequency) or ultrasonic waves in the environment [1]. They are used in hydroacoustic exploration, medical devices, in the construction and engineering industry in non-destructive testing devices: flaw detectors, concrete scopes, thickness gauges. As their active parts are used piezoelectric elements of plane, cylindrical and spherical shape, usually made of polarized piezoceramic materials [2]. To increase the efficiency of the piezoelectric transducers, varying the frequency range and the type of perturbed waves, several interconnected piezoelectric elements can be used, possibly with different types of polarization. To match the impedances of the piezoceramic and the controlled object, transition matching layers made of elastic materials are sometimes added. If vibration damping is required, viscoelastic layers are used. To ensure the integrity and the necessary conditions for fixing the working elements of the piezoelectric transducers can be embedded in the housing. Thus, piezoelectric transducers can be a complex object made of materials with different physical and mechanical properties: piezoceramics with different directions of polarization, elastic, viscoelastic, conductive materials, etc.

Piezoceramic spheres, cylinders and flat bodies are often used in acoustoelectric devices to receive and emit an acoustic signal, including non-stationary [3] (Fig. 1). To ensure a more effective acoustic response to electrical perturbation compared with homogeneous bodies, multilayer electro-mechanical transducers are used.

Fig. 1. Piezoceramic actuators of cylindrical, spherical and flat shape

Questions about the oscillations of electromechanical transducers are relatively often raised in the literature due to their widespread use. The principles of operation and scope of piezoceramic transducers are described in [1, 2, 4, 5 etc.]. In [6, 7] the problems of oscillations of homogeneous and non-homogeneous cylinders were solved. Stationary and nonstationary oscillations of piezoceramic bodies with curved surfaces were studied in [8].

The oscillation problems of multilayer transducers are found in single works. The harmonic axisymmetric oscillations of multilayer piezoceramic cylinders without internal electrodes were studied in [9]. Nonstationary oscillations of the piezoceramic sphere without internal electrodes too were considered in [10]. Nonstationary oscillations in cylinders with piezoceramic radially polarized layers were considered in [11]. In [12] plane multilayer element with elastic reinforcing layer taking into account the impact of ideal acoustic medium was investigated.

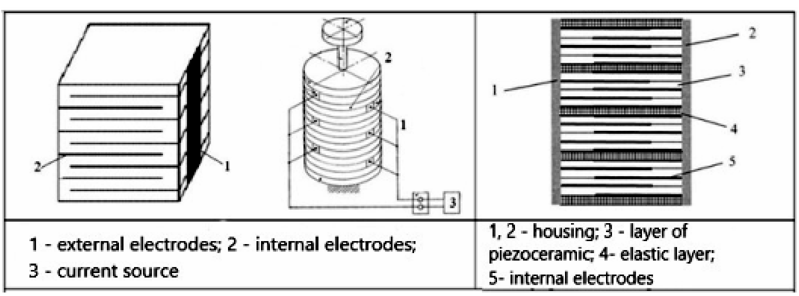


Fig. 2. The structure of the multilayer converter

In resonant modes of operation, energy dissipation in piezoceramic elements is taken into account by introducing complex material constants [1, 5]. In [13] this approach extends to nonstationary modes of operation by excluding the time variable due to the Laplace transform.

An analysis of the literature shows that the study of propagation of perturbations in electroelastic multilayer bodies is indeed an urgent and important task. It arises in the process of designing devices that operate on the basis of the piezoelectric effect. Therefore, it is necessary to study the propagation of electromechanical perturbations in multilayer piezoceramic bodies of different geometries. To model such devices, it is sufficient to use approaches based on direct numerical methods. For studying of the thickness and radial oscillations, that provide the operating frequency of the piezoelectric element, it is convenient to use the finite-difference method. The paper implements a universal approach to the oscillations description of bodies of flat, cylindrical and spherical shape, which follows from the general equations of electroelastic axisymmetric oscillations using the parameter *N*.

1. Statement of the problem

There are investigated thickness (radial) oscillations of multilayer hollow plates, cylinders and spheres, consisting of n piezoceramic polarized by thickness layers with interface surfaces $R_0 <\ldots< R_k <\ldots< R_n$, where R_n - external, R_0 – internal body surface.

The general equation of motion and the equation for the electrical induction of the k-th layer due to displacement and the electric potential for bodies of different geometries has the form

$$c_{33}{}^{k}\frac{\partial^2 u^k}{\partial r^2}+\frac{N}{r}c_{33}{}^{k}\frac{\partial u^k}{\partial r}+\frac{N^2}{r^2}[c_{13}{}^{k}(1-\frac{1}{N})-c_{11}{}^{k}+\frac{1}{2}(N-1)(c_{11}{}^{k}-$$
$$-c_{12}{}^{k})]u^k+(e_{33}{}^{k}-e_{31}{}^{k})\frac{N}{r}\frac{\partial\varphi^k}{\partial r}+e_{33}{}^{k}\frac{\partial^2\varphi^k}{\partial r^2}=\rho\frac{\partial^2 u^k}{\partial t^2}; \tag{1}$$

$$e_{33}{}^{k}\frac{\partial^2 u^k}{\partial r^2}+\frac{N}{r}(e_{31}{}^{k}+e_{33}{}^{k})\frac{\partial u^k}{\partial r}+N(N-1)e_{31}{}^{k}\frac{u^k}{r^2}-$$
$$-\varepsilon_{33}{}^{k}\frac{N}{r}\frac{\partial\varphi^k}{\partial r}-\varepsilon_{33}{}^{k}\frac{\partial^2\varphi^k}{\partial r^2}=0. \tag{2}$$

Material relations in layers with radial direction of polarization

$$\sigma^k{}_r=c_{33}{}^{k}\frac{\partial u^k}{\partial r}+Nc_{13}{}^{k}\frac{u^k}{r}+(-1)^{k+1}e_{33}{}^{k}\frac{\partial\varphi^k}{\partial r};$$

$$\sigma^k{}_\theta=c_{13}{}^{k}\frac{\partial u^k}{\partial r}+N[c_{11}-\frac{1}{2}(N-1)(c_{11}-c_{23})]\frac{u^k}{r}+(-1)^{k+1}e_{31}{}^{k}\frac{\partial\varphi^k}{\partial r};$$

$$D^k{}_r=(-1)^{k+1}(e_{33}{}^{k}\frac{\partial u^k}{\partial r}+Ne_{31}{}^{k}\frac{u^k}{r})-\varepsilon_{33}{}^{k}\frac{\partial\varphi^k}{\partial r},\ k=1\ldots n. \tag{3}$$

In (1)-(3) $u=u_r$ – radial (thickness) displacements, σ_r,σ_θ – radial and circumferential stresses, D_r – electrical induction, φ – electric potential,

$c_{ij}^{k} = c_{ij}^{E}$ – modules of elasticity of the material at a constant electric field, e_{ij} – piezoelectric modules, $\varepsilon_{33}^{k} = \varepsilon_{33}^{S}$ – dielectric constant at constant deformation, ρ – material density.

In the case of opposite to radial direction of polarization, the sign of the piezoelectric modules changes to the opposite. Previously, in [11] it was established that for maximum amplitude piezoelectric element must have an odd number of layers and layers must be opposite polarized. Going from a single-layer to a multi-layer element, we increase its sensitivity almost in proportion to the number of layers, but in addition we need to control the mechanical and electrical strength of the element [2]. Energy dissipation in piezoelectric transducers is not taken into account. In the future, in non-stationary modes of operation, it can be considered as attenuation by Rayleigh or by the method proposed in [13].

For elastic isotropic layers we have the equation of oscillations

$$\rho \frac{\partial^2 u^k}{\partial t^2} = (\lambda + 2\eta)\left(\frac{d^2 u^k}{dr^2} + \frac{N}{r}\frac{du^k}{dr} - \frac{N}{r^2}u^k \right), \tag{4}$$

where $\lambda = \dfrac{\mu E}{(1+\mu)(1-2\mu)}$, $\eta = \dfrac{E}{2(1+\mu)}$ – Lame parameters. For taking into account the viscoelastic properties of the elastic layer, we introduce oscillation propagation velocity $v_\kappa = \sqrt{\dfrac{\rho}{\lambda + 2\eta}}$ and the Rayleigh attenuation parameter b_k :

$$v_\kappa^2 \frac{\partial^2 u^k}{\partial t^2} + b_k \frac{\partial u^k}{\partial t} = \frac{d^2 u^k}{dr^2} + \frac{N}{r}\frac{du^k}{dr} - \frac{N}{r^2}u^k. \tag{5}$$

The parameter b_k depends on the frequency of perturbation and modal damping of the corresponding forms of natural oscillations, given as a percentage of the critical damping [14].

The initial conditions are superimposed on the displacements and their velocities

$$u(r,t=0) = \overset{0}{f}(r), \quad \frac{\partial u_r}{\partial t}(r,t=0) = \overset{1}{f}(r). \tag{4}$$

Stresses or displacements are set on the outer surfaces

$$u(R_a,t) = U_a(t) \quad \wedge \quad \sigma_r(R_a,t) = p_a(t), \quad a = 0,n. \tag{5}$$

The conditions of full contact between the layers $k = 1...n-1$ are fulfilled

$$u^k(R_k) = u^{k+1}(R_k), \quad \sigma_r^k(R_k) = \sigma_r^{k+1}(R_k), \quad \varphi^k(R_k) = \varphi^{k+1}(R_k). \tag{6}$$

The interface surfaces and the outer surfaces are electroded, the potential difference is set on the electrodes

$$\varphi(R_k) = (-1)^{k+1} V(t), \qquad k = 0..n. \tag{7}$$

To make the solution general, we enter dimensionless variables [8, 11, 12]. Dimensionless form of the original equations does not change them.

2. Numerical solution method

We introduce a partition by spatial coordinate:

$\Omega=\{r_{m(k-1)+i}=R_{k-1}+(i-1)\Delta_k,\ \Delta_k=(R_k-R_{k-1})/m,\ k=1,...,n,\ i=1,...,m+1\}$.

In the transition to the central differences, equations (1), (2) are transformed into a system of equations $(a_k=c_{13}^k(1-\frac{1}{N})-c_{11}^k+\frac{1}{2}(N-1)(c_{11}^k-c_{12}^k))$:

$$\rho^k\frac{d^2u_{m(k-1)+i}}{dt^2}=(\frac{c_{33}^k}{\Delta r^2}-\frac{Nc_{33}^k}{2\Delta_k r_{m(k-1)+i}})u_{m(k-1)+i-1}+(\frac{N^2a_k}{r_{m(k-1)+i}^2}-2\frac{c_{33}^k}{\Delta r^2})u_{m(k-1)+i}+$$
$$+(\frac{c_{33}^k}{\Delta r^2}+\frac{Nc_{33}^k}{2\Delta r r_{m(k-1)+i}})u_{m(k-1)+i+1}+(\frac{e_{33}^k}{\Delta r^2}+\frac{N(e_{33}^k-e_{31}^k)}{2\Delta_k r_{m(k-1)+i}})\varphi_{m(k-1)+i+1}- \quad (9)$$
$$-2\frac{e_{33}^k}{\Delta r^2}\varphi_{m(k-1)+i}+(\frac{e_{33}^k}{\Delta r^2}-\frac{N(e_{33}^k-e_{31}^k)}{2\Delta r r_{m(k-1)+i}})\varphi_{m(k-1)+i-1};$$

$$(e_{33}^k-\frac{N(e_{31}^k+e_{33}^k)\Delta r}{2r_{m(k-1)+i}})u_{m(k-1)+i-1}+e_{33}^k(\frac{N(N-1)\Delta_k^2}{r_{m(k-1)+i}^2}-2)u_{m(k-1)+i}+$$
$$+(e_{33}^k+\frac{N(e_{31}^k+e_{33}^k)\Delta r}{2r_{m(k-1)+i}})u_{m(k-1)+i+1}-(-\frac{N\varepsilon_{33}^k\Delta r}{2r_{m(k-1)+i}}+\varepsilon_{33}^k)\varphi_{m(k-1)+i-1}+ \quad (10)$$
$$+2\varepsilon_{33}^k\varphi_{m(k-1)+i}-(\frac{N\varepsilon_{33}^k\Delta r}{2r_{m(k-1)+i}}+\varepsilon_{33}^k)\varphi_{m(k-1)+i+1}=0,$$

which in matrix form is written as

$$\rho\frac{d^2U}{dt^2}=AU+B\Phi+F_1,\quad D\Phi=CU+F_2, \quad (11)$$

where $U=\{u_{m(k-1)+i-1}\}$, $\Phi=\{\varphi_{m(k-1)+i-1}\}$, $k=1,\dots,n$, $i=1,\dots,m$.

Displacements on external surfaces

$$u_1=(p_0(t)-\frac{c_{33}^1}{2\Delta_1}(4u_2-u_3)-\frac{e_{33}^1}{2\Delta_1}(-3\varphi_1+4\varphi_2-\varphi_3))/(\frac{Nc_{13}^1}{R_0}-3\frac{c_{33}^1}{2\Delta_1});$$
$$u_{nm+1}=(p_n(t)+\frac{c_{33}^n}{2\Delta_n}(4u_{nm}-u_{nm-1})-$$
$$-\frac{e_{33}^n}{2\Delta_n}(3\varphi_{nm+1}-4\varphi_{nm}+\varphi_{nm-1}))/(3\frac{c_{33}^n}{2\Delta_n}+\frac{Nc_{13}^n}{R_n}). \quad (12)$$

Displacements on the section surfaces are found from the conjugation conditions (6)

$$u_{mk+1}=(\frac{c_{33}^{k+1}}{2\Delta_{k+1}}(4u_{mk+2}-u_{mk+3})+\frac{e_{33}^{k+1}}{2\Delta_{k+1}}(-3\varphi_{mk+1}+4\varphi_{mk+2}-\varphi_{mk+3})-$$

$$+\frac{c_{33}^k}{2\Delta_k}(4u_{mk}-u_{mk-1})+\frac{e_{33}^k}{2\Delta_k}(-3\varphi_{mk+1}+4\varphi_{mk}-\varphi_{mk-1}))/c_\kappa, \qquad (13)$$

where $c_\kappa = 3\frac{c_{33}^k}{2\Delta_k}+\frac{Nc_{13}^k}{R_k}+3\frac{c_{33}^{k+1}}{2\Delta_{k+1}}-\frac{Nc_{13}^{k+1}}{R_k}$.

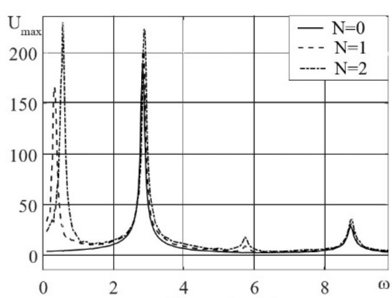
An explicit and implicit difference scheme is used for time integration. When using an implicit difference scheme (Newmark's algorithm))

$$\ddot{u}^{p+1}=\frac{u^{p+1}-u^p}{\xi^2\Delta t^2}-\frac{1}{\xi^2\Delta t}\dot{u}^p-\frac{1-\xi}{\xi}\ddot{u}^p, \qquad (14)$$

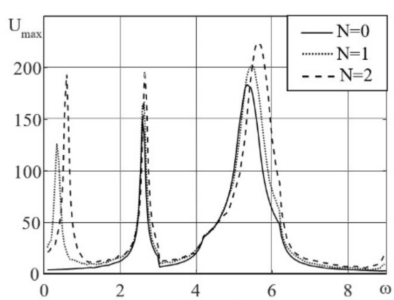
where ξ – the parameter of the scheme, equations (11) (12) (13) form a single matrix, from which all unknowns in the internal points and contour points are determined simultaneously. For an explicit scheme on each time layer, the displacements at the internal points are first determined, and then by (12) (13) we find the unknowns on the contour and the boundaries of the section. An important issue in the application of an explicit scheme is the choice of the step of partitioning in spatial and temporal coordinates, as the condition of stability of the explicit scheme must be fulfilled. Practice shows that it is usually enough to take $\Delta_x \geq 10\Delta t$. For an implicit scheme, steps of the same order are taken, ξ =0,5.

3. Obtained results

Nonstationary oscillations of PZT-4 [5, 8, 11, 12] ceramic elements at zero initial conditions are investigated. External radiuses are $R_0=2\,\text{cm}$, $R_n=3\,\text{cm}$. Consider the case n = 5, i.e. piezoelectric elements consist of five layers of oppositely polarized ceramic PZT-4. The thickness of the layers is the same $h_i=(R_n-R_0)/n=2\text{mm}$, where n - number of layers. We compare the oscillations of the elements without elastic layers and taking into account the external elastic layer with a thickness of h = 2 mm. For elastic material we take $E=1{,}1\cdot 10^{11}\,\text{MPa}$, $\mu = 0.25$ (aluminum). The results are given in dimensionless form.



(a)



(b)

Fig. 3. The value of the maximum displacements in bodies of different geometries depending on the frequency of perturbation: (a) without elastic layer; (b) with elastic layer

An important issue in the design and operation of a piezoelectric element is the determination of its natural (resonant) frequencies. On Fig. 3 the maximum values of displacements for piezoelectric elements without an outer and with an outer elastic layer under non-stationary harmonic loading are compared. The maximum values of displacements occur during loading, the frequency of which coincides with or is close to its own. In cylinders and spheres, the first natural frequency corresponds to radial oscillations of the middle surface, the second frequency is the frequency of thickness oscillations and differs little for bodies of different geometries. The presence of an elastic layer increases the first frequency and lowers the second frequency, as well as enhances the third. Due to this property, such piezoelectric elements become multimode and belong to broadband, i.e. the range of operating frequencies for the element is significantly expanded.

One of the basic operation modes of such elements is a pulse load, which allows you to send a signal to the test medium and receive a response after the attenuation of oscillations in the element. The electric perturbation is given in the form of a half-wave sinusoid

$$V(t)=\begin{cases}V_0\sin\omega t, & 0<t<\pi/\omega;\\ 0, & 0<t<\pi/\omega.\end{cases}$$

We take ω=1, $V_0=2kV$. At such initial data the unit of dimensionless time approximately corresponds to 1 mcs, the unit of dimensionless movement corresponds to 55 mcm. Determining the attenuation parameter b_k is a difficult practical task, as it depends not only on the material, but also on the shape of the structure and the type of perturbation. In practice, the formula $b_k=\dfrac{2\pi\omega_a\omega_b}{(\omega_a+\omega_b)Q_k}$ is used [14], where (ω_a,ω_b) - the frequency range under consideration, Q_k - quality factor (the ratio of the resonant frequency to the width of the resonant curve at 1/2), which is considered constant in the range (ω_a,ω_b). Quality factor is one of the most important characteristics of the oscillator and is determined experimentally.

In connection with the above, we consider cases with arbitrary values b_k, which allows us to establish the dependence of the damping time of oscillations in the element on the value b_k. Note that for the pulse mode rapid attenuation of perturbations is a useful property, so for elastic matching layers materials with pronounced viscoelastic characteristics are selected.

Consider a flat piezoelectric element 1cm thick with free outer surfaces. With such a geometry, oscillations occur only due to the moving of the wave along the thickness of the element. Analyzing the results presented in Fig. 4, we see that the presence of an elastic layer increases the amplitude of natural oscillations, that occur after removal of the external load almost twice (for other loads, the growth is less). The period of oscillations increases from 2.2 mks during the travel of the elastic wave along the thickness of the elastic layer to 2.4 mks, which corresponds to the first natural frequency for the flat element ω = 2.62 (Fig. 3b). Due to the damping properties of the elastic layer, the amplitude of oscillations decreases according to the change of b_k. At 300 mks (*t*=300) using $b_k = 0.01$ oscillations completely disappear, at $b_k = 0.05$ - decreased by 96%, at $b_k = 0.1$ - decreased by 82% relatively to the maximum absolute amplitude of 77 mkm. It should be noted that the oscillation amplitude of a single-layer element is 16-20% of the amplitude of oscillations of a five-layer element, i.e. the conversion of electrical energy into mechanical energy in multilayer elements is much more efficient.

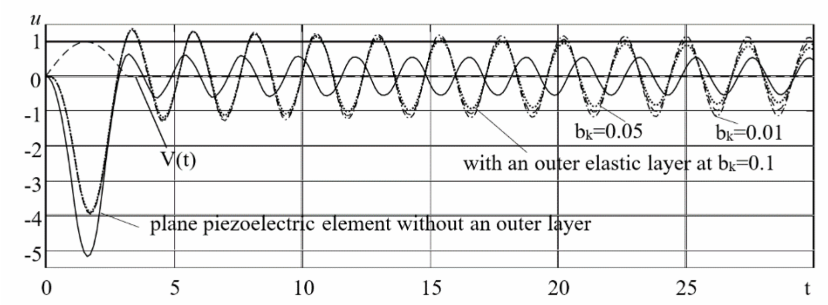


Fig. 4. Oscillations of the outer surface of a plane piezoelectric element ($N = 0$) without an outer layer and with an outer elastic layer with different damping coefficients

The oscillations of the cylindrical element are shown on Fig. 5. The presence of an elastic layer reduces the maximum amplitude and frequency of oscillations, as the stiffness of the element has increased. The oscillation period changed from 19 to 17 mks. The oscillations are a combination of thickness and radial oscillations, as a result of which the amplitude has increased 8 times compared to a flat element. We see that for $b_k = 0.01$ at 300 mks the amplitude decreased by 30%, at $b_k = 0.05$ - by 73%, at $b_k = 0.1$ - by 92%. The maximum amplitude of oscillations was 0.55 mm. As the ratio of radius to cylinder thickness increases, the amplitude and period of oscillations also increase.

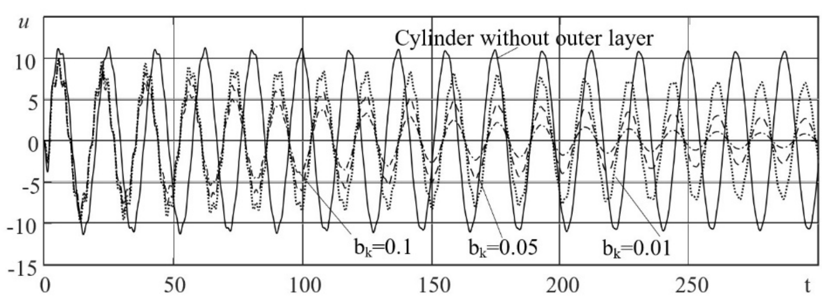


Fig. 5. Oscillations of the outer surface of a cylindrical piezoelectric element ($N = 1$) without an outer layer and with an outer elastic layer at different damping coefficients

Consider the oscillations of a multilayer piezoceramic sphere (Fig. 6). The period of oscillations, taking into account the elastic layer, changed from 11.1 to 10.5mks. The maximum amplitude of oscillations was 0.5mm. For 300mks at $b_k = 0.01$ amplitude decreased by 22%, at $b_k = 0.05$ - by 59%, at $b_k = 0.1$ - by 89%.

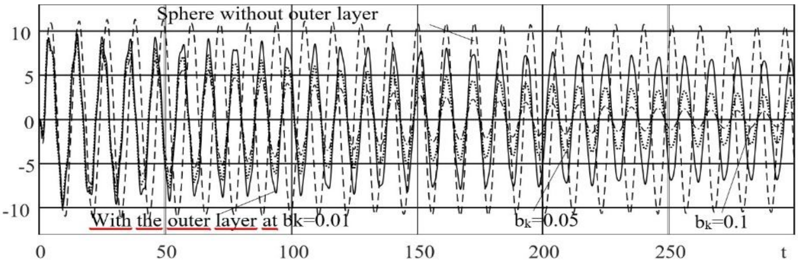


Fig. 6. Oscillations of the outer surface of a spherical piezoelectric element ($N = 2$) without an outer layer and with an outer elastic layer with different damping coefficients

Conclusion

Because of piezoelectric elements are often embedded in the housing and supplemented with matching layers to protect against mechanical damage, it is necessary to study their effect on the oscillations of the element. The proposed approach to the study of nonstationary oscillations of piezoceramic electromechanical converters (actuators) is easily implemented in any computer program.

The first natural frequency for cylinders and spheres corresponds to the radial mode of oscillations, while the second natural frequency for cylinders and spheres coincides with the first for flat bodies and corresponds to the thickness mode. The transient processes in the elements under impulse loads and the influence of the outer elastic layer (housing or matching layer) are studied, taking into account the Rayleigh attenuation. It is found that the presence of an elastic layer in a flat element increases the amplitude and period of natural oscillations of the piezoelectric element, and in cylindrical and spherical decreases them. An important property of the elastic layer is that its presence amplifies the third and dampens the fourth natural frequency. It should be noted that the obvious advantage in the amplitude of oscillations for the sphere and the cylinder is lost at a distance from the element, because the intensity of the emitted signal decreases inversely proportional to the distance. But due to the comprehensive direction of the signal, the probability of finding defects or other obstacles increases significantly.

Thus, the influence of the outer elastic layer with viscoelastic properties on the oscillations of the multilayer element is significant and should not be neglected. To reduce the impact, we need to reduce the thickness of the housing compared to the thickness of the element. The use of multilayer elements with electroded interface surfaces and variable direction of polarization of the layers increases the conversion efficiency of electrical energy into mechanical energy almost in proportion to the number of layers.

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ПЕРЕХІДНІ ПРОЦЕСИ В П'ЄЗОКЕРАМІЧНИХ БАГАТОШАРОВИХ АКТУАТОРАХ З ВРАХУВАННЯМ ЗОВНІШНЬОГО В'ЯЗКОПРУЖНОГО ШАРУ

В роботі розвивається узагальнений підхід до дослідження товщинних (радіальних) збурень, що виникають в п'єзокерамічних пластинах, циліндрах, сферах при електричних навантаженнях. Розглядаються багатошарові перетворювачі змінного напрямку поляризації з електродованими поверхнями розділу. Запропонований підхід дозволяє досліджувати коливання тіл з шарами, виконаними з різних електропружних та пружних матеріалів. Чисельна реалізація виконана за допомогою скінченних різниць. Описано коливання багатошарових пластин, циліндрів та сфер з зовнішнім пружним або в'язкопружним підкріплюючим шаром та без нього при імпульсних та гармонічних нестаціонарних навантаженнях. Встановлено, що для плоского шару зовнішній шар підвищує амплітуду та період вільних коливань після зняття навантаження, а для циліндрів та куль - понижує. Наявність пружного шару підсилює третю та гасить четверту власну частоту перетворювача, чим розширює частотний діапазон його роботи.

Ключові слова: п'єзокерамічні багатошарові перетворювачі; нестаціонарні коливання; електричне збурення; п'єзоелектричні актуатори; пружні, в'язкопружні шари; товщинні, радіальні моди коливань.

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ПЕРЕХОДНЫЕ ПРОЦЕССЫ В ПЬЕЗОКЕРАМИЧЕСКИХ МНОГОСЛОЙНЫХ АКТУАТОРАХ С УЧЕТОМ ВНЕШНЕГО ВЯЗКОУПРУГОГО СЛОЯ

В работе развивается обобщенный подход к исследованию толщинных (радиальных) возмущений, возникающих в пьезокерамических пластинах, цилиндрах, сферах при электрических нагрузках. Рассматриваются многослойные преобразователи переменного направления поляризации с электродированными поверхностями раздела. Предложенный подход позволяет исследовать колебания тел со слоями, выполненными из различных электроупругих и упругих материалов. Численная реализация выполнена с помощью конечных разностей. Описаны колебания многослойных пластин, цилиндров и сфер с внешним упругим или вязкоупругим подкрепляющим слоем и без него при импульсных и гармонических нестационарных нагрузках. Установлено, что для плоского слоя внешний слой повышает амплитуду и период свободных колебаний после снятия нагрузки, а для цилиндров и сфер - понижает. Наличие упругого слоя усиливает третью и гасит четвертую собственную частоту преобразователя, чем расширяет частотный диапазон его работы.

Ключевые слова: пьезокерамические многослойные преобразователи; нестационарные колебания; электрическое возмущение; пьезоэлектрические актуаторы; упругие, вязкоупругие слои; толщинные, радиальные колебания.

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Григор'єва Л.О. **Перехідні процеси в п'єзокерамічних багатошарових актуаторах з врахуванням зовнішнього в'язкопружного узгоджувального шару** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2020. – Вип.105. – С. 255-266. – Англ.

Досліджуються коливання багатошарових п'єзокерамічних плоских, циліндричних та сферичних тіл з врахуванням зовнішнього в'язкопружного шару.
Іл. 6. Бібліогр. 14 назв.

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Grigoryeva L.O. **Transient responses in Piezoceramic Multilayer Actuators Taking into Account External Viscoelastic Layer** // Strength of Materials and Theory of Structures: Scientific-and-technical collected articles. – K.: KNUBA, 2020. – Issue 105. – P. 255-266.

The oscillations of multilayer piezoceramic flat, cylindrical and spherical bodies taking into account the outer viscoelastic layer are investigated.
Fig. 6. Ref. 14.

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Григорьева Л.О. **Переходные процессы в пьезокерамических многослойных актуаторах с учетом внешнего вязкоупругого слоя** // Опір матеріалів і теорія споруд: наук.-тех. збірн. – К.: КНУБА, 2019. – Вип. 103. – С. 255-266. – Англ.

Рассматривается влияние нагрева на параметры собственных колебаний параболических оболочек вращения.
Табл. 6. Іл. 10. Библиогр. 20 назв.

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